Hierarchical Decision Making with Supply Chain Applications

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Abstract

Hierarchical Decision Making with Supply Chain Applications

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Hierarchical decision making is a decision system, where multiple decision makers are involved and the process has a structure on the order of levels. It gains interest not only from a theoretical point of view but also from real practice. Its wide applications in supply chain management are the main focus of this dissertation.

The first part of the work discusses an application of continuous bilevel programming in a remanufacturing system. Under intense competitive pressures to lower production costs, coupled with increasing environmental concerns, used products can often be collected via customer returns to retailers in supply chains and remanufactured by producers, in order to bring them back into "as-new" condition for resale. In this part, hierarchical models are developed to determine optimal decisions involving inventory replenishment, retail pricing and collection price for returns. Based on the simplified assumption of a single manufacturer and a single retailer dealing with a single recoverable item under deterministic conditions, all of these decisions are examined in an integrated manner. Models depicting decentralized, as well as centralized policies are explored. Analytical results are derived and detailed sensitivity analysis is performed via an extensive set of numerical computations.

In the second part of this dissertation, a discrete bilevel problem is illustrated by investigating a biofuel production problem. The issues of governmental incentives, industry decisions of price, and farm management of land are incorporated. While fixed costs are natural components of decision making in operations management, such discrete phenomena have not received sufficient research attention in the current literature on bilevel programming, due to a variety of theoretical and algorithmic difficulties. When such costs are taken

into account, it is not easy to derive optimality conditions and explore convergence properties due to discontinuities and the combinatorial nature of this problem, which is NP-hard. In order to solve this problem, a derivative-free search technique is used to arrive at a solution to this bilevel problem. A new heuristic methodology is developed, which integrates sensitivity analysis and warm-starts to improve the efficiency of the algorithm.

0.1 List of Symbols Used

Notation used for literature review		
x	=	vector of decision variables controlled in an upper level problem(ULP)
y	=	vector of decision variables controlled in a lower level problem(LLP)
s	=	vector of slack variables controlled in a LLP
λ	=	vector of Lagrange multipliers in a LLP
z	=	vector of decision variables in a bilevel problem, $z = (x, y, s, \lambda)$
X	=	linear feasible set for decision variables controlled in an ULP
Y	=	linear feasible set for decision variables controlled in a LLP
F	=	objective function for an ULP
G	=	constraint functions for an ULP
f	=	objective function for a LLP
g	=	constraint functions for a LLP
C_x	=	objective coefficients for decision variables x in a linear ULP
C_y	=	objective coefficients for decision variables y in a linear ULP
c_x	=	objective coefficients for decision variables x in a linear LLP
c_y	=	objective coefficients for decision variables y in a linear LLP
B	=	right hand side of constraints in a linear ULP
b	=	right hand side of constraints in a linear LLP
A_x	=	constraint matrix for decision variables x in a linear ULP
A_y	=	constraint matrix for decision variables y in a linear ULP
a_x	=	constraint matrix for decision variables x in a linear LLP
a_y	=	constraint matrix for decision variables y in a linear LLP
d	=	directional derivative
d_x	=	directional derivative for x
d_y	=	directional derivative for y
P	=	coefficient matrix in a quadratic upper level problem
Q	=	coefficient matrix in a quadratic lower level problem
R	=	set of real numbers
Z	=	set of integers
$L(x, y, \lambda)$	=	Lagrangian function

k	=	index of iterations
r	=	radius of neighborhood
ϵ	=	tolerance
$\vartheta_o(x)$	=	best-case objective value of the ULP for a fixed x (optimistic)
$\vartheta_p(x)$	=	worst-case objective value of the ULP for a fixed x (optimistic)
$\Phi(x)$	=	penalty function of the objective value in the upper level problem
κ	=	maximum value of the upper level variable
$\Gamma(x)$	=	feasible region of the lower level variable given x
$\Re(x)$	=	reaction set for the lower level variable given x
ϵ^k	=	step length at the kth iteration
$\mathrm{V}(x, r)$	=	the neighborhood of x with a radius of r
M	=	a large positive constant
k	=	iteration index
INT	=	interior
BD	=	boundary
$\bar{F}$	=	upper bound for the upper level objective function value
$\underline{F}$	=	lower bound for the upper level objective function value
F'	=	the first derivative for the upper level objective function
f'	=	the first derivative for the lower level objective function
$\underline{f}$	=	lower bound for the lower level objective function value
$\mathcal{B}$	=	index set for basic variables
$\mathcal{N}$	=	index set for nonbasic variables
		Notation used for biofuel production model
V	=	set of biofuels
E	=	set of farms
C	=	set of food crops
D	=	set of nonfood crops
$D(v)$	=	subset of nonfood crops used to make biofuel v
D'	=	subset of nonfood crops with subsidies
τ_v	=	government tax credit given to industry for biofuel v (\$/ton)
p_{de}	=	price paid by industry to farm e for the land assigned to produce nonfood crop d(\$/ton)

x_{ce}	=	area allocated to food crop c by farm e (acres)
xn_{de}	=	area allocated to nonfood crop d by farm e (acres)
xf_e	=	area set aside as fallow by farm e (acres)
q_{de}	=	binary variable to decide whether farm e will plant nonfood crop
α_{dve}	=	conversion rate from the land assigned to nonfood crop d in farm e to biofuel v (ton/acre)
γ	=	set-aside payment for fallow land (\$/acre)
ρ	=	minimum percentage of set-aside land expected by the government for nonfood crops
θ	=	percentage of arable land required to be set aside for nonfood crops or fallow
σ_e	=	total arable land available on farm e (acres)
u_v	=	upper bounds on production of biofuel v from certain nonfood crops (ton)
π_{ve}	=	threshold for the industry price (\$/ton)
m_{ce}	=	gross margin for food crop c grown on farm e (\$/acre)
c_{de}	=	net production cost for nonfood crop d on farm e (\$/acre)
χ_{de}	=	upper bound on percentage of land permitted for nonfood crop d on farm e
δ	=	maximum percentage of land expected by government for fallow
t_{de}	=	fixed cost for farm e for planting nonfood crop d (\$)

Notation used for remanufacturing model(retailer)

d	=	demand rate of the product (units/time unit)
S_r	=	fixed ordering cost for retail stock replenishment(\$/order)
h_r	=	inventory holding cost of the product (\$/unit/time unit)
h_{rr}	=	inventory holding cost of the used (returned) product (\$/unit/time unit)
r_c	=	unit reimbursement to customers for returns (\$/unit)
p_s	=	unit selling price of the product (new or remanufactured) (\$/unit)
x	=	rate at which customers return the used item to the retailer (units/time unit)
X	=	total quantity of returns in a replenishment cycle (units)

Notation used for remanufacturing model(manufacturer)		
m	=	manufacturing or remanufacturing rate of the product(unit/time unit)
S_m	=	fixed manufacturing/remanufacturing setup cost per replenishment lot ($/setup)
S_{rm}	=	total fixed cost of shipping a replenishment lot of new products to the retailer and transporting the returned items back to the manufacturing facility ($/cycle)
S_i	=	fixed ordering cost of inputs for manufacturing and remanufacturing ($/lot)
h_m	=	inventory holding cost of new or remanufactured product ($/unit/time unit)
h_i	=	inventory holding cost of input materials necessary for the production of a unit of the new product($/unit/time unit)
h_{ir}	=	inventory holding cost of input materials necessary for remanufacturing a unit of the used product($/unit/time unit)
r_m	=	transfer price paid to retailer by manufacturer for collecting used products($/unit)
p_w	=	wholesale price charged to retailer for the new product ($/unit);
c_s	=	variable transportation cost of shipping new product to the retailer($/unit)
c_r	=	variable cost of transporting, cleaning, preparation, etc for returned items($/unit)
c_m	=	variable cost of manufacturing new product ($/unit)
c_{rm}	=	variable cost of remanufacturing a returned used product into a new one($/unit)

Notation used for remanufacturing model(common to both)		
T	=	inventory replenishment cycle time, common to retailer and manufacturer (time units)
Q	=	replenishment quantity consisting of new and/or remanufactured items (units)

1. Introduction to Hierarchical Decision Making (HDM)

1.1 Background

Hierarchical decision making is a decision making system, where multiple decision makers are involved and the process has a hierarchical structure, i.e., decision making on the order of levels. Usually, decision makers in the higher level make decisions first. After observing the decisions in the higher level, decision makers in the lower level react with their optimal strategies. With the assumption of perfect information about the lower level, the decision makers in the higher level could anticipate the strategies from the lower level before their decision making. As a result, the decision makers will take responses from the lower level into consideration before the upper level decisions are made.

We call the decision makers in the upper level problem *the leaders*. Correspondingly, the decision makers in the lower level are called *the followers*. The leaders usually are the players who have more power and more information in this system. For example, government in public economics, regional system operators in the electricity market, and system planners in transportation lend to act as leaders. On the contrary, the followers often have less power and less information and are put into reactive situations. Comparatively, industry sectors, power generators and arbitragers in the electricity market and the travellers in the transportation system generally act as followers.

The hierarchical decision making process in economics is formally referred to as a *Stackelberg game*. Therefore, the discussion of the development of hierarchical decision making should date back to the introduction of the Stackelberg game. The concept was first introduced by Von Stackelberg[74]. As outlined in the Economics literature, the Stackelberg game, traditionally, can be solved by backward deduction. However, backward deduction is restricted by problem size and characteristics of decision variables. With widespread applications of Stackelberg games in the real word, hierarchical decision making systems become more and more popular, and more and more complex as well. Decision systems may not only include continuous variables, but also extend to include discrete variables, timing concerns and uncertainty decisions, which add difficulties to computation and interpretation efforts. New mathematical models and systematic study of solution methods which are able to address such evolving situations have become necessary.

Hierarchical programming is a mathematical framework to represent Stackelberg games. It has multiple optimization problems at different levels. If the number of levels is limited to two, then this type of hierarchical programming is called a *Bilevel Problem*. To simplify the terminology, we use **BP** to denote “bilevel problem” or “bilevel programming” in the following text. Actually, no matter how multi-players act in the lower level or how complex the system is, all hierarchical programming problems are able to be restructured as a two-level problem, which means that a large-scale system could always be decomposed into several smaller and manageable subproblems. As a simplified expression of hierarchical problems[6], **BP** also can be treated as a special case of *mathematical programs with the optimization problem in the constraints*, which is used by [18, 19, 20]. The discussion of **BP** becomes the core of solving multi-level/hierarchical programming problems since **BP** is a simplified hierarchical problem.

1.2 General Formulation and Categories

The terminology of bilevel programming was first introduced by Candler[24]. In the past 30 years, the area has grown fast with research on linear and nonlinear programming. Interested readers might refer to the books such as [71], [10] and [31]. The general formulation of **BP** is:

$$\begin{aligned}
\min_{x} \quad & F(x,y) \\
\text{s.t.} \quad & G(x,y) \leq 0 \\
& y \in \operatorname*{argmin}_{y'}\{f(x,y') : g(x,y') \leq 0\},
\end{aligned} \tag{1.2.1}$$

where the problem has two levels. $x \in X \subseteq R^{n_1}$ is the set of *the upper level decision variables*, decided by the upper level decision maker and $y \in Y \subseteq R^{n_2}$ is the set of *the lower level decision variables*, decided by the lower level decision maker. Similarly, the functions $F : R^{n_1 \times n_2} \to R$ and $f : R^{n_1 \times n_2} \to R$ are called *the upper-level objective function* and *the lower-level objective function*, separately, while $G : R^{n_1 \times n_2} \to R^{m_1}$, and $g : R^{n_1 \times n_2} \to R^{m_2}$ are *the upper-level constraints* and *the lower-level constraints*, respectively.

According to the different characteristics of objective functions and constraints, **BP** can take various forms. For example, the upper level objective function in (1.2.1) can take the form $F(x)$ without the lower level decision variable y. It is also possible that the upper level decision variables do not influence the constraints in the lower level so that the lower level constraint functions can be written as $g(y)$, while the problem (1.2.1) is general enough to accommodate a wide range of problems. With the variation in the structure of the problems, the corresponding algorithms change significantly and computational complexity of implementation can differ. Therefore, we will give sufficient attention to special cases of (1.2.1) when we delve into algorithms.

If the objective function and constraints in (1.2.1) are linear, **BP** can be classified as a Linear Bilevel Problem(**LBP**). Otherwise, it is called a Nonlinear Bilevel Problem(**NBP**). In each type, some typical problems with certain features are discussed in the next sections. Besides, if there exists discrete variables in the problem, whether in objective functions or in constraints, whether in the upper level or in the lower level, these kinds of problems are called Discrete Bilevel Problems(**DBP**).

1.2.1 Linear Bilevel Programming

If the objective functions F, f and the constraints G, g are all linear, (1.2.1) can be categorized as a *linear bilevel programming problem*(**LBP**). Bialas and Karwan[17] summarize geometric characteristics and track the development of related algorithms. They further categorize **LBP** into two classes: *linear resource control problems* and *linear price control problems.* The *linear resource control problem* is a type of **BP** where the decision variables in the upper level are not the coefficients in the lower level problem:

$$\begin{aligned} \min_{x} \quad & F(x,y) = C_x x + C_y y \\ \text{s.t.} \quad & G(x,y) = A_x x + A_y y - B \leq 0 \\ & y \in \operatorname*{argmin}_{y'} \{ f(x,y') = c_x x + c_y y' : g(x,y') = a_x x + a_y y' - b \leq 0 \}. \end{aligned} \tag{1.2.2}$$

This kind of problem may be applicable in many areas where limited resources are distributed among different entities, for a example, governmental budgets allocation among various sectors.

The *linear price control problem* has an upper level objective function determined by both x and y. The mathematical expression is:

$$\begin{aligned} \min_{x} \quad & F(x,y) = C_x x + C_y y \\ \text{s.t.} \quad & G(x,y) = A_x x + A_y y - B \leq 0 \\ & y \in \operatorname*{argmin}_{y'} \{ f(x,y') = x^T y' : g(x,y') = a_x x + a_y y' - b \leq 0 \}. \end{aligned} \tag{1.2.3}$$

Note that the inner product in the lower level objective function makes the upper level decision control the coefficients in the lower level. Given the decision in the upper level problem, the lower level becomes a linear problem. This problem has a very wide application are in the analysis of tax and subsidy programs.

1.2.2 Nonlinear Bilevel Programming

If F or G are nonlinear in x, or if f or g are nonlinear in y, then (1.2.1) turns into a *nonlinear bilevel problem*(**NBP**). Since **LBP** is NP-hard, the further complexity of **NBP** has limited development in this area. However, the *Quadratic Bilevel Problem* (**QBP**) has received a lot of attention due to its special structure, which makes most algorithms implementable and robust. Generally, a **QBP** can be expressed as

$$\begin{aligned}
\min_{x} \quad & F(x,y) = C_x x + C_y y + (x,y)^T P(x,y) \\
\text{s.t.} \quad & G(x,y) = A_x x + A_y y \leq B \\
& y \in \underset{y'}{\operatorname{argmin}}\{f(x,y) = c_x x + c_y y^{'} + (x,y^{'})^T Q(x,y^{'}) : g(x,y^{'}) = a_x x + a_y y^{'} - b \leq 0\}.
\end{aligned} \tag{1.2.4}$$

1.2.3 Discrete Bilevel Problem

If a **BP** has discrete variables, the problem is called a *Discrete Bilevel Problem*(**DBP**). These discrete variables can be integer-valued or binary, or represent other discrete choices, and they can appear on either of the two levels of the **BP**. The existence of discrete variables adds computational complexity to the problem.

1.2.4 Other Categories of BP

One special case in **BP** is the *min-max problem.* The leader(*minimizer*) tries to find the optimal values of its decision variables to minimize an objective function, maximized with respect to the follower(*maximizer*)'s variables. The decision makers consider the worst case scenario. Meanwhile, both the maximizer and the minimizer have the same objective function $F(x,y) = f(x,y)$ and are subject to the same joint constraints $G(x,y) = g(x,y)$. Then the min-max problem can be written as:

$$\begin{aligned} \max_x \min_y \quad & F(x,y) \\ \text{s.t.} \quad & G(x,y) \leq 0. \end{aligned} \tag{1.2.5}$$

Another special case is a Stackelberg game with multiple agents at each level. This is a very common problem in economic markets of operations and contains problems with oligopolistic competition where a Nash equilibrium may be sought among all the players at the same level.

Cooperative games, where the upper level and the lower level can cooperate to attain the goal, is also a special case of **BP**. It also belongs to the area of multi-objective optimization. Usually, a centralized planner is assumed before the optimization of both participants. We will examine it with models in our remanufacturing system in Chapter 3. Meanwhile, we also focus on the non-cooperative game with applications in biofuel production. Non-cooperative means that there exists conflicts between the objective of the upper level and that of the lower level.

1.3 Characteristics of BP and its Solutions

1.3.1 Characteristics of BP

We can relax the feasible region for (1.2.1) without considering the optimal reaction of y to x. Then the *relaxed feasible region* for the pair (x, y) is defined as

$$\Gamma = \left\{(x,y) \in R^{n_1} \times R^{n_2} : G(x,y) \leq 0 \text{ and } g(x,y) \leq 0\right\}, \tag{1.3.1}$$

which is generally assumed to be bounded and nonempty so that the original problem is not trivial and does have a optimal solution. For a given $\bar{x}$, we can have a *lower-level feasible set* corresponding to $\bar{x}$ as

$$\Gamma(\bar{x}) = \left\{ y \in R^{n_2} : g(\bar{x}, y) \leq 0 \right\}. \tag{1.3.2}$$

If the lower level specifies its optimal reaction to a given $\bar{x}$, the set of optimal y is called *the lower-level reaction set* as:

$$\Re(\bar{x}) = \operatorname{argmin} \left\{ f(\bar{x}, y') : y' \in \Gamma(\bar{x}) \right\}. \tag{1.3.3}$$

This set groups all the best strategies of y for any given $\bar{x}$. Consequently, we can have the reaction set of y defined in their feasible region for all the given x defined in the upper-level feasible set as:

$$IR = \left\{ (x, y) \in R^{n_1} \times R^{n_2} : G(x, y) \leq 0, y \in \Re(x) \right\}. \tag{1.3.4}$$

The set is called as *Induced Region* or *Inducible Region.* If the upper level decision makers choose actions to optimize their objective functions within this inducible region, the solutions will be optimal for the original bilevel problem. Notice that the inducible region is usually neither convex nor connected.

At some given x, the lower level may have multiple solutions which are all optimal for the lower level problem with the same objective function. This is the same as saying that the reaction set of the lower level problem $\Re(x)$ is not a singleton. In this case, if the leader has full control over the lower level feasible set $\Gamma(x)$ so that he can choose the action which benefits himself most, we call the scenario an *optimistic situation.*

A solution (x^*, y^*) is the *local optimistic solution* (**LOS**) if the lower level optimum corresponding to the value of x^* is always less than that from the other lower level optima for x in the neighborhood $V(x^*, r)$ of x^* with a positive radius r. Let us define $\vartheta_o(x) = \min_y \left\{ F(x, y) : y \in \Re(x) \right\}$. As a result, the sufficient conditions for a **LOS** are:

$$\begin{aligned}
& x^* \in V(x^*, r) \cap X \\
& y^* \in \Re(x^*) \\
& G(x^*, y^*) \leq 0 \\
& \vartheta_o(x^*) \leq \vartheta_o(x) \text{ for all } x \in V(x^*, r) \cap X.
\end{aligned} \tag{1.3.5}$$

If the above requirement is also met in all of X, we call it a *global optimistic solution*(**GOS**), which means

$$\begin{aligned}
& x^* \in X \\
& y^* \in \Re(x^*) \\
& G(x^*, y^*) \leq 0 \\
& \vartheta_o(x^*) \leq \vartheta_o(x).
\end{aligned} \tag{1.3.6}$$

Relatively, if the coordination between the leader and the follower does not exist, or the leader is risk-averse and avoids any loss at the least, then we redefine

$$\vartheta_p(x) = \max_y \{F(x, y) : y \in \Re(x)\}.$$

A point (x^*, y^*) is called a *Local Pessimistic Solution*(**LPS**), if it follows the following conditions within $V(x^*, r)$:

$$\begin{aligned}
& x^* \in V(x^*, r) \cap X \\
& y^* \in \Re(x^*) \\
& G(x^*, y^*) \leq 0 \\
& \vartheta_p(x^*) \leq \vartheta_p(x) \text{ for all } x \in V(x^*, r) \cap X;
\end{aligned} \tag{1.3.7}$$

Similarly, a *Global Pessimistic Solution*(**GPS**) has the following properties:

$$
\begin{aligned}
& x^* \in X \\
& y^* \in \Re(x^*) \\
& G(x^*, y^*) \leq 0 \\
& \vartheta_p(x^*) \leq \vartheta_p(x).
\end{aligned}
\tag{1.3.8}
$$

The particular choice among these options is determined by the power distributed between the leader and the follower. If the follower is strong, the leader can only take the action to reduce its loss in the worst case. However, if the leader is much stronger, the control he has leads to the best scenario he wants.

1.3.2 Karush-Kuhn-Tucker Conditions

The solution methods to optimize **BP**, especially **NBP**, rely heavily on Karush-Kuhn-Tucker (**KKT**) optimality conditions. **KKT** conditions for the lower level problem can be defined if the lower level problem is convex and regular. We can integrate KKT optimality conditions into the upper level problem so that the **BP** becomes a single-level problem. The formulation is:

$$
\begin{aligned}
\min_{x,y} \quad & F(x, y) \\
\text{s.t.} \quad & G(x, y) \leq 0 \\
& g(x, y) \leq 0 \\
& \lambda g(x, y) = 0 \\
& \lambda \geq 0 \\
& \nabla_y L(x, y, \lambda) = 0,
\end{aligned}
\tag{1.3.9}
$$

where

$$
L(x, y, \lambda) = f(x, y) + \sum_{i=1}^{m_2} \lambda_i g_i(x, y). \tag{1.3.10}
$$

To ensure that an optimal solution to the general nonlinear bilevel programming problem can be obtained by (1.3.9), we set up the following sufficient optimality conditions for the lower level problem [67]:

1. The bilevel problem is well posed, i.e., at any given $\bar{x}$, the optimal lower level solution $y(\bar{x})$ and the corresponding multipliers $\lambda(\bar{x})$ are unique. Otherwise, the leader cannot pose his decision within the set of $y(x)$, which causes an ill-posed problem;

2. The gradients of all the active constraints in the lower level are linearly independent, i.e., at any given $\bar{x}$, the corresponding Lagrange multipliers $\lambda(\bar{x})$ are unique.

3. Strict complementarity holds, i.e., the multipliers of any active inequality lower level constraints are positive.

4. The second-order sufficiency conditions are satisfied, which means

$$\left\{d^T \nabla_y^2 L(x, y(x), \lambda(x))d > 0 : \forall d \in E(x), d \neq 0\right\}, \tag{1.3.11}$$

   where $E(x) = \{d \in R^{m_2} | \nabla_y L(x, y, \lambda)d = 0\}$ is the tangent space at $y(x)$.

It should be noted that the new problem is not always exactly equivalent to the original problem, only in the optimistic scenario. There does not exist any efficient way to apply this approach to the pessimistic case. Also, even with the ideal assumptions, (1.3.9) is still hard to solve. The major difficulty exists because of the complementary slackness condition $\lambda \cdot g(x, y) = 0$ in the KKT conditions of the lower-level problem.

1.4 Differences between HDM and Other Multi-Objective Models

Hierarchical Decision Making describes a decision making system with multiple stages, each of which has one or more decision makers. Although Goal Programming Problems also have multiple objectives, they differ from Hierarchical Programming Problem in the sense that goal programming usually only has one decision maker. The decision makers in goal programming can balance the trade-off among several objectives so that in the end the maximum utility can be realized. If the different levels of a hierarchical decision making framework can be integrated in a systematic manner, or in other words, there exists a centralized decision maker, the hierarical programming problem becomes a goal programming problem. However, in most cases, the decision makers at different levels have their own perspectives and their own separate objectives.

Another model, Analytical Hierarchy Process, is also close to Hierarchical Decision Making, in that it also has a hierarchical structure, which is defined as goal, criteria and decision making units. The major use of Analytical Hierarchy Process is to conduct comparisons, evaluate and choose among several decision units according to multiple criteria. Priority weights are assigned to different criteria. With the evaluation of these alternative decision units on these criteria, the overall scores for the decision units could be calculated and compared. The applications of AHP involve planning, resource allocation, priority setting, and selection among alternatives. Other application areas include forecasting, total quality management, business process re-engineering, quality function deployment, and the Balanced Scorecard. Again, under AHP, there is usually one decision maker. Even in the presence of multiple stakeholders, the AHP framework seeks to reconcile their responses to be consistent. Also, the choices being compared in an AHP framework are only discrete; whereas continuous decisions variables, such as price can be included in hierarchical programming models.

2. Literature Review

In this chapter, we start with a review of the literature on solution methods for hierarchical decision making. Then, we put emphasis on applications of hierarchical decision making. Finally, the chapter is concluded with a summary to indicate the motivation of this study.

2.1 Solution Methods for Continuous Bilevel Programming Problems

In this section, we review the features and the development of methodologies in the area of **BP**. Our focus is on continuous **BP**s for now and postpone the discussion of **DBP**s. We categorize the methods into two groups: vertex-enumeration based algorithms and gradient based algorithms.

Vertex-enumeration based algorithms require solving the lower level problem or the relaxed problem explicitly to obtain the vertices. By enumerating all vertices, a global solution can be found. The global solution might not be guaranteed if we only enumerate some of the vertices. Gradient based algorithms require the evaluation of the first derivatives or even the second derivatives. Some approximation methods are applied. Furthermore, the methods in this category rarely can guarantee a global solution. We review all the methods belonging to these two classes separately in the next two subsections.

2.1.1 Vertex-enumeration Based Algorithm

In this subsection, we summarize the direct enumeration method, the branch-and-bound method and the complementarity pivoting method. All three methods are based on the enumeration of all or part of all the possible solutions during optimization. A global optimum cannot be guaranteed whenever any solution procedure attempts to devise a system to only partially enumerate the solution space.

a) Direct Vertex Enumeration

As its name suggests, direct vertex enumeration directly calculates all the vertices of the feasible region. In this part, we review all the methods which require enumerating some or all the points to get optimal solutions. This type of method has especially wide use in linear bilevel programming because one of the significant features of **LBP**s is that the feasible sets are polyhedral when all the constraints are linear. The feasible region (1.3.1) is examined. At least one of the vertices in the region is the optimal solution for the bilevel problem if the set is nonempty. This method is straightforward, however, computational efforts are highly affected by the size of the problem, especially with the number of variables in the model.

Papavassilopoulos[59] proposes several algorithms to solve an **LBP** without constraints in the upper level:

$$\begin{aligned} \min_{x} \quad & F(x,y) = C_x x + C_y y \\ s.t. \quad & y \in \underset{y}{\operatorname{argmin}}\{f(x,y) = c_y y : \text{s.t. } g(x,y) = a_x x + a_y y - b \le 0\}. \end{aligned} \tag{2.1.1}$$

The first algorithm solves the lower level problem with respect to y at a given x^k, that is:

$$y^k = \underset{y}{\operatorname{argmin}}\{c_y y : \text{s.t. } a_x x^k + a_y y \le b\}$$

, then checks all the neighboring vertices of the current optimal point (x^k, y^k) to find the next start point (x^{k+1}, y^{k+1}) if it qualifies with both an improvement in the upper level objective value and stays in the reaction set. Meanwhile, the objective value of the upper level problem at this point provides a lower bound, which can be enforced as a new constraint into the optimization of the subproblem. The best point selected from the neighboring vertices should be the one that leads to the largest change in the objective function or the one that lies on the edge as close as the coefficient of the upper level problem. The algorithm continues until all the vertices have been examined. The best point giving the smallest upper level objective value is a global optimal solution to the original bilevel problem.

The second method presented in [59] separates all the constraint sets into different subsets according to the binding properties. Combined with the **KKT** condition of the lower level problem, the new problem is:

$$\begin{aligned} \min_{x,y,\lambda} \quad & F(x,y) = C_x x + C_y y \\ s.t. \quad & c_y + a_y^T \lambda = 0 \\ & a_x x + a_y y \leq b \\ & \lambda^T (a_x x + a_y y - b) = 0 \\ & \lambda \geq 0. \end{aligned} \tag{2.1.2}$$

Let $I = \{1, 2, \dots m_2\}$ define the set of all the indices of the constraints, along with I_1^j which denotes jth combination of binding constraints, where $j = 1, 2, \dots, 2^{m_2}$. A complement subset I_2^j are also defined. Then the above constraints can be divided into 2 subsets of constraints in 2^{m_2} ways. The jth such partition of the constraints is listed as follows:

$$\begin{aligned} & c_y + \sum_{i \in I_1^j} a_{yi} \lambda_i^j = 0 \\ & \lambda^T (a_x x + a_y y - b) = 0 \\ & a_{xi} x + a_{yi} y = b_i, i \in I_1^j \\ & a_{xi} x + a_{yi} y \leq b_i, i \in I_2^j \\ & \lambda_i^j \geq 0, i \in I_1^j \\ & \lambda_i^j = 0, i \in I_2^j. \end{aligned} \tag{2.1.3}$$

A point (x^j, y^j) that satisfies the above constraints describes one of the vertices in the reaction set of the original **BP**. As a result, any feasible point could be written as an affine function of the vertices that make a certain subset of the constraints hold, i.e.,

$$x = \mu_1 x^1 + \mu_2 x^2 + \ldots + \mu_N x^N = \sum_j \mu_j x^j$$

$$y = \mu_1 y^1 + \mu_2 y^2 + \ldots + \mu_N y^N = \sum_j \mu_j y^j$$

$$\mu_1 + \mu_2 + \ldots + \mu_N = 1.$$

The equivalent problem is

$$\begin{aligned} \min \quad & F(x,y) = C_x \sum_j \mu_j x^j + C_y \sum_j \mu_j y^j \\ \text{s.t.} \quad & \mu_j c_y + \sum_{i \in I_1^j} a_{yi} \mu_j \lambda_i^j = 0, j = 1, \ldots, N \\ & \sum \mu_j = 1. \\ & \mu_j \geq 0, j = 1, \ldots, N \end{aligned} \tag{2.1.4}$$

Note that the computational complexity increases with the number of constraints in the lower level problem.

The third algorithm in [59] starts from solving the upper level problem subject to the relaxed feasible region Γ (1.3.1), that is

$$(\bar{x}^k, \bar{y}^k) \in \operatorname*{argmin}_{x,y} \{F(x,y) : \text{s.t. } a_x x + a_y y \leq b\}.$$

Then it gets the optimal solution $(\bar{x}^k, \tilde{y}^k)$ by solving the lower level problem within the lower level feasible region(1.3.2) at a fixed $x = \bar{x}^k$:

$$\tilde{y}^k \in \operatorname*{argmin}_{y} \left\{ f(\bar{x}^k, y) : \text{s.t. } a_x \bar{x}^k + a_y y \leq b \right\}.$$

The algorithm checks whether the solution satisfies $\tilde{y}^k = \bar{y}^k$. If so, $(\bar{x}^k, \bar{y}^k)$ is the optimal solution. Otherwise, we find the next vertex $(\bar{x}^{k+1}, \bar{y}^{k+1})$ within the reaction set until no better upper objective value can be found.

Among the three algorithms, if the solution is already around the global solution, the third algorithm will outperform the others. But if only a good guess of x^k exists, the first one is better. As to the second one, [59] suggests combining it with the first algorithm to find the optimal solution fast.

Bialas and Karwan[16, 17] introduce the"K-th best" algorithm method, which is quite close to the last method mentioned in [59]. Take the same steps to get $(\bar{x}^k, \bar{y}^k)$ and $(\bar{x}^k, \tilde{y}^k)$. If $\tilde{y}^k = \bar{y}^k$, the solution is the global solution to the original problem. If not, search the adjacent extreme points in the neighbourhood of $(\bar{x}^k, \tilde{y}^k)$ to get the relative smallest objective value in the upper level, which is expected to be larger than that for $(\bar{x}^k, \tilde{y}^k)$. Then this point is called $(\bar{x}^{k+1}, \bar{y}^{k+1})$, and the above process is repeated until $\tilde{y}^K = \bar{y}^K$, which is the "K-th best".

Candler and Townsley [25] also focus on (2.1.1) and develop an equivalent problem as:

$$\begin{aligned} \min_{x,\tilde{y}} \quad & C_x x + C_y \tilde{y} \\ \text{s.t.} \quad & a_x x + \tilde{a}_y \tilde{y} - b \leq 0 \end{aligned} \tag{2.1.5}$$

where $\tilde{a}_y$ is an optimal basis of a_y with nonnegative reduced cost and $\tilde{y}$ is the solution, with the components corresponding to the columns in $\tilde{a}_y$, since the solutions of the equivalent problem are feasible for the original problem. Whenever no degeneracy appears, the algorithm searches through moving from one optimal basis to another until there is no improvement in the objective function.

For a general **LBP**, Tuy et al.[79] outline the method of polyhedral annexation to get a global solution. The general **LBP** can be written as a reverse convex constrained problem:

$$\begin{aligned}
\min_{x\in X} \quad & F(x,y) = C_x x + C_y y \\
\textbf{s.t.} \quad & A_x x + A_y y \le B \\
& a_x x + a_y y \le b \\
& c_y y \le \varphi(x),
\end{aligned} \tag{2.1.6}$$

where $\varphi(x)$ is the optimal objective value of the optimized problem, i.e.,

$$\varphi(x) = \min_{y} \left\{ c_y y : a_x x + a_y y \le b, y \ge 0 \right\},$$

and is a convex polyhedral function. The constraint $c_y y \le \varphi(x)$ is called a *reverse convex constraint*. Without considering the last reverse convex constraint, we get the optimal solution (x^k, y^k), which denotes one of the polyhedron vertices. If (x^k, y^k) satisfies the reverse convex constraint, i.e., $c_y y^k = \varphi(x^k)$, we can conclude that it is the optimal solution. If not, we rewrite the objective function and constraints in terms of the nonbasic variables. The method of finding the next vertex from the current vertex is to construct a new polyhedron by cutting off the current vertex from the previous polyhedron, that is adding a new constraint $F(x,y) \le F(x^k, y^k) - \epsilon$ while ensuring the new vertex is not included in the region of $c_y y \le \varphi(x)$. Whenever the convex region fully contains the polyhedron, the last solution is the global optimal solution.

b)Branch-and-Bound Method

Falk[36] first applies a branch-and-bound method to solve a linear minmax problem

$$\max_{x} \min_{y} \left\{ F(x,y) = C_x x + C_y y : G(x,y) = A_x x + A_y y - B \le 0 \right\}, \tag{2.1.7}$$

where both the objective function and the constraints are linear. The basic idea is to solve two subproblems iteratively in each stage, where the first subproblem

$$\bar{F}^k = \max_{x,y} \{F(x,y) = C_x x + C_y y : A_x x + A_y y - B \leq 0\} \tag{2.1.8}$$

provides an upper bound for F at the solution (x^k, y^k), and the second subproblem

$$\underline{F}^k = \min_{y} \left\{F(x^k,y) = C_x x^k + C_y y : A_x x^k + A_y y - B \leq 0\right\} \tag{2.1.9}$$

yields a lower bound for F. If $\bar{F}^k = \underline{F}^k$, then (x^k, y^k) is the optimal solution to the original **BP**. Otherwise, we construct stage $(k+1)$ by branching on each basic variable x_j^k, where j is in the index set of basic variables B^k, for the first subproblem of stage k. Each child node has the form

$$\max_{x,y} \left\{F(x,y) = C_x x + C_y y : A_x x + A_y y - B \leq 0, x_j^k = 0, \right\} \tag{2.1.10}$$

for some j, that is, the basic variable x_j^k is now required to be nonbasic. The process continues recurrsively until a solution is found.

In addition, [36] also considers constructing subproblems using the complementarity conditions of the problem (1.3.9). The search tree is developed based on this condition. In fact, most branch-and-bound methods used in **BP** rely on this idea, which is formally presented in [7] and [9]. The branching scheme pertains to locating the segment in the inducible region of the **BP** problem.

In [7], Bard discusses a convex **BP** with multiple followers in the lower level. For the generalized model (1.2.1), the objective function $F(x,y)$ and the constraints $G(x,y)$ are convex in all their arguments. A point in the inducible region is defined by the solution (x^0, y^0) of the relaxed combined problem:

$$\min_{x,y} \{F(x,y) : x \in X, \nabla_y f(x,y) + \lambda \nabla_y g(x,y) = 0, g(x,y) \leq 0, \lambda \geq 0\},$$

which provides a lower bound $\underline{F} = F(x^0, y^0)$. The relaxation is obtained by omitting the complementarity constraints from (1.3.9). As before, the solution to the lower level problem

$$\min_y \left\{ f(x^k, y) : g(x^k, y) \le 0 \right\}$$

is obtained by $(x^k, \bar{y}^k)$. $F(x^k, \bar{y}^k) = \underline{F}$ implies that it is the optimal solution to the original problem. Otherwise, we branch on constraint $g_i(x, y)$, which is selected by

$$\operatorname*{argmin}_i \left\{ \frac{-\nabla F(x^k, y^k)^T \nabla g_i(x^k, y^k)}{||\nabla F(x^k, y^k)|| \cdot ||\nabla g(x^k, y^k)||} \right\}$$

.

The upper bound is updated by solving the resulting child problem:

$$\min_{x,y} \{F(x, y) : x \in X, \nabla_y f(x, y) + \lambda \nabla_y g(x, y) = 0, g(x, y) \le 0 \text{ and } g_i(x, y) = 0\} \quad (2.1.11)$$

If the objective value is greater than the current upper bound, the branch would be fathomed. Next, the ongoing path is determined by the assignment of all the remaining constraints into the following subsets:

$$S_1 = \{i : \text{undetermined}\}$$

$$S_2 = \{i : g_i(x, y) = 0, i \notin S_1\}$$

$$S_3 = \{i : \lambda_i = 0, i \notin S_1\}.$$

With the new path, a new upper bound is attained at the solution of the following problem:

$$
\begin{aligned}
\min_{x,y} \quad & F(x,y) \\
& x \in X, \\
& \nabla_y f(x,y) + \lambda^T \nabla_y g(x,y) = 0, \\
& g_i(x,y) = 0, i \in S_2, \\
& \lambda_i = 0, i \in S_3.
\end{aligned}
\tag{2.1.12}
$$

Optimality is reached if this bound equals $\underline{F}$. While this is a branch-and-bound method, the strategy of identifying the active sets S_1, S_2, S_3 requires significant bookkeeping. In [34], Edmunds and Bard devise strategies to efficiently implement this method.

A Branch-and-Bound method and an active set strategy are also applied to solve the bilevel problem with a linear upper level problem and a quadratic lower level problem in [9]. Since the single problem converted by substituting the **KKT** conditions of the lower level problem into the upper level problem is a linear problem with complementarity constraints, the globally optimal solution can be guaranteed. The basic idea is similar to that described above for [7], and the node to be branched is the one with the largest complementarity product $\lambda_i g_i(x^k, y^k)$.

[40] presents a way to apply a branch-and-bound method to **LBP**s of the form (1.2.2) and (1.2.3). The discussion still focuses on the tightness of constraints in the lower level problem, and new binary variables η are introduced to represent whether the constraints are tight:

$$
\eta_i : \begin{cases} 0 & \text{if } g_i(x,y) < 0 \\ 1 & \text{if } g_i(x,y) = 0 \end{cases}
\tag{2.1.13}
$$

for $i = \{1 \ldots m_2\}$,

$$\eta_{m_2+j} : \begin{cases} 0 & \text{if } y_j < 0 \\ 1 & \text{if } y_j = 0 \end{cases} \tag{2.1.14}$$

for $j = \{1 \dots n\}$. [40] further shows the following certain logical relationship about η_i holds:

$$\text{if } (c_y)_j > 0 \ , \sum_{i|a_{ij}>0} \eta_i \geq 1$$

$$\text{if } (c_y)_j < 0, \sum_{i|a_{ij}<0} \eta_i + \eta_{m_2+j} \geq 1$$

then a new branch-and-bound method based on the value of η_i is applied to get the optimal solution in a linear case. With a determined η_i or a subset of η, according to the binding constraints, some y in the lower level could be expressed by the other (x, y) values. It results in the subproblems with all the parameters updated correspondingly:

$$\begin{aligned} \min_{x} \quad & \tilde{F}(x, \tilde{y}) = \tilde{C}_x x + \tilde{C}_y \tilde{y} \\ & \tilde{G}(x, \tilde{y}) = \tilde{A}_x x + \tilde{A}_y \tilde{y} - \tilde{B} \leq 0 \end{aligned} \tag{2.1.15}$$

$$\tilde{y} \in \underset{\tilde{y}'}{\operatorname{argmin}} \{ \tilde{f}(x, \tilde{y}') = \tilde{c}_x x + \tilde{c}_y \tilde{y}' : \tilde{g}(x, \tilde{y}') = \tilde{a}_x x + \tilde{a}_y \tilde{y}' - \tilde{b} \leq 0 \}. \tag{2.1.16}$$

At every node, the algorithm needs to solve three subproblems: (1) the problem (2.1.15) with all the constraints from (2.1.16) to get the solution $(\bar{x}, \bar{y})$ (2) The problem (2.1.16) to get the solution $(\bar{x}, \tilde{y})$. (3) the original lower level problem at the given $\bar{x}$ to get $(\bar{x}, \tilde{y}^o)$. In each branch-and-bound process, we not only make sure that the solutions are rational by checking $\bar{y} = \tilde{y}$ and $\bar{y} = \tilde{y}^o$, but also make sure that there is enough improvement in the objective function of the upper level problem.

The disjunctive nature of the complementarity slackness conditions is captured by Fortuny-

Amat and McCarl[39]. When solving a quadratic bilevel problem:

$$\begin{aligned}
\min_{x} \quad & F(x,y) = C_x x + C_y y + (x,y)^T P(x,y) \\
& G(x,y) = A_x x + A_y y - B \leq 0 \\
& y \in \operatorname*{argmin}_{y'} \{ f(x,y') = c_y y' + (x,y')^T P(x,y') : g(x,y') = a_x x + a_y y' - b \leq 0 \}.
\end{aligned} \tag{2.1.17}$$

They add zero-one vectors η_1 and η_2 and reformulate (2.1.17) in a single problem as follows:

$$\begin{aligned}
\min_{x,y,\eta_1,\eta_2} \quad & F(x,y) \\
\text{s.t.} \quad & G(x,y) \leq 0 \\
& g(x,y) + s_1 = 0 \\
& L_y(x,y) + s_2 = 0 \\
& s_1 \leq M\eta_1 \\
& \lambda \leq M(e - \eta_1) \\
& y \leq M\eta_2 \\
& s_2 \leq M(e - \eta_2) \\
& \eta_1, \eta_2 \in \{0,1\},
\end{aligned} \tag{2.1.18}$$

where M is a large positive parameter. Branch-and-bound can be employed to solve this mixed integer problem and the global optimum can be guaranteed.

Al-Khayyal[3] applied a branch-and-bound algorithm to the bilevel problem with a concave objective function and quadratic constraints in the lower level. Upper bounds and lower bounds are obtained through rectangular partitions, which confine arguments to small intervals, corresponding to upper bounds and lower bounds for the objective function. The complementarity condition $\lambda_i g_i(x,y) = 0$ is rewritten as

$$0 = \sum_i \lambda_i g_i(x,y) = \sum_i \left[\lambda_i - \max\{0, \lambda_i + g_i(x,y)\} \right],$$

then converted to the quadratic form:

$$0 = \sum_i \left[\lambda_i^2 - z_i\right], \text{where } z_i \geq \left[max\left\{0, \lambda_i + g_i(x,y)\right\}\right]^2.$$

The upper level objective function is replaced by $F(x,y) + \sum_i z_i$. The lower level constraint functions g have a quadratic form and by defining x in certain regions such as $\overline{m}^k \leq x^k \leq \underline{m}^k$, we can divide the whole feasible region into several rectangular intervals, which provide the upper bounds and lower bounds for the upper level objective functions.

The algorithm is further extended by Jaumard et al.[42] to the situation with convex objective functions in both levels, quadratic upper level constraints and affine lower level constraints.

c)Parametric Complementarity Pivot Algorithm

The sequential linear complementarity problems(**SLCP**) method is a type of parametric complementarity pivot(**PCP**) algorithm. It was introduced by Bialas and Karwan [17] as a continuation of the work in [15]. The basic idea is presented in the following. After combining the upper level problem with the KKT conditions of the lower level and adding slack variables, a standard **LBP** can be rewritten as :

$$\begin{aligned}
\min \quad & C_x x + C_y y \\
\text{subject to} \quad & -s_1 = -b + a_y y + a_x x \\
& -s_2 = c_y - a_x \lambda \\
& \lambda^T s_1 = y^T s_2 = 0 \\
& x, y, \lambda, s_1, s_2 \geq 0.
\end{aligned} \tag{2.1.19}$$

A parameter κ is introduced to change the objective function to a constraint and the problem into a linear complementarity problem:

$$
\begin{aligned}
\min \quad & \kappa \\
\text{subject to} \quad & -s_1 = -b + a_y y + a_x x \\
& -s_2 = c_y - a_x \lambda \\
& \lambda^T s_1 = y^T s_2 = 0 \\
& \kappa \geq C_x x + C_y y \\
& x, y, \lambda, s_1, s_2 \geq 0.
\end{aligned} \tag{2.1.20}
$$

Instead of solving a nonconvex nonlinear problem, we solve the parametric LCP(κ):

$$
\begin{bmatrix} -s_1 \\ -s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} -b \\ c_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \kappa + \begin{bmatrix} 0 & a_y & a_1 \\ -a_y^T & 0 & 0 \\ 0 & -C_y & -C_x \end{bmatrix} \begin{bmatrix} \lambda \\ x \\ y \end{bmatrix}. \tag{2.1.21}
$$

In the kth iteration, κ^k is defined by

$$
\kappa^k = C_x x^{k-1} + C_y x^{k-1} - \gamma |C_x x^{k-1} + C_y x^{k-1}|,
$$

where γ is a small positive constant and (x^{k-1}, y^{k-1}) is the optimal solution in the $(k-1)$th LCP corresponding to κ^{k-1}. The global solution is claimed to be found if LCP(κ^k) has no solution. Correspondingly, the solution (x, y) associated with κ^{k-1} is the optimal solution.

2.1.2 Gradient-Based Algorithms

In this section, we will review the literature on solving **BP**s with gradient-based methods. We put special emphasis on three methods: direct descent method, trust region method and penalty method.

For all three, the gradient of the objective function in the upper level with respect to its decision variables is evaluated by using partial derivatives:

$$
\nabla_x F(x, y(x)) = \nabla_x F(x, y) + \nabla_y F(x, y) \nabla_x y(x). \tag{2.1.22}
$$

In some special cases such as the **BP** without constraints in the lower level, it is not difficult to calculate the gradient $\nabla_x y(x)$ by directly differentiating an explicit form of the function $y(x)$ [33]. However, there exists either complicated constraints or multiple variables in the lower level problem, and the calculation of partial gradients becomes extremely difficult.

In [51], Kolstad and Lasdon propose a method to calculate the gradients $\nabla_x y$. Distinguished by whether the solution is between bounds or at a bound, $y(x)$ is partitioned as $y_{\mathcal{B}1}$, $y_{\mathcal{N}1}$ respectively. Binding constraints and nonbinding constraints are denoted by $g_{\mathcal{B}2}$ and $g_{\mathcal{N}2}$. Correspondingly, we have the Lagrange multipliers $\lambda_{\mathcal{B}2}$ and $\lambda_{\mathcal{N}2}$. For any component of x, say x_i, the derivatives satisfy

$$
\begin{aligned}
& dy_{\mathcal{N}2}/dx_i = 0 \\
& d\lambda_{\mathcal{N}2}/dx_i = 0 \\
& \begin{bmatrix} \nabla_y^2 L & (\nabla_y g_{\mathcal{B}2})^T \\ (\nabla_y g_{\mathcal{B}2})^T & 0 \end{bmatrix} \cdot \begin{bmatrix} dy_{\mathcal{B}1}/dx_i \\ d\lambda_{\mathcal{B}2}/dx_i \end{bmatrix} = - \begin{bmatrix} (\nabla_y(\partial L/\partial x_i))^T \\ \partial g_{\mathcal{B}2}/\partial x_i \end{bmatrix}.
\end{aligned}
\tag{2.1.23}
$$

This method to calculate the gradients is efficient if the lower level has mostly inactive constraints or simple bounds on the variables. Originally, a procedure for computing derivatives needed to solve a linear system of size $n_2 + m_2$. However, this method just requires solving a linear system of size equal to the sum of the number of active constraints and the number of between-bounds-variables.

a)Direct Descent Method

With the Lagrangian expression developed in (1.3.10), Savard and Gauvin[67] propose a steepest descent method to solve the bilevel problem without upper level constraints. For the optimal solution $(x^*, y(x^*))$, the following relationship holds:

$$F(x^*, y(x^*)) = \nabla_x F(x^*, y(x^*))d_x + \nabla_y F(x^*, y(x^*))d_y(x^*, d_x) \geq 0, \tag{2.1.24}$$

where $d_y(x^*, d_x)$ is the directional derivative for y, related to both the value of x^* and the directional derivative d_x. As a result, in order to find the steepest descent direction $d = (d_x, d_y)$, the algorithm needs to solve the following linear quadratic bilevel program:

$$\begin{aligned} \min_{d_x} \quad & \nabla_x F(x,y)d_x + \nabla_y F(x,y)d_y(x,d_x) \\ \text{s.t.} \quad & ||d_x|| \leq 1 \end{aligned} \tag{2.1.25}$$

where the value of $d_y(x, d_x)$ is obtained by solving the quadratic program:

$$\begin{aligned} \min_{d_y} \quad & (d_x^T, d_y^T)\nabla^2_{xy} L(x,y,\lambda)(d_x, d_y) \\ \text{s.t.} \quad & \nabla_y g_B(x,y)d_y \leq -\nabla_x g_B(x,y)d_x \\ & \nabla_y g_{NB}(x,y)d_y = -\nabla_x g_{NB}(x,y)d_x \\ & \nabla_y f(x,y)d_y \leq -\nabla_x f(x,y)d_x + \nabla_x L(x,y,\lambda)d_x. \end{aligned} \tag{2.1.26}$$

Using the existing algorithms to solve quadratic bilevel problem [9, 42], d_x can be determined in each iteration. If (2.1.25) provides a positive objective value, a local optimum is reached. Otherwise, along with the direction d_x, the step length ϵ is calculated by

$$F(x + \epsilon d_x, y(x + \epsilon d_x)) \leq F(x, y(x)).$$

The algorithm continues from the new startpoint $(x + \epsilon d_x, y(x + \epsilon d_x))$ in the next iteration until it stops at the local optimal solution. In order to get the global optimal solution, some global schemes need to be included.

Vicente et al.[85] propose and develop several new terminologies such as extreme induced region points(EIR points), extreme induced direction(EIR direction) and local star induced region point(LSIR point). EIR points are obtained by solving the optimization problem of replacing the lower level problem with its KKT conditions. The EIR direction is defined as the direction between two EIR points. The LSIR point is an EIR point, at which there exists an EIR direction pointing to the adjacent EIR point that gives a better objective value. Thus, the LSIR point cannot be a local optimum. The basic algorithm is described as constructing $D^k = \{\nabla F(z^k)^T d < 0 \text{ and } d \text{ is an EIR direction}\}$ at an EIR point $z^k = (x^k, y^k)$ in kth iteration. If D^k is a null set, the local optimum is arrived; Otherwise, if a better objective function can be found by $d^k \in D^k$ at the adjacent EIR point, the current check point moves to that point as z^{k+1}; otherwise, z^k is an LSIR point.

Vicente et al.[85] go further into the modified steepest descent algorithm by searching the descent direction $d^k = (d_x^k, d_y^k)$ and calculating the exact step size ϵ^k so that $z^{k+1} = (x^{k+1}, y^{k+1}) = (x^k, y^k) + \epsilon^k(d_x^k, d_y^k)$. The direction $d^k = (d_x^k, d_y^k)$ can be obtained by the sequential LCP method[43]. Another hybrid method takes advantages of both of the above two methods. The algorithm can converge to the local optimum if the objective function in the upper level is concave. However, a global optimum cannot be guaranteed.

The work that integrates the descent method and a bundle method is done by Falk and Liu[37]. They propose two methods: leader predominate algorithm and adaptive leader predominate algorithm. The former uses a bundle method on the upper-level problem and combined with a trust region method to ensure converge to certain points. The latter incorporates a quasi-Newton method into a bundle method in the inner problem. The quasi-Newton method relies on the second-derivative indicator. The local structure of the current estimated solution is very helpful in the scheme. Global convergence and local superlinear convergence to regular points have been proved. Both methods subtract the subgradient information to compute directional derivatives, however, they are all confined to local solutions instead of global solutions.

b)Trust Region Method

The core of a trust region method is to approximate the original problem $H(z)$ with a linear or a quadratic function $\tilde{H}(z)$, that is considered accurate enough in a neighborhood of the current point with $||z - \tilde{z}|| \leq r$. This neighborhood is referred to as the trust region.

The effectiveness of the new approximation model is measured by the ratio:

$$\frac{H(z) - H(\tilde{z})}{\tilde{H}(z) - \tilde{H}(\tilde{z})}. \tag{2.1.27}$$

The value of this ratio determines the radius and the step length from the current point to the next point. It is a measure of how well the approximation represents the original problem. The next iteration may continue to stay at the current search point or move to a new point. Additionally, the radius could remain the same, increase, or decrease. All these changes are determined by the above ratio (2.1.27).

A bilevel program of the form

$$\begin{aligned} \min_{x} \quad & F(x, y) \\ \text{s.t.} \quad & f(x, y)^T (y - y') \leq 0, \ \forall y' \in \Gamma(x) \end{aligned} \tag{2.1.28}$$

is considered in Marcotte et al's work [58]. At the kth iteration based on the current solution (x^k, y^k), the algorithm forces x to lie within the distance r^k, replacing F and f by their respective first-order Taylor series expansion $\tilde{F}$ and $\tilde{f}$ to approximate the original problem, that is

$$\begin{aligned} \min_{x} \quad & \tilde{F}(x, y) \\ \text{s.t.} \quad & \tilde{f}(x, y)^T (y - y') \leq 0, \ \forall y' \in \Gamma(x) \\ & ||x - x^k|| \leq r^k. \end{aligned} \tag{2.1.29}$$

As before, the lower level problem is replaced by its KKT conditions with the complementarity condition transferred to a mixed-integer problem. The subproblem in each iteration is:

$$\begin{aligned}
\min_{x} \quad & \tilde{F}(x,y) \\
\text{s.t.} \quad & ||x - x^k|| \leq r^k \\
& a_x x + a_y y - b \leq 0 \\
& \lambda a_y = \tilde{F}(x,y) \\
& \lambda \geq 0 \\
& \lambda \leq M\eta \\
& a_x x + a_y y - b \leq M(e - \eta^T).
\end{aligned} \tag{2.1.30}$$

The solution to the above problem is $(\bar{x}^k, \bar{y}^k)$, which can be substituted into (2.1.31) to get the ratio.

$$\frac{f(x^k, y(x^k)) - f(\bar{x}^k, y(\bar{x}^k))}{f(x^k, y(x^k)) - f(\bar{x}^k, y(\bar{x}^k))} \tag{2.1.31}$$

where $y(x^k)$ and $y(\bar{x}^k)$ are the best reaction of the follower to the choice of x^k and $\bar{x}^k$, respectively.

Colson et al.[29] approximate F, G and g with linear functions $\tilde{F}$, $\tilde{G}$ and $\tilde{g}$, respectively, and approximate f with a quadratic model $\tilde{f}$. That is, the upper level problem becomes

$$\begin{aligned}
\min \quad & \tilde{F}(x,y) = F(x^k, y^k) + \nabla_x F(x^k, y^k)(x - x^k) + \nabla_y F(x^k, y^k)(y - y^k) \\
\text{s.t.} \quad & \tilde{G}(x,y) = G(x^k, y^k) + \nabla_x G(x^k, y^k)(x - x^k)
\end{aligned} \tag{2.1.32}$$

and the lower level problem becomes

$$\begin{aligned}
\min \quad & \tilde{f}(x,y) = f(x^k, y^k) + \nabla_x f(x^k, y^k)(x - x^k) + \nabla_y f(x^k, y^k)(y - y^k) \\
& + \tfrac{1}{2} \begin{pmatrix} x - x^k & y - y^k \end{pmatrix} D^2 f(x,y) \begin{pmatrix} x - x^k \\ y - y^k \end{pmatrix} \\
\text{s.t.} \quad & \tilde{g}(x,y) = g(x^k, y^k) + \nabla_x g(x^k, y^k)(x - x^k) + \nabla_y g(x^k, y^k)(y - y^k).
\end{aligned} \tag{2.1.33}$$

To rewrite the problem as a mixed integer problem incorporating the KKT conditions in the lower level problem, the following subproblem yields the solution $(\bar{x}^k, \bar{y}^k)$ in the iteration k:

$$\begin{aligned}
\tilde{F}(x,y) \quad & = \nabla_x F(x^k, y^k)x + \nabla_y F(x^k, y^k)y \\
\text{s.t.} \quad & \left\|x - x^k\right\| \leq r^k \\
& \nabla_x G(x_k)x \leq G(x_k) - \nabla_x G(x_k)x_k \\
& \nabla_x g(x^k, y^k)x + \nabla_y g(x^k, y^k)y \leq g(x^k, y^k) - \nabla_x g(x^k, y^k)x^k + \nabla_y g(x^k, y^k)y^k \\
& \lambda \geq 0 \\
& \lambda \leq M\eta \\
& -\tilde{g}(x,y) - \nabla_x g(x^k, y^k)x - \nabla_y g(x^k, y^k)y \leq M(e - \eta) \\
& \nabla^2_{xy} f(x^k, y^k)(x - x^k) + \nabla^2_{yy} f(x^k, y^k)(y - y^k) + \nabla_y f(x^k, y^k) + \lambda \nabla_y g(x^k, y^k) = 0 \\
& \eta \in \{0, 1\}\,.
\end{aligned} \tag{2.1.34}$$

When we fix $\bar{x}^k$ and solve the lower level problem, we can get that the optimal reaction from the follower is $y(\bar{x}^k)$. The ratio of achieved versus predicted reduction is

$$\frac{F(x^k, y(x^k)) - F(\bar{x}^k, y(\bar{x}^k))}{\tilde{F}(x^k, y^k) - \tilde{F}(\bar{x}^k, \bar{y}^k)},$$

and the algorithm will continue the search from the new start point $(\bar{x}^k, y(\bar{x}^k))$ or stay at the same point (x^k, y^k) searching with a smaller step length.

Trust region methods are designed for single-level nonlinear programming problems and cannot be stand-alone methods to solve a **BP**. They are usually used to solve **BP** after a certain reformulation on the problem. The above two papers both implement the trust region method after combining the **KKT** conditions of the lower level with the upper level problem and incorporating the mixed integer method from [39]. It is also possible to apply a penalty method with the trust region method implemented to accelerate the computational performance[26].

c)Penalty Approach

Due to the nonconvexity of the lower level problem, it can guarantee neither continuity nor differentiability at every point. In order to smooth the lower level objective function, a penalty method can be employed. A penalty approach adds a penalty term to the original objective function so that the new objective function is smoothed and can be solved easily.

If the lower level problem is linear, the duality gap is zero when it reaches optimality. It is not difficult for the leader to anticipate the reaction of the follower during its decision-making if it can directly penalize the duality gap of the follower's problem in its objective function. Two assumptions are required to take this penalty approach.

1. If x^* is an optimal solution for the leader, then the reaction set for this x^* is a singleton.
2. The feasible region for λ and for x are non-empty bounded polyhedral, i.e. polytope.

It can be proved that with these assumptions, lower level optimality can be reached by increasing penalty parameter monotonically. The method is first introduced by Anandalingam and White [5] to get a global optimal solution in a linear static Stackelberg game:

$$\begin{aligned} \min_{x} \quad & F(x,y) = C_x x + C_y y \\ \text{s.t.} \quad & G(x,y) = A_x x + A_y y - B \leq 0 \\ & y \in \underset{y'}{\operatorname{argmin}}\{f(x,y') = c_x x + c_y y' : g(x,y') = a_x x + a_y y' - b \leq 0\}. \end{aligned} \tag{2.1.35}$$

For any given x, $a_x x$ and $c_x x$ become constants. The dual problem for the lower level problem can be expressed as:

$$\begin{aligned} \min_{\lambda \geq 0} \quad & \lambda^T (b - a_y y) \\ \text{s.t.} \quad & \lambda^T a_y \geq c_y. \end{aligned} \tag{2.1.36}$$

If the duality gap $\Phi(y,\lambda) = \left[c_y y - \lambda^T (b - a_x x)\right]$ is treated as a penalty term and appended into the upper level problem with a penalty parameter t, then the new problem is:

$$\begin{aligned} \max_{x,y,\lambda} \quad & \tilde{F}(x,y,\lambda,t) = (C_x x + C_y y) - t \cdot \Phi(y,\lambda) \\ \text{s.t.} \quad & A_x x + A_y y - B \geq 0 \\ & a_x x + a_y y - b \geq 0 \\ & \lambda^T a_y \geq c_y \\ & x \geq 0, y \geq 0, \lambda \geq 0. \end{aligned} \tag{2.1.37}$$

Therefore, the original bilevel problem transfers to a single problem, which can be solved easily. Later on, White and Anandalingam[88] further extend their work by using a sequential t generation and cone splitting algorithm to find the global optimum under the same framework of penalizing the dual-gap. The cone splitting algorithm generates a better cone in the current local optimal solution so that the size of the feasible region under examination is reduced. They also study a way to find starting points not only for the penalty algorithm but for other algorithms such as the branch and bound method. However, the effectiveness depends on the correlation between the objectives. The penalty method does not always outperform the branch-and-bound method in the linear case although it is much better than the K-th best method according to their numerical results.

Another main penalty approach that has been used in **BP** is the barrier method. The barrier method is also called the interior penalty method, which constructs a barrier function appropriately defined on the interior of the feasible region. Therefore, the method transforms the parametric constrained optimization problem for the lower level/ the combined problem in **BP** into an unconstrained problem.

Aiyoshi and Shimizu [1] propose a barrier penalty method based on the lower level feasible region. The penalty function $\Phi(g(x,y))$is defined as

$$\Phi(g(x,y)) \begin{cases} > 0 & \text{if } y \in INT\Gamma(x) \\ \to \infty & \text{if } y \to BD\Gamma(x). \end{cases} \tag{2.1.38}$$

The augmented lower level problem is represented by:

$$\min_y \tilde{f}(x,y) = f(x,y) + t^k \Phi(g(x,y)), \quad t^k > 0. \tag{2.1.39}$$

With the assumptions mentioned, the **KKT** conditions for the lower level problem are combined with the upper level problem to get a single problem:

$$\begin{aligned} \min_{x,y} \quad & F(x,y) \\ \textbf{s.t.} \quad & G(x,y) \le 0 \\ & \nabla_y \tilde{f}(x,y) = 0. \end{aligned} \tag{2.1.40}$$

Under the convexity assumption, it can be shown that there exists a strictly decreasing sequence $\{t^k\} \to 0$ for which the solution to (2.1.40) is the same as the original problem. The same method can be applied to solve a Min-Max problem [70].

The method in [2] not only considers constructing an interior barrier function $\Phi(g(x,y))$ of (2.1.38) for the lower-level problem but also $\Phi_2(G(x,y))$ for the upper level problem defined as:

$$\Phi_2(G(x,y)) \begin{cases} > 0 & \text{if } (x,y) \in INT\Gamma \\ \to \infty & \text{if } (x,y) \to BD\Gamma. \end{cases} \tag{2.1.41}$$

It also constructs an exterior barrier function $\Phi_1(||\nabla_y \tilde{f}(x,y,t)||)$ based on the follower's first-order optimality condition:

$$\Phi_1(||\nabla_y \tilde{f}(x,y,t)||)\begin{cases} = 0 & \text{if } (x,y) \in \left\{(x,y) \in \bigcup_{x \in R^N} x \times INT\Gamma(x) | \nabla_y \tilde{f}(x,y,t) = 0\right\} \\ > 0 & \text{if } (x,y) \in \left\{(x,y) \in \bigcup_{x \in R^N} x \times INT\Gamma(x) | \nabla_y \tilde{f}(x,y,t) \neq 0\right\} . \end{cases} \tag{2.1.42}$$

Then, the bilevel problem converts to a single level problem without constraints:

$$\min_{x,y} \tilde{F}(x,y,t,t_1,t_2) = F(y,x) + t\Phi(g(x,y)) + t_1\Phi_1(||\nabla_y \tilde{f}(x,y,t)||) + t_2\Phi_2(G(x,y)). \tag{2.1.43}$$

Here t, t_1 and $t_2 \geq 0$ are the penalty parameters. This penalty method effectively avoids the difficulty of dealing with the complementarity slackness conditions which arise when replacing the follower's problem with the corresponding optimality conditions. The only concern is ill-conditioning, which may be avoidable by using different initial points.

A double penalty method where both in the upper and lower level objective functions are penalized also can be applied to the NBP problem. Loridan and Morgan[55] first propose a double penalty function method for the **BP** with non-singleton reaction sets with pessimistic concerns. Instead, Ishizuka and Aiyoshi[41] prove that the method can be easily implemented for optimistic **NBP**. The basic principle is the same in both cases. They define the lower level constraints for (x, y) as

$$S_1 = \{(x,y) : x \in X, Y \in Y, g(x,y) \leq 0\},$$

and the upper level constraints combined with the optimal reaction of the lower level variable define IR as S_2:

$$S_2 = \{(x,y) : x \in X, Y \in \Re(x), g(x,y) \leq 0, G(x,y) \leq 0\}.$$

Then the nonnegative valued continuous penalty functions are:

$$\Phi_i(x,y)\begin{cases} = 0 & \text{if } (x,y)\in S_i, i=1,2 \\ > 0 & \text{otherwise.}\end{cases} \tag{2.1.44}$$

The algorithm also constructs a penalty function defined on the lower level feasible region:

$$\Phi_0(x,y)\begin{cases} = 0 & \text{if } y\in INT\Gamma(x) \\ \to +\infty & \text{if } y\to BD\Gamma(x).\end{cases} \tag{2.1.45}$$

With a positive penalty parameter, the augmented objective function for the lower level problem is

$$\tilde{f}(x,y;t) = f(x,y) + t\Phi_2(x,y) + \frac{1}{t}\Phi_0(y).$$

Additionally, an exterior penalty function for the upper level problem is constructed as:

$$\tilde{F}(x,y;t) = F(x,y) + t\Phi_2(x,y) + t\Phi_1(x,y).$$

For each t, the approximate **BP** problem is formulated as:

$$\begin{aligned} \min_{x,y} \quad & \tilde{F}(x,y;t) \\ \text{s.t.} \quad & x\in X, \\ & y\in \operatorname*{argmin}_{y'} \tilde{f}(x,y';t), \end{aligned} \tag{2.1.46}$$

The algorithm is similar to the one in [2].

Another penalty method is proposed by Shimizu and Lu in [72]. The optimal-value function of the lower level on the feasible region $\Gamma(x)=\{y : g(x,y)\le 0, y\in Y\}$ is

$$\bar{f}(x) = f(x,y^*) = \min_{y\in\Gamma(x)} f(x,y).$$

We set κ_1 as an upper bound for the lower level problem and get an equivalent problem with y replaced by an upper level decision variable $\tilde{y}$:

$$\begin{aligned}
\min_{x,\tilde{y},\kappa_1} \quad & F(x,\tilde{y}) \\
\text{s.t.} \quad & G(x,\tilde{y}) \leq 0 \\
& g(x,\tilde{y}) \leq 0 \\
& f(x,\tilde{y}) - \kappa_1 = 0 \\
& \bar{f}(x) - \kappa_1 = 0.
\end{aligned} \tag{2.1.47}$$

The exterior penalty method is applied:

$$\begin{aligned}
\min_{x,\tilde{y},\kappa_1} \quad & F(x,\tilde{y}) - t(f(x,\tilde{y}) + h(x) - 2\kappa_1) \\
\text{s.t.} \quad & G(x,\tilde{y}) \leq 0 \\
& g(x,\tilde{y}) \leq 0 \\
& f(x,\tilde{y}) - \kappa_1 \leq 0 \\
& \bar{f}(x) - \kappa_1 \leq 0.
\end{aligned} \tag{2.1.48}$$

By adding another artificial variable κ_2, (2.1.48) can be reformulated as

$$\begin{aligned}
\min_{x,\tilde{y},\kappa_1,\kappa_2} \quad & \kappa_2 - t\,[f(x,\tilde{y}) + h(x) - 2\kappa_1] \\
\text{s.t.} \quad & F(x,\tilde{y}) - \kappa_2 \leq 0 \\
& G(x,\tilde{y}) \leq 0 \\
& g(x,\tilde{y}) \leq 0 \\
& f(x,\tilde{y}) - \kappa_1 \leq 0 \\
& \bar{f}(x) - \kappa_1 \leq 0,
\end{aligned} \tag{2.1.49}$$

then solving (2.1.49) is equivalent to solving the original problem.

All of the above penalty methods are used for solving **NBP**. For large enough values of the penalty parameters, the methods attain local optima if the penalty function is convex and might get global solution.

2.1.3 Discrete Bilevel Problem Algorithm

Due to the difficulty of dealing with discrete variables, the amount of literature in the area of the mixed-integer bilevel problem is quite limited. Most discussions focus on the linear case and few exist in nonlinear cases with favorable problem structures.

To solve a **BLP** which has binary decision variables in both levels, Bard and Moore[10] use the previously discussed methodology of [7] and combine an active set method and a branch-and-bound method. The original bilevel problem is

$$\min_{x\in\{0,1\}} F(x,y) = C_x x + C_y y \tag{2.1.50}$$

with the lower level problem as:

$$\begin{aligned} \min_{y\in\{0,1\}} \quad & f(y) = c_y y \\ \text{s.t.} \quad & a_x x + a_y y \leq b. \end{aligned} \tag{2.1.51}$$

They reformulate and repeat solving the following subproblem with parametric constraints

$$\begin{aligned} \min_{(x,y)\in\{0,1\}} \quad & f(y) = c_y y \\ \text{s.t.} \quad & a_x x + a_y y \leq b \\ & F(x,y) = C_x x + C_y y \leq \kappa \\ & \sum_{j=1}^{n_1} x_j \geq l. \end{aligned} \tag{2.1.52}$$

The parameter κ is an upper bound for the objective function of the upper level problem, which is similar to a cut added to enforce a satisfactory result for the leader. Another right-hand-side parameter l restricts the number of x variables being set as 1, which offers a good rule to branch. The value of both variables get updated in each iteration. This subproblem means to find good points in the follower's rational reaction region according to a specific active set for the leader. The depth-first branch-and-bound and active set method are identical to those in [7].

Edmund and Bard [35] further develop a branch-and-bound method for a class of BLP with a discrete upper level problem and a convex nonlinear lower level problem. As in [34], they replace the lower level problem with its KKT conditions and integrate with the upper level problem to get a mixed-integer nonlinear single problem. Then they relax the problem by eliminating the complementarity slackness conditions and replacing the integer conditions with $0 \leq x \leq 1$. Based on the solution of the relaxed problem, they conduct a branch-and-bound search tree by finding the live node with the maximum complementarity or integrality violation. They also point out the potential of applying this method to solve DBP with a discrete lower level problem.

[45] and [44] model the gas shipper problem on a mixed integer bilevel problem. Then they solve the problem with a penalty function. Dempe and Kalashnikov [31] solve a mixed integer bilevel problem with integer variables located in the lower level problem by moving the integer variables to the upper level problem. Recent work [66] by Saharidis and Lerapetritou is based on Benders decomposition and solves a mixed integer **BLP**. In each iteration, a master problem and a subproblem are generated. The master problem relaxes the original problem by removing integer variables while the subproblem provides bounds given fixed values for some variables in the master problem. The master problem in the next iteration uses additional cuts produced according to Lagrangian information from the current subproblem.

2.2 Applications of Hierarchical Decision Making

The economic interpretation underlying hierarchical decision making implies that the upper level sends a signal to the lower level by setting up the price or allocating limited resources so that the lower level can adjust its strategy. Since the upper level problem usually has more power in the game system, government or other dominant players make decisions first. Instead, some subordinate sectors and other dominated players respond sequentially. Therefore, we mention the following applications with emphasis on the supply chain applications.

2.2.1 Agriculture Problems

Candeler and Norton [24] mention the model regarding the regulation of milk by the government in the Netherlands. Government decides a milk subsidy and duties on import in the upper level problem to maximize the overall benefits from the perspective of consumer, government and farms. The lower level problem deals with the revenue of the monopoly for producing milk, butter and cheeses from the perspective of farmers.

In [23], policies of Mexican agriculture are suggested to be examined such as subsidies on fertilizer use, subsidies on irrigation investment loans, support prices on wheat and corn and water taxes, as well, in an upper level problem of bilevel problem. Correspondingly, the outputs regarding employment, farm income, corn/wheat production and governmental expenses could be set to be optimized in the upper level problem. The response of Mexican agriculture is modeled in the lower level problem. Specifically, Candler et al. illustrate with an example of an irrigation problem concerned with water policy.

Fortuny-Amat and McCarl [39] discuss the case where the farmers can make decisions on fertilizer use in the lower level problem. The alternative combinations include whether or not fertilizer application equipment is loaned with the fertilizer, whether or not prices use FOB at the fertilizer plant or delivered to the farm. The fertilizer supplier decides the product price and production variations in the upper level problem.

Bard [8] first proposes a biofuel model as a bilevel problem. However, he does not consider any integer variables, which plays a key role in decision making for farms and government.

2.2.2 Resource Allocation Problems

This type of application problem addresses the issue of allocating limited resources to different uses to create maximum benefit. However, since various decision makers at different levels control various resources and have different profits, the objectives cannot be accumulated together directly and bilevel models are applied.

Bracken and McGill [19] first summarize several applications of bilevel problems in military applications: strategic offense or defense force structure optimization, strategic bomber force structure and basing optimization, optimization of weapon mix and targeting for attrition processes, strategic defense optimization to achieve post-attrack production capabilities and optimization of aircraft deployment and sortie allocation. The lower and upper level problems are assignment problems of resources for defensive and the offensive decision makers and minimizing the cost of the force, seperately, to satisfy their individual targets. Their work is extended by [22].

Bracken and McGill [20] also introduce the application of bilevel problems to balance the conflict of resouces needed for production and marketing. They illustrate with an example in the airline industry.

Anandalingam and Apprey [4] propose a model to deal with water conflicts in India and Bangladesh. Recent work by Smith et al. [73] points to renewed interest in this area for a new product development application.

2.2.3 Pricing Problems

Cote et al. [30] model price and fare optimization in the airline industry with a bilevel model. In the upper level problem, the leader airline decides the fare vector. At the lower level, passengers respond by choosing among different airline companies(the leader or the leader's competitor), different classes and different flows to minimize their costs.

2.2.4 Transportation Problems

BP appears frequently in network design problems. One typical problem is the toll setting problem, where the leader is the manager(the owner) of a highway, who plans the toll setting to maximize his profits. The travellers(the users of the hightway) decide their usage of the highway to minimize their costs, which consist of time cost, gas cost and toll fees ([57], [12],[13] and [53]). Ben-Ayed et al. [12, 13] discuss Tunisia's inter-regional highway by using a bilevel linear model, which is the first application of bilevel problem in transportation design systems. [53] proposes a model for a single product, and the model is extended to multiple commodities[21].

Dempe and Kalashnikov[31, 45, 44] present a discrete bilevel model to compute a cash-out penalty when the supply and the demand are not equal for a natural gas shipping company.

2.3 Literature Review for HDM Problems in Closed-loop Supply Chains

Fleischmann et al. [38], Guide et al. [27], Rubio et al. [65] and Pokharel and Mutha [60] provide thorough surveys of existing research involving remanufacturing. A significant portion of the work on product recovery addresses inventory control and related matters. In one of the earliest such works, Schrady [68] presents a deterministic model for repairable items and derives EOQ type fixed lot sizes for recovery and reorder. Richter [62, 63] studies a similar scenario and develops a different control approach utilizing the relationships between control parameters and the return rate. Schrady's model is extended to the multiple items case by Mabini et al., who suggests numerical solution methods. More recently, Koh et al. [50] propose a deterministic recovery model with a fixed return rate deriving optimal policies under limited remanufacturing capacity, and Tang and Teunter [77] have embellished the economic lot scheduling problem via the incorporation of returns. As opposed to these deterministic approaches, stochastic recovery models have also received a substantial amount of research attention. Cohen et al. [28] study the case where a fixed proportion of returns are received after the passage of a fixed amount of time, treating recoverable and serviceable inventories in an identical manner. Their work has been extended by Kelle and Silver [47, 46], who outline a purchasing policy under random returns. More recently, van der Laan et al. [81, 82] present a general model for remanufacturing and disposal involving four control parameters, leading to approximations for deriving the average cost of an (s, Q) type inventory model for remanufacturing. Subsequently, the differences between discounted cash flow vs. average cost based remanufacturing models have been examined in detail (Teunter and van der Laan, 2002 [76]; van der Laan, 1996 [83]; van der Laan et al., 2003 [80]). In conjunction with remanufacturing, issues concerning product disposal have also received a significant amount of research attention, although this is not a major concern here. Amongst other works, various types of remanufacturing systems with PUSH and PULL types of disposal strategies have been studied [48, 56].

The major thrust of the extant remanufacturing literature is on the timing and sizing decisions for manufacturing and remanufacturing activities, with primary attention on inventory related matters. Efforts towards integrating the decisions of inventory replenishment, product pricing and customer incentive for returning used items (in the form of a cash refund or a discount coupon) in a remanufacturing environment have been relatively rare. As a notable exception, in a recent study Savaskan et al. [69] have dealt with the questions of pricing and return incentives from a game theoretic perspective, in examining alternative reverse logistics structures for the collection of recoverable products. Bhattacharya et al. [14] conduct the integration of optimal order quantities in different channels, reflecting the various relationships among retailer, manufacturer and remanufacturer. Also, Vorasayan and Ryan [86] outline procedures for deriving the pricing and quantity decisions for refurbished products.

Inventory control decisions, which are intertwined with such questions, however, have been only superficially treated in the pricing related research. This study is an attempt to address this deficiency in the current body of work involving remanufacturing. We address some of the major issues concerning inventories, pricing, used product collection, materials procurement, product delivery and planning for manufacturing and remanufacturing in an integrated manner. Specifically, we develop procedures for developing such integrated policies towards achieving a well-coordinated supply chain, incorporating a lean production process. Our emphasis is on the mathematical modeling of product remanufacturing under a scenario involving a single retailer and a single manufacturer, dealing with a single recoverable product. Furthermore, as mentioned earlier, the models developed here attempt to establish an integrated policy, simultaneously specifying decisions concerning inventory replenishment at various stages of the supply chain, retail pricing, as well as the appropriate incentive level for inducing customer returns of used items, from the perspectives of, maximizing the profits of, respectively, the retailer, the manufacturer and the entire supply chain. For simplicity of analysis and implementation, we assume a deterministic environment. Furthermore, the product's demand and return rates are modeled as simple linear functions.

2.4 Literature Review for HDM Problems in Biofuel Supply Chains

The majority of the biofuel literature focuses on the political decisions and economic cost and benefit analysis of biofuel production according to the practice in different countries.

Rozakis et al. [64] use a partial-equilibrium model to discuss the biofuel cost of chains operating under different policy scenarios in France. The possibilities of biofuel cost reduction are analyzed. Monte Carlo simulation is implemented to deal with uncertainty, and minimal tax exemption levels for the viability of the activity are obtained. The study by Kruse et al. [52] uses a stochastic model to find out the effects of extension of the tax credits and import tariff on future growth in biofuels. The trade-off between maximizing farm income and minimizing goverment costs are balanced by setting up the appropriate tax credits and import tariff. Rajagopal and Zilberman [61] summarize the literature on the biofuel problem from the perspective of environment, economics and policy-setting.

Regarding the application of bilevel programming to the biofuel problem, the study by Candler et al. [23] points out the potentials and the difficulties. Bard et al. [11] formulate a leader-follower game that helps decision makers arrive at a rational policy for encouraging biofuel production. In this dissertation, we base our work on [11] and incorporate fixed costs in the farm's problem to further extend the model. Doing so requires that we include binary variables in the lower level, which greatly increases the computation complexity. However, the resulting model is more consistent with the practice. Fixed costs are a significant part of biofuel production costs, and therefore our models make a very important contribution to the field by incorporating it into the model. These costs can be related to equipment, transportation, or other ramp-up costs associated with switching to a new type of crop and/or harvesting methods.

2.5 Conclusions of Literature Review

Motivated by the real issues in supply chain management, this dissertation studies pricing, lot sizing in remanufacturing and biofuel production with hierarchical decision making models. In the remanufacturing system we investigate the price setting decision and the inventory strategy simultaneously. In the cooperative game, the manufacturer and the retailer can work together to avoid side-payment and double-marginalization. The hierarchical decision making model becomes a single problem with a multi-objective function if we assume an ideal case with a centralized planner. However, it might not always be the case in supply chain. According to asymmetrical information acquisition and power distributed in the different levels, noncooperative games/Stackelberg games are more common. Through comparing different strategies in different hierarchical decision frameworks, we exploit and study the decision making and the profits in this closed loop supply chain.

Meanwhile, we also look at the biofuel production model, in which the participants are the government, the industry and farms. As suppliers, the farms have less power and only respond to the government's decision. Thus, we develop models as noncooperative Stackelberg games. The government chooses policy decision variables, and the farmers decide the behavior strategy. Our efforts complement this work by providing a methodology for solving a mixed integer linear **BP**. Due to the special structure of the problem, we propose two methods. The first method is based on a derivative-free search method. Since there are several "jumps" due to the inclusion of integer variables, it is too difficult for a gradient-based method to obtain a global optimal solution. However, we still exploit the gradient techniques locally by applying sensitivity analysis information to speed the computation. Another method proposed is based on a nonlinear algorithm. We consider a way to calculate the derivative for the objective function in the upper level with respect to the decision variable belonging to that level. The chain rule is used to incorporate the gradient information, which is obtained through solving the lower level problem at any given x directly.

3. Hierarchical Decision Making Problems in Closed-loop Supply Chains

3.1 Background

Under intense competitive pressure to lower production costs, coupled with increasing environmental concerns, more and more manufacturers today are focusing on possibilities of product recovery and reuse. Practices of waste paper and scrap metal recycling, reuse of containers, and, more recently, recovery of electronic components have given rise to the notion of environment friendly, or "green" manufacturing. The Xerox Green World Alliance remanufactured and recycled more than 90%, 25% and 100%, respectively, of remanufactured print cartridges, new toners and plastic parts. This program has led to significant environmental and financial benefits for Xerox. First, it has prevented 128 million pounds of waste materials from entering landfills. Secondly, the savings in terms of energy have been estimated to be $320,000$ megawatt-hours in 2005, due to reuse of parts, in addition to those resulting from the associated reduction in materials purchasing costs [89].

The efficient incorporation of used products and/or materials considerations into manufacturing processes and supply chain contracts have proven to be important from the standpoint of "waste free" goals with "sustainable initiatives". One issue a manufacturer faces involves the process of collecting the returns. In such a reverse channel structure, our attention is confined to the case of customer returns at the retail level, which is cited to be the most effective method for used products collection according to Savaskan et al. [69]. This is a common practice for items such as disposable cameras and mobile phones, where manufacturers utilize retailers for collecting the used products. For instance, the Eastman Kodak Company receives returned single-use cameras from large retailers who also develop film for customers. On the average, 76% of the weight of a disposed camera is reused in the production of a new one (Savaskan et al., 2004[69]). Generally, retailers are responsible for providing incentives to customers and finally are reimbursed by the manufacturer to ensure that end-of-life products are returned for remanufacture in a timely manner. The various pricing decisions made in this context significantly affect the profitability of both stages of the supply chain.

Another concern for the manufacturer, as well as the retailer pertains to inventory issues. Economies of scale may dictate that manufacturers collect and take back returns at periodic intervals, requiring retailers to have storage space for holding the products returned by customers. By the same token, manufacturers also need to allocate storage space for such items. Needless to say that a product's retail price, customer incentive for returns (both determined by the retailer), as well as the transfer price paid by the producer to the retailer for collecting returns are likely to shape the inventory policies for returned items at both the retailer's and the manufacturer's ends. This study is an attempt to examine these issues from an integrated supply chain perspective.

3.2 Assumptions

The supply chain under study consists of a single retailer and a single manufacturer involved in the production and sale of a single recoverable product. Customers are refunded a part of the purchase price by the retailer as an incentive to return used products, which can be restored to "as new" condition for resale through a remanufacturing process deployed by the manufacturer. The manufacturing/remanufacturing environment is a batch production system where each batch of the product may consist of a mix of remanufactured and new manufactured items within a single setup. The used items, after cleaning, restoration, etc. are completely reincorporated in the existing production process, so that remanufacturing and new product manufacturing rates are the same, although their variable costs may differ.

For coordination purposes, the lot-for-lot policy is in effect for input materials ordering, manufacturing and remanufacturing, product delivery and retail inventory replenishment, with a common cycle time of T. In other words, the necessary input materials procurement, production (including remanufacture), delivery and retail stock replenishment cycles are one and the same. This lot-for-lot feature is commonly found in JIT based lean manufacturing systems, where minimal levels of material and product inventories are desired.

All input materials for manufacturing or remanufacturing are treated as a composite bundle. In each case, the total bundle of inputs necessary for producing (or remanufacturing) a unit of the end product is defined as a "unit". All of the input materials (for manufacturing and remanufacturing) are ordered on a lot-for-lot basis with a single procurement order prior to the setup of a batch.

The retailer is responsible for collecting returned items and holding them in inventory until picked up by the producer. In our decentralized models, the manufacturer pays the retailer a unit transfer price for the returned items, in order to induce the latter to engage in the collection activity. Without loss of generality, it is assumed that the retailer's cost of this collection effort is negligibly small, although the cost of holding the returned products in inventory at the retail level is taken into account. Under the centralized scenario, the used product transfer price and the producer's wholesale price become irrelevant for avoiding double marginalization. In the decentralized models, the retailer sets the item's selling price and the unit reimbursement to customers for returns. The wholesale price, where applicable, is the same for new or remanufactured items.

We assume that the market demand, the customer return rate and all lead times are deterministic. Thus, a production batch of Q units consists of $Q-X$ new items and X units of remanufactured product as shown in Figure (C.1), which depicts the process flow schema of the supply chain under consideration. Figure (C.2) shows the various inventory-time plots at the retail and manufacturing facilities. Without loss of generality, these plots are constructed with the assumption that the setup and transit times, as well as the cleaning and refurbishment times for the recovered items are zero. Before setting up a production batch, the X units of returned items collected during the cycle are transported back to the plant for remanufacturing. Therefore, the value of the quantity $Q - X$ is known prior to each setup. After completion of the manufacturing and remanufacturing process, the replenishment lot of Q is delivered to the retailer for sale. All transportation costs are paid by the producer.

Consistent with classical microeconomic theory, we model the retail demand rate, d, as a decreasing function of its selling price, p_s, i.e. $d = A - Bp_s$. Furthermore, the product's return rate, x, and the total units returned, X, during a cycle are expressed, respectively, as $x = ar_c - bp_s$ and $X = Tx = Qx/d$. The parameters A, B, a, and b are known, or can be estimated empirically. It is reasonable to assume that the average rate of used product returns is likely to increase as the return incentive, r_c, as well as the overall demand level, d, increase (or, alternately, as the retail price decreases). Furthermore, as mentioned earlier, we adopt linear structures for both d and x for simplicity of analysis and implementation.

3.3 Profit Analyses

3.3.1 The Retailer's Profit

The retailer has two sources of revenue, captured by the first two terms in the below profit function. The first of these represents the revenue from the sales of new products and the second term expresses the net revenue, through reimbursements from the manufacturer, for collecting the used items. The next term represents the average ordering cost and the remaining two terms show, respectively, the costs of holding new product and returned item inventories per time unit at the retailer's end (see Figures (C.2(a) and (b))). Its profit per time unit can be expressed as

$$\Pi_r = (p_s - p_w)d + (r_m - r_c)x - S_r\frac{d}{Q} - h_r\frac{Q}{2} - h_{rr}\frac{Qx}{2d}. \qquad (3.3.1)$$

Substituting $x = ar_c - bp_s$ and $d = A - Bp_s$ into (3.3.1), the retailer's average profit per time unit can be rewritten as

$$\Pi_r = (p_s - p_w)(A - Bp_s) + (r_m - r_c)(ar_c - bp_s) - S_r(\frac{A - Bp_s}{Q}) - \frac{Q}{2}\left[h_r + h_{rr}\frac{(ar_c - bp_s)}{(A - Bp_s)}\right]. \tag{3.3.2}$$

3.3.2 The Manufacturer's Profit

Thus, in order to develop the manufacturer's profit function, we need to determine the average inventories at the manufacturing facility. From Figure (C.2(c)), the average inventory of the finished product at the manufacturer's end is

$$(\frac{Q}{2})(\frac{Q}{m})/(\frac{Q}{d}) = \frac{Qd}{2m}.$$

Also, from Figure (C.2(d)) it can be shown that the average inventories of the input materials necessary for remanufacturing and manufacturing purposes, respectively

$$\frac{x^2 Q}{2md} + \frac{Q}{2m}[d - 2x + x^2/d].$$

Incorporating these results, the profit per time unit for the manufacturer can be expressed as

$$\begin{aligned}\Pi_m =& (p_w - c_s)d - \frac{d}{Q}(S_m + S_{rm} + S_i) - (r_m + c_r)x \\ & - \frac{h_m Q d}{2m} - \frac{h_{ir} Q x^2}{2md} - \frac{h_i Q}{2m}(d - 2x + \frac{x^2}{d}) - c_m(d - x) - c_{rm}x.\end{aligned} \tag{3.3.3}$$

The first term in (3.3.3) shows the manufacturer's revenue based on the wholesale price, less the variable shipping cost to the retailer. The second term includes the fixed costs involving production set up, transportation of new products to and used items from the retailer and ordering of input raw materials. The third term expresses the reimbursement cost to retailer, as well as the variable transportation, cleaning and preparation costs for the returned items. The next three terms represent the holding costs, respectively, for the finished product and input materials inventories necessary for remanufacturing and manufacturing. The final two terms in (3.3.3) are the variable costs per time unit for manufacturing and remanufacturing, respectively. Substituting for d and x into (3.3.3), and collecting terms, the manufacturer's profit per time unit is rewritten as follows:

$$\begin{aligned} \Pi_m =&(p_w - c_s - c_m)(A - Bp_s) - \frac{A - Bp_s}{Q}(S_m + S_{rm} + S_i) \\ &- (r_m + c_r + c_{rm} - c_m)(ar_c - bp_s) \\ &- \frac{Q}{2m}\left[h_m(A - Bp_s) + \frac{h_{ir}(ar_c - bp_s)^2}{(A - Bp_s)}\right] \\ &- \frac{Q}{2m}h_i\left[A - Bp_s - 2(ar_c - bp_s) + \frac{(ar_c - bp_s)^2}{A - Bp_s}\right]. \end{aligned} \tag{3.3.4}$$

3.3.3 The Supply Chain's Profit

Suppose that the retailer and the manufacturer agree to cooperate towards formulating a jointly optimal integrated policy for the supply chain as a whole. The focus of such a centralized policy, where both parties are willing to freely share their cost and other relevant information, is to maximize the profitability of the entire system, rather than that of either party. In this centralized approach, we propose that in order to avoid double marginalization, the parameters wholesale price p_w and manufacturer's rebate for returned items rm need not be considered and are omitted. Thus, combining (3.3.1) and (3.3.4), without an explicit wholesale price and a direct manufacturer's reimbursement to the retailer for product returns, the total supply chain profit is

$$\begin{aligned}\Pi_s =&(p_w - c_s - c_m)(A - Bp_s) - \frac{A - Bp_s}{Q}(S_m + S_{rm} + S_i + S_r) \\ &- (r_c + c_r + c_{rm} - c_m)(ar_c - bp_s) \\ &- \frac{Q}{2m}\left\{m\left[h_r + h_{rr}\frac{ar_c - bp_s}{A - Bp_s}\right] + (h_{ir} + h_i)\left[\frac{(ar_c - bp_s)^2}{(A - Bp_s)}\right]\right\} \\ &- \frac{Q}{2m}\left\{(h_m + h_i)(A - Bp_s) - 2h_i(ar_c - bp_s)\right\}.\end{aligned} \tag{3.3.5}$$

3.4 Development of Hierarchical Decision Making Models and Analysis

3.4.1 Retailer Controlled Model with p_w Given

In some industries, due to intense competition, the wholesale price for the manufacturer is determined by the existing market conditions and is, consequently, treated as a constant parameter. The exposition in this subsection pertains to such cases.

Suppose that the retailer can set the integrated optimal order quantity and pricing policy for sales and returns, independent of the manufacturer, with the assumption that it wields sufficient power as a dominant member of the supply chain. The system is described by the following model:

$$\max_{Q, r_c, p_s} \Pi_r. \tag{3.4.1}$$

In such a case, subject to certain assumptions to ensure joint-concavity, the necessary condition for optimality could be applied to calculate the optimal decision from retailer.

The first order optimality conditions are shown below, obtained by setting $\frac{\partial \Pi_r}{\partial Q}$, $\frac{\partial \Pi_r}{\partial r_c}$ and $\frac{\partial \Pi_r}{\partial p_s}$, respectively, equal to 0; i.e.

$$Q = \sqrt{\frac{2S_r(A - Bp_s)}{h_r + h_{rr}(ar_c - bp_s)/(A - Bp_s)}} \tag{3.4.2}$$

$$r_c = \frac{1}{2}\left[r_m + \frac{b}{a}p_s - \frac{Qh_{rr}}{2(A - Bp_s)}\right] \tag{3.4.3}$$

$$(A-Bp_s)^2[B(p_w-2p_s)+A-b(r_m-r_c)+S_r\frac{B}{Q}]-\frac{Q}{2}h_{rr}[B(ar_c-bp_s)-b(A-Bp_s)] = 0. \tag{3.4.4}$$

The above conditions (3.4.2), (3.4.3) and (3.4.4) can be solved simultaneously by any standard equation solving software, in order to obtain Q, r_c, and p_s . Since the return rate cannot exceed the demand rate of the item, i.e. $d \geq x$, we can then easily show that any feasible solution must satisfy the requirement: $p_s \leq (A - ar_c)/(B - b)$. Hence, the roots of p_s in equation (3.4.4) that are negative or violate this feasibility condition are disregarded in this and subsequent models for computational purposes.

Proposition 3.4.1. *Q, r_c and p_s obtained from (3.4.2), (3.4.3) and (3.4.4) represent the global optimum if the following conditions are satisfied:*

a)

$$\frac{d}{Q} \geq (\frac{ah_{rr}^2}{16S_r})^{1/3}$$

b)

$$\frac{ah_{rr}^2(Ab - aBr_c)(Ab - aBr_c - bd)}{2d^4} + \frac{aBh_{rr}[4S_r(Ab - aBr_c) + Q^2a - 2dbS_r]}{2Q^2d^2}$$
$$+ \frac{2aS_r(B^2S_r - 4BdQ + b^2dQ)}{Q^4} \leq 0$$

A proof of the above proposition is provided in the Appendix A.

3.4.2 Manufacturer Controlled Model with p_w Given

If the manufacturer, instead of the retailer, is in a position of dictating supply policy, it would prefer to implement a production and delivery policy (assuming the lot-for-lot operating framework) that is optimal from its own perspective. In this case, the supplier's wholesale price is treated as a given parameter. The retailer, nevertheless, is likely to be free to set its own selling price and the level of incentive to induce customers to return the used products, given the manufacturer's preferred replenishment lot size.

Note that the item's selling price, p_s, and the customer return reimbursement, r_c are set by the retailer. Then the manufacturer's optimal batch size and the consequent retailer's policy variable values are established by the scheme of a sequential game, denoted as:

$$\begin{aligned} \max_{Q} \quad & \Pi_m \\ \text{s.t.} \quad & (r_c, p_s) \in \left\{ \operatorname*{argmax}_{r_c, p_s} \Pi_r \right\} \end{aligned} \tag{3.4.5}$$

which can be solved by combining the optimality conditions (3.4.3) and (3.4.4) resulting from the retailer's problem as constraints with the optimality condition with respect to the order quantity in the upper level problem. The set of equations (3.4.6) shown below

$$Q = \sqrt{\frac{2m(A - Bp_s)(S_m + S_{rm} + S_i)}{h_m(A - Bp_s) + \frac{h_{ir}(ar_c - bp_s)^2}{A - Bp_s} + h_i \left[A - (B - 2b)p_s - 2ar_c + \frac{(ar_c - bp_s)^2}{A - Bp_s}\right]}} \tag{3.4.6}$$

$$r_c = \frac{1}{2}\left[r_m + \frac{b}{a}p_s - \frac{Qh_{rr}}{2(A - Bp_s)}\right]$$

$$(A - Bp_s)^2 \left[B(p_w - 2p_s) + A - b(r_m - r_c) + S_r \frac{B}{Q}\right] - \frac{Q}{2} h_{rr} \left[B(ar_c - bp_s) - b(A - Bp_s)\right] = 0$$

can be solved simultaneously. Neither the retailer nor the manufacturer would benefit from any deviation from the optimal solution above, which represents the equilibrium state of the whole system.

Proposition 3.4.2. *The manufacturer would adopt a remanufacturing strategy only when the condition:*

$$r_m + c_r + c_{rm} \leq c_m - \frac{Q}{2m}\left[\frac{h_{ir}x}{d} + h_i(\frac{x}{d} - 2)\right]$$

is satisfied.

(see the Appendix A for proof).

3.4.3 Manufacturer Controlled Model with p_w Unknown

Under monopolistic market conditions, manufacturers may lower the wholesale price in order to encourage retailers to increase their order quantities. As discussed before, under a decentralized policy, the retailer determines its order quantity as one of the decision variables along with the selling price and the customer return incentive. It will make these decisions after the observation of a wholesale price set by the manufacturer. Initially, the manufacturer would anticipate the optimal response from the retailer when it decides on the wholesale price, resulting in the following model:

$$\begin{aligned} \max_{p_w} \quad & \Pi_m \\ \text{s.t.} \quad & (r_c, p_s, Q) \in \left\{ \underset{r_c, p_s, Q}{\operatorname{argmax}} \, \Pi_r \right\} \end{aligned} \tag{3.4.7}$$

As before, the optimality condition of the lower level problem (3.4.2)-(3.4.4) could be combined with the upper level problem as

$$\begin{aligned} \max_{p_w} \Pi_m = {} & (p_w - c_s - c_m)(A - Bp_s) - \frac{A - Bp_s}{Q}(S_m + S_{rm} + S_i) \\ & - (r_m + c_r + c_{rm} - c_m)(ar_c - bp_s) \\ & - \frac{Q}{2m}\left\{h_m(A - Bp_s) + \frac{h_{ir}(ar_c - bp_s)^2}{(A - Bp_s)}\right\} \\ & - \frac{Q}{2m}h_i\left[A - Bp_s - 2(ar_c - bp_s) + \frac{(ar_c - bp_s)^2}{A - Bp_s}\right] \end{aligned} \tag{3.4.8}$$

subject to

$$Q = \sqrt{\frac{2S_r(A - Bp_s)}{h_r + h_{rr}(ar_c - bp_s)/(A - Bp_s)}}$$

$$r_c = \frac{1}{2}\left[r_m + \frac{b}{a}p_s - \frac{Qh_{rr}}{2(A - Bp_s)}\right]$$

$$(A - Bp_s)^2[B(p_w - 2p_s) + A - b(r_m - r_c) + S_r\frac{B}{Q}] - \frac{Q}{2}h_{rr}[B(ar_c - bp_s) - b(A - Bp_s)] = 0$$

If the manufacturer, instead of the retailer, has control of the order quantity, the model above may be written as a bilevel problem, as shown below:

$$\begin{aligned} \max_{Q,p_w} \quad & \Pi_m \\ \text{s.t.} \quad & (r_c, p_s) \in \left\{\operatorname*{argmax}_{r_c,p_s} \Pi_r\right\}, \end{aligned} \tag{3.4.9}$$

which, as before, is the same as:

$$\begin{aligned} \max_{p_w,Q} \Pi_m = {} & (p_w - c_s - c_m)(A - Bp_s) - \frac{A - Bp_s}{Q}(S_m + S_{rm} + S_i) \\ & - (r_m + c_r + c_{rm} - c_m)(ar_c - bp_s) \\ & - \frac{Q}{2m}\left[h_m(A - Bp_s) + \frac{h_{ir}(ar_c - bp_s)^2}{(A - Bp_s)}\right] \\ & - \frac{Q}{2m}h_i\left[A - Bp_s - 2(ar_c - bp_s) + \frac{(ar_c - bp_s)^2}{A - Bp_s}\right] \end{aligned} \tag{3.4.10}$$

subject to:

$$r_c = \frac{1}{2}\left[r_m + \frac{b}{a}p_s - \frac{Qh_{rr}}{2(A - Bp_s)}\right]$$

$$(A - Bp_s)^2\left[B(p_w - 2p_s) + A - b(r_m - r_c) + S_r\frac{B}{Q}\right] - \frac{Q}{2}h_{rr}\left[B(ar_c - bp_s) - b(A - Bp_s)\right] = 0.$$

This constrained nonlinear problem may be solved by one of several widely available optimization software packages, such as “fmincon” in MATLAB.

3.4.4 Centralized Model for Supply Chain Optimality

Suppose that the retailer and the manufacturer agree to cooperate towards formulating a jointly optimal integrated policy, involving inventory replenishment, retail pricing and customer return reimbursement decisions, for the supply chain as a whole. The focus of such a centralized policy, where both parties are willing to freely share their cost and other relevant information, is to maximize the profitability of the entire system, rather than that of either party. We illustrate in the next section that this centralized joint optimization approach can be economically attractive from the standpoint of both the parties through an equitable profit sharing methodology. In this centralized approach, we propose that in order to avoid double marginalization, the parameters wholesale price p_w and manufacturer's rebate for returned items r_m need not be considered and are omitted. With the deduction in (3.3.5), the first order optimality conditions of the total supply chain profit yield the optimal values of the replenishment lot size, Q, unit customer reimbursement for returns, r_c and the unit selling price, p_s, which maximize the total supply chain profit under the proposed centralized policy, as shown below:

$$Q = \sqrt{\frac{2m(A - Bp_s)(S_r + S_m + S_{rm} + S_i)}{TH}}, \tag{3.4.11}$$

where

$$\begin{aligned} TH =& m\left[h_r + h_{rr}\frac{(ar_c - bp_s)}{(A - Bp_s)}\right] + (h_{ir} + h_i)\left[\frac{(ar_c - bp_s)^2}{(A - Bp_s)}\right] \\ &+ (h_m + h_i)(A - Bp_s) - 2h_i(ar_c - bp_s) \end{aligned} \tag{3.4.12}$$

$$r_c = \frac{[(h_{ir} + h_i)bp_s + 2h_i d - h_{rr}m] - \frac{2m(A-Bp_s)}{Q}(c_r + c_{rm} - c_m - \frac{a}{b}p_s)}{2a(h_{ir} + h_i) + \frac{4m(A-Bp_s)}{Q}} \tag{3.4.13}$$

$$
\begin{aligned}
&(A - Bp_s)^2 \left[A - B(2p_s - c_m - c_s) + b(r_c + c_r + c_{rm} - c_m) + \frac{B}{Q}(S_r + S_m + S_{rm} + S_i) \right] \\
&- \frac{Q}{2m} \left\{ (h_{ir} + h_i) \left[B(ar_c - bp_s)^2 - 2b(ar_c - bp_s)(A - Bp_s) \right] \right\} \\
&- \frac{Q}{2m} \left\{ h_{rr} m(aBr_c - bA) - Bh_m(A - Bp_s)^2 + h_i(2b - B)(A - Bp_s)^2 \right\} = 0.
\end{aligned}
\tag{3.4.14}
$$

Once again, conditions (3.4.11), (3.4.13) and (3.4.14) can be solved via any appropriate equation solving software, such as fmincon in MATLAB, for determining the centrally controlled inventory replenishment, retail pricing and return reimbursement decisions.

Proposition 3.4.3. *Q, r_c and p_s obtained from solving (3.4.11), (3.4.13) and (3.4.14) are globally optimal if the following conditions, in addition to conditions (a) and (b) under Proposition (3.4.1) are satisfied:*

(c)

$$
\begin{aligned}
&\frac{4(S_m + S_{rm} + S_i)(db - xB)^2(h_{ir} - h_i)}{mQ^2d^2} \\
&- \left[-\frac{B(S_m + S_{rm} + S_i)}{Q^2} + \frac{Bh_m - h_i(2b - B)}{2m} + \frac{(2db - Bx)x(h_{ir} - h_i)}{2md^2} \right]^2 \geq 0
\end{aligned}
$$

(d)

$$\frac{a^2B^2Q(h_{ir}-h_i)(h_i+h_m)^2}{4m^3d} - \frac{a^2B^2Qxh_i(h_{ir}-h_i)(h_i-h_m)}{m^3d^2}$$
$$-\frac{a^2BQx^2(h_{ir}-h_i)(Bh_{ir}h_i-6bh_{ir}h_i+Bh_{ir}h_m-5Bh_i^2+6bh_i^2-Bh_ih_m)}{4m^3d^3}$$
$$-\frac{a^2B^2Qx^3h_i(h_{ir}-h_i)^2}{m^3d^4} + \frac{a^2B^2Qx^4(h_{ir}-h_i)^3}{4m^3d^5}$$
$$+\frac{a^2B^2(h_{ir}-h_i)(S_m+S_{rm}+S_i)^2}{Q^3md} - \frac{a^2B^2(h_{ir}-h_i)(h_i-h_m)(S_m+S_{rm}+S_i)}{Qm^2d}$$
$$+\frac{2a^2B^2xh_i(h_{ir}-h_i)(S_m+S_{rm}+S_i)}{Qm^2d^2} - \frac{a^2B^2x^2(h_{ir}-h_i)^2(S_m+S_{rm}+S_i)}{Qm^2d^3} \leq 0$$

(see the Appendix A for proof).

3.5 A Numerical Illustration and Discussion

To illustrate our models outlined above, a numerical example is provided below. The following information pertaining to the two parties in the supply chain are available.

Retailer:

h_r = \$0.015/unit/day,	h_{rr} = \$ 0.002/unit/day,
$A = 120$,	$B = 3.0$
$a = 15$,	$b = 0.1$
S_r = \$50/order.	

That is, the daily demand rate is $d = 120 - 3p_s$ and the daily return rate is $x = 15r_c - 0.1p_s$.

Manufacturer:

S_m = \$300/batch,	S_{rm} = \$200/batch,	S_i =\$30/batch
h_m =\$0.01/unit/day,	h_i =\$0.009/unit/day,	h_{ir} =\$0.007/unit/day
c_s =\$2/unit,	c_m = \$8/unit,	c_{rm} = \$2/unit
c_r =\$1.20/unit,	p_w = \$20/unit,	r_m = \$2.80/unit
m =100 units/day.		

All the results obtained from the various perspectives are summarized in Table (B.1,B.2). It can be easily verified that the chosen parameters satisfy the joint concavity conditions (a), (b), (c) and (d). Hence, all the solutions shown in Table (B.1,B.2) are globally optimal.

From this table it is clear that if the retailer has sufficient policy implementation power in the supply chain, it attempts to keep the replenishment lot size comparatively small (i.e. 428.46 units), in view of its relatively low fixed ordering cost. Furthermore, through its retail pricing (p_s = \$30.032/unit), in conjunction with a customer return reimbursement price of \$1.485/unit, it prefers to achieve daily market demand and customer return rates of 29.904 and 19.391 units, respectively, that attempt to balance the gains from sales and returns against the ordering and inventory carrying (for both new and used items) costs. The maximum attainable daily profit for the retailer is, thus, \$318.361, resulting in a profit of \$299.918/day for the manufacturer. Note that as every unit of the returned product represents a net gain of \$1.315 (i.e. the difference between the amount, r_m, compensated by the manufacturer and the customer reimbursement, r_c) for the retailer, it attempts to achieve a relatively high used item return rate of about 64%.

If, on the other hand, the manufacturer is in a position to exert a greater level of negotiating power in the supply chain, its individual optimal policy would dictate a significantly larger replenishment batch of 6083.1 units, due to the relatively high fixed setup and transportation costs. In spite of a more than six-fold increase in the lot size, however, the selling price and return reimbursement, set by the retailer in response, are both only slightly lower than their values under its own optimal policy, i.e. \$29.90 and \$1.40 per unit, respectively. It is interesting to note that, consequently, the retail demand rate increases slightly to 30.200

units/day and the average product returns decline slightly to 18.00 units/day. The returns, however, now decline slightly to 59.6% of sales. Not unexpectedly, implementing the manufacturer's optimal replenishment policy reduces the retailer's profit to \$275.698/day, whereas the manufacturer's profit increases to \$377.596/day. Nevertheless, in terms of total supply chain profitability, the difference between adopting any one party's optimal policy over the other's amounts to only about 5.36%.

Table (B.1) and Table (B.2) show that if the retailer and the manufacturer decide to cooperate through the sharing of necessary information and adopt a jointly optimal policy that maximizes the total supply chain profit, instead of optimizing either party's position, both parties stand to gain considerably from such an approach. As mentioned earlier, the centralized model attempts to avoid double marginalization, i.e. the manufacturer does not explicitly charge the retailer a wholesale price, nor does it explicitly offer the latter a reimbursement for collecting the returns (implying that $p_w = r_m = 0$). Without these cost factors, the centrally controlled approach results in a maximum supply chain profit of \$ 830.78/day, representing a more than 34.4% improvement in total system profitability, compared to the retailer's optimal policy, or over 27.2% improvement vis-a-vis the manufacturer's optimal policy. As expected, the jointly optimal replenishment quantity now is 1562.4 units, which is less than the manufacturer's optimal batch size, but larger than the retailer's optimal order quantity. More interestingly, the retail price is reduced to \$23.3/unit and the return reimbursement is decreased to \$ 0.5/unit, respectively, resulting in a considerably larger demand rate of 50.2 units/day, as well as a smaller average product return rate of 5.5 units/day (i.e. about 10.96% of items sold are returned by customers). The implication of our centralized model is that under a jointly optimal policy, relatively fewer products sold are remanufactured items. Under the given set of problem parameters, it appears desirable to increase the overall market demand through a lower retail price. Also, there is a lesser emphasis on collecting customer returns for remanufacturing. The centralized model reduces the incentive for customer returns, which maximizes the total supply chain profitability.

The absence of a wholesale price and an explicit incentive for the retailer to collect returned items raises some interesting questions concerning a fair and equitable sharing of the total gain resulting from the centralized cooperative policy shown in Table(B.1) and Table(B.2). Although this can be achieved in several possible ways, we propose a profit sharing plan under a scenario where the retailer is the more powerful member of the supply chain and can dictate the implementation of its own optimal policy. The task of the manufacturer is then to offer sufficient incentive to the retailer in order for the latter to adopt the results of this procedure. Note that under its own individual optimal policy, the retailer's share is 51.5% of the total profit for both parties. Therefore, it would be reasonable if the retailer is allocated the same percentage of the total supply chain profit of \$ 830.78/day yielded by the centralized model. In other words, the retailer's share of the total profit is \$ 427.78/day and that of the manufacturer is \$ 402.999/day. With this profit sharing arrangement, each party's daily profit is more than 30% larger than that achieved under the retailer's optimal policy. Thus, it is economically attractive for both parties to adopt the jointly optimal policy yielded by our centralized model. If the manufacturer is more powerful of the two parties, the terms of a corresponding profit sharing arrangement, can also be derived easily along similar lines.

Finally, Table(B.1) and Table(B.2) also show the results for the decentralized models where under monopolistic competition, the manufacturer can set its wholesale price, which is now treated as a decision variable. Compared with the results for a given wholesale price, the retailer's individual optimal policy dictates increasing both the selling price from \$30.032/unit to \$31.680/unit and the customer return reimbursement from \$ 1.485/unit to \$ 1.498/unit. Consequently, the order quantity is reduced from 428.46 units to 388.392 units. These changes indicate that the retailer would expend less effort to increase market demand and would tend to compensate by attempting to increase its revenue from returns. This appears to be a rational response to a higher wholesale price. Also, as expected, the manufacturer's share of the total supply chain profit now increases from 48.5% to 61.11%, while, the total profits for the supply chain declines to $586.83/day. These effects, not unexpectedly, tend to be magnified when the supplier is in a position to dictate the adoption of its own optimal policy by the retailer. Now the total supply chain profit shrinks further to $512.792/day, although the manufacturer's relative share of this, as well as its own daily profit go up substantially, albeit at the expense of the retailer.

3.6 Sensitivity Analysis

Based on the numerical example cited above, we conduct a focusedsensitivity analysis to explore the effect of varying some selected parameters on system performance and the decision variables, e.g., the total supply chain profit and the retailer's share of it. We vary each of the selected parameters in the range of 60% to 140% of their original values cited in section 3.5 and record the percentage changes in the objective function value and decision variables of interest. The results outlined below are about only those parameters that significantly affect the selected target variables. "Significant" is defined as a larger than ±10% impact. Finally, the economic insights underlying the analyses are highlighted.

3.6.1 Selling Price

As can be seen in the Figures (C.3) through (C.6), the major factors which affect the selling price of the new product are the parameters A and B in the demand function. The effects of varying these two variables are similar in all the four situations examined, regardless of whether the decision-making is decentralized or centralized or whether the wholesale price is exogenous or endogenous. The selling price is an increasing function of the relative market share (A) while it is a decreasing function of the elasticity of the demand(B). As can be expected, the larger the market share, the higher the price the retailer can set. Also if the product demand is relatively price-sensitive, the retailer tends to set a lower price to increase demand, leading to higher profitability. However, if the new product is less price sensitive, there appears to be some sacrifice in the profit margin in order to increase profit. In both the retailer controlled and the manufacturer controlled situation, as observed in Figures (C.4) and (C.5), the wholesale price has a very strong impact on the decision of setting the retail price. A higher wholesale price tends to lead to a higher retail price. Since a higher wholesale price results in higher costs for the retailer, a higher selling price seems to be necessary on its part.

3.6.2 Reimbursement Price

With respect to the reimbursement level for the returned product, the centralized decision maker is only concerned with the market share, A, and the production cost, c_m, (refer to Figure (C.7)). If the market share is sufficiently high, the retailer has little incentive to increase the reimbursement price. In other words, the reimbursement price appears to be quite stable if the retailer enjoys a relatively large market share. Nevertheless, as its market share declines, the retailer tends to lack interest in collecting the returned products. Consequently, the reimbursement price will be reduced quickly. Similar effects of the market size on the reimbursement price can be found in the decentralized, the retailer controlled and the manufacturer controlled situations.

Under decentralized decision making(refer to Figure (C.8) through (C.10), the reimbursement, r_m, the retailer obtains from the manufacturer for collecting the returns determines the reimbursement price, r_c, set by the former to induce customers to return the used product. The positive relationship between these decision variables reflects the source of creating incentives for obtaining end-user returns in the closed-loop supply chain.

Figure (C.10) indicates that if the wholesale price is not given exogenously in the manufacturer controlled situation, the coefficient, a, of the return price in the return rate function, $x = ar_c - bp_s$, has a slightly negative impact on the reimbursement price. Since the coefficient, a, along with the reimbursement price, r_c, decide the return rate, the special relationship observed here illustrates that the customer return rate is likely to remain relatively stable in the manufacturer controlled scenario.

3.6.3 Order Quantity

The analysis of the effect of varying individual model parameters on the order quantity is more complex. As can be seen in Figure (C.11) through (C.14), the parameters which have a significant influence on the order quantity differ remarkably depending on the operating environment. As before, the relative market share measure, A, appears to have a major impact on the order quantity under all cases examined. The parameter A has positive relationships with the order quantity in the centralized, as well as the retailer controlled situations, whereas, this relationship exhibits a curvature and a negative relationship in the manufacturer controlled situation. This observation appears to be somewhat counterintuitive. Under centralized control (refer to Figure (C.11) for the new product, s_m, in traditional EOQ model, and varies positively with the production rate, m. The holding cost rate for the used and returned product, h_{rr}, has a negative relationship with the order quantity in the centralized situation.

In the retailer controlled scenario (refer to Figure (C.12), its relative market share, A, and the price-sensitivity parameter, B, included in the demand rate function are major factors. At the same time, the changes in setup cost for returned product, S_r, and the holding cost rate, h_r, appear to affect the order quantity. In addition the wholesale price also tends to impact the order quantity.

In the manufacturer controlled scenario (refer to Figures(C.13) and (C.14)) parameters appear to be interrelated and complex in their interaction. Most of these, nevertheless, do not appear to affect the order quantity within the range of parameter variations considered.

3.6.4 Supply Chain Profits

As before, changes in the parameters, A and B, which define the market demand rate appear to appreciably impact the supply chain profit in all of the four operating environments as can be seen in Figure (C.15)-(C.18). The total supply chain profit changes positively with changes in market share, A, and negatively with changes in the price-demand sensitivity parameter, B. The production cost rate, as the cost for the input for the whole system, tends to affect the supply chain profit in the centralized situation (see Figure (C.15)). In the decentralized situations (as Figures (C.16) and (C.17) above), changes in the wholesale prices, p_w, has a slightly negative effect on total supply chain profit. In the manufacturer controlled situation, the reimbursement price, r_m, paid by the manufacturer to the retailer to encourage the retailer to collect the returns, appears to significantly impact supply chain profit. This implies that selecting the price to encourage the retailer to collect the returns, under the manufacturer controlled situation should be given some attention. Otherwise, it is likely to reduce the profitability of the entire supply chain.

3.6.5 Retailer's Share in Supply Chain Profit

The parameters in the demand rate function, A and B, and the wholesale price, p_w, have major impacts on the retailer's profit share ratio in both the retailer controlled and the manufacturer controlled situations (see Figure (C.19) -(C.21)), if p_w is not given. For the latter case, as can be seen in Figure (C.20), increasing the reimbursement price, r_m, set by the manufacturer is likely to increase the retailer's efforts in collecting the returns. If the wholesale price is not given exogenously, the only major factor that seem to have an impact on the retailer's share in the supply chain profit is the unit production cost. None of the other parameters appear to have any significant influence in this regard.

3.7 Summary and Conclusions

In this study, we have developed mathematical models under a deterministic scenario, for simultaneously determining the production/delivery lot size, the retail price and the customer return reimbursement level for a single recoverable product in a two-echelon supply chain consisting of a single retailer and a single lean manufacturer. Items returned by customers at the retail level are refurbished and totally reintegrated into the manufacturer's existing production system for remanufacturing and are sold eventually as new products. As in many lean manufacturing (JIT) environments, we assume a lot-for-lot operating mode for production, procurement and distribution, as an effective mechanism for supply chain coordination.

Decentralized models are developed and solved for determining profit maximizing optimal policies from the perspectives of both members of the supply chain. A centralized, jointly optimal procedure for maximizing total supply chain profitability is also presented. A numerical example illustrates that the centralized approach is substantively superior to individual optimization, due to the elimination of double marginalization. The example also outlines a fair and equitable proportional profit sharing scheme, which is economically desirable from the standpoint of either member of the supply chain, for the purpose of implementing the proposed centrally controlled model.

Of necessity, the simplifying assumptions made here (e.g. deterministic parameters and the lot-for-lot modality), are the major limitations of this study. Embellishments by future researchers, such as relaxation of the lot-for-lot assumption, incorporation of uncertainty, more realistic and complex demand and product return functions, multiple products, manufacturers, etc. will, undoubtedly, lead to more refined remanufacturing and related models. Furthermore, future efforts in this area should consider the development of integrated decision models under stochastic conditions, which are likely to be more realistic from an implementation standpoint. Nevertheless, the results obtained in this study are likely to be of some value to practitioners as broad guidelines for integrated pricing, recoverable product collection, production planning and inventory control decisions, as well as for designing more streamlined, well-coordinated supply chains towards gaining competitive advantage. We also hope that our efforts will prove to be useful for researchers in shedding light on some of the intricate and inter-related aspects of product remanufacturing towards developing more effective decision making models for supply chain and reverse logistics management.

Finally, we have outlined the parametric conditions, (a), (b), (c) and (d), under Proposition (3.4.1) and Proposition (3.4.3), for joint concavity, that are sufficient for yielding globally optimal solutions for the various operating scenarios examined here. If, however, in a given case, one or more of these conditions are violated, the respective solutions yielded by our models represent merely local optima. The quality of such locally optimal solutions with respect to truly global optima remains a matter for future research.

4. Discrete Hierarchical Decision Making Problems in Biofuel Supply Chains

4.1 Introduction of the Biofuel Problem

Biofuel is a type of fuel derived from biomass and used for power and automotive transport. Many crops such as wheat, corn, rapeseed and sunflower can be the resources to produce biofuel. Recently, with more concerns over rising oil prices, gas emission effects and rural development interests significantly drive the biofuel use and development. The legislation, HB3543 signed by Oregon Governor Ted Kulongoski in July 2007, requires "all gasoline sold in the state to be blended with 10% bioethanol and all diesel fuel sold in the state to be blended with 2% biodiesel" [75]. In order to break its dependence on imported oil, the U.S. government now provides more funding, including $179 million for the President's Biofuels Initiative[78] and $375 million by the Department of Energy[32], to reduce the cost and improve the efficiency of biofuel production.

In this paper, we base our analysis closely on a model proposed in [11]. Government, industry and farms form a supply chain through which material, product and currency flow. Generally, there are three levels of decision making in this biofuel production problem:

- As a leader, the government plays an active role in encouraging biofuel production. Under economical, environmental and social considerations, the government would like to provide direct incentives such as tax exemptions to industry. Naturally, the cost to the government is expected to be minimized while ensuring the required amounts of production.
- Due to the high production cost of biofuel, industry expects more tax exemption from goverment to ensure its own profit. Otherwise, industry has no incentive to replace oil with biofuel products. The industry will determine the price paid to the farmers at the gate.

- Because of surging demand for biofuels, farms benefit by switching from producing food crops to nonfood crops, which are the raw materials for biofuel production. Farms choose among three options: planting food crops, growing nonfood crops or leaving a part of the land fallow. The strategic plan needs to be set for the allowable land to be better utilized to realize the maximum profits.

Models involving such decisions for biofuel production are typically formulated as bilevel programming problems. At each level, a certain objective function needs to be optimized without forcing lower levels to make suboptimal decisions.

- In the upper level problem, the government needs to minimize its total expenditure by setting the optimal tax exemption level provided to the industry. Meanwhile, the industry will determine the price paid to the farmers at the gate.
- In the lower level problem, farms decide the optimal assignments of allowable lands to maximize their profits.

However, in order to ensure sufficient food and enough fuel, a license fee might be charged if the farms sell crops as a biofuel resource. In addition, transportation and equipment costs for nonfood crops may be incurred because of the new destination and purpose of the final products. We categorize all these costs into fixed costs. As a result, the increased importance of the biofuel development program with fixed cost stimulated our interest to improve modeling and solution methods used in the past. The introduction of binary variables is essential in the agronomic practice and policy making, despite adding significant complexity to the computation of the optimal solution.

In this study, we examine an instance with two types of biofuel: ester and ethanol. Having only two such considerations allows us to use a grid search algorithm to handle the upper level problem, as was proposed in [3]. For each instance of the lower level problem, we use a cutting plane algorithm to handle the mixed-integer linear programming problem. By using sensitivity analysis and warm-starts, we illustrate the efficient solution of a numerical example. We also investigate another approach based on nonsmooth nonlinear optimization. We treat the dependence on the lower level as a nonlinearity in the upper level, and apply a sequential quadratic programming method to solve government model, leading to the introduction of quadratic penalty term to smooth farm's model and the incorporation of nonlinear mixed integer cuts. The efficiency of the extended method also has been examined.

4.2 Models

Using the notation introduced at the beginning of this dissertation, we introduce the upper and lower level models.

4.2.1 The Goverment Model (Upper Level)

The upper level problem consists of the government model:

$$\min_{\tau,p,xn} \sum_{v\in V,d\in D,e\in E} \alpha_{dve} xn_{de} \tau_v - \gamma \sum_{e\in E,d\in D'} xn_{de} \tag{4.2.1}$$

$$\text{subject to} \quad \sum_{e\in E,d\in D} xn_{de} \geq \rho\theta \sum_{e\in E} \sigma_e \tag{4.2.2}$$

$$\sum_{e\in E,d\in D} \alpha_{dve} xn_{de} \leq u_v \qquad \forall v \in V \tag{4.2.3}$$

$$p_{de} = \max_{v\in V}\{(\tau_v - \pi_{dv})\alpha_{dve}\} \qquad \forall d \in D(v), e \in E \tag{4.2.4}$$

$$p_{de} \geq 0, \tau_v \geq 0 \qquad \forall v \in V, d \in D, e \in E \tag{4.2.5}$$

The objective (4.2.1) is to minimize the total value of tax credits provided by the government after the savings to the government of not having to pay set-aside payments due to the planting of certain nonfood crops on what would otherwise be fallow land. The conversion rate α considers the rate of transferring the nonfood crops to biofuel for the industry as well as the yield rate for each farm, which can vary significantly due to soil fertility and plant nutrition. Inequality (4.2.2) ensures the minimum percentage of the total arable land to be used for food crops. Constraints (4.2.3) limit the production of biofuel v to no more than u_v. These constraints explain the need for a certain amount of biofuel for the government, but ensure that those biofuels with storage issues, such as ethanol, do not get produced excessively. The next constraint (4.2.4) represents the industry's problem to price the nonfood crops appropriately. The quantity π_{dv} represents a threshold, which should be set large enough to guarantee that the industry plants which convert the nonfood crops to biofuel will make profit. The price, p_{de}, is the price offered per unit of area and is equal to the price offered per unit of weight of nonfood crop(independent of farm) multiplied by the yield of the nonfood crop per unit area on farm e. Although not expressed in (4.2.1)-(4.2.5), the variables xn are assumed to be in the set of optimal solutions for the lower level problem parameterized by p, discussed next.

4.2.2 The Farms Model (Lower Level)

The lower level problem consists of the farms model:

$$\max_{xc,xn,xe,q} \sum_{e\in E,c\in C} m_{ce}xc_{ce} + \sum_{e\in E,d\in D}(p_{de}-c_{de})xn_{de} + \gamma\sum_{e\in E} xf_e - \sum_{e\in E,d\in D} q_{de}t_{de} \tag{4.2.6}$$

$$\text{subject to } \sum_{c\in C} xc_{ce} + \sum_{d\in D} xn_{de} + xf_e \leq \sigma_e, \forall e\in E \tag{4.2.7}$$

$$\sum_{d\in D} xn_{de} + xf_e = \theta\sigma_e, \forall e\in E \tag{4.2.8}$$

$$xc_{de} + xn_{de} \leq \chi_{de}\sigma_e, \forall e\in E, d\in D \tag{4.2.9}$$

$$xn_{de} \leq q_{de}\sigma_e, \forall e\in E, d\in D \tag{4.2.10}$$

$$xf_e \leq \delta\sigma_e, \forall e\in E \tag{4.2.11}$$

$$xc_{ce} \geq 0, xn_{de} \geq 0, xf_e \geq 0, q_{de}\in\{0,1\}, \forall c\in C, d\in D, e\in E \tag{4.2.12}$$

Function (4.2.6) is the objective function of the agriculture sector to maximize its profits by assigning the total arable land among food crops, nonfood crops and fallow lands. Inequality (4.2.7) confines the assignments to the total available land. Among them, a certain amount of land, called set-aside land, is only used for nonfood crops and for keeping fallow in (4.2.8). Agronomic considerations are reflected in (4.2.9). For example, previous year's production levels, soil nutrient levels, or the amount of labor and equipment available can limit the amount of land available for a type of crop. As such, for each nonfood crop, an upper bound may also be given as a percentage of available land on each farm. (4.2.10) is used to implement capacity constraints, where the value of q_{de} determines whether or not land is available at farm e to grow crop d. (4.2.11) gives the upper bound on the amount of land left fallow. (4.2.12) ensures that all the continous variables are nonnegative and q_{de} are binary variables. Without the last term in the objective function (4.2.6) and the capacity constraints (4.2.10), the model would turn into the biofuel model without fixed cost. The inclusion of fixed costs affects the optimization methodology and the complexity.

4.3 Solution Methods

Two solution methods, grid search and nonsmooth nonlinear programming, are introduced in [11] and varying degrees of efficiency and success are reported. The grid search method can be applied even if the objective function is neither continuous nor differentiable. However, the solution is at a low level of accuracy and the whole process is computationally intensive. The nonlinear approach has the potential to reach a higher level of accuracy, but [11] does not report much success with it as the algorithm is highly sensitive to the initial point and the magnitude of the smoothing term.

In this paper, we chose to investigate the extension of these two solution methods to the problem introduced in the previous section. In order to solve the discrete bilevel programming problem obtained by introducing fixed costs into the biofuel model, we pay particular attention to the handling of the integer variables. As a first solution method, we apply a pattern search approach as a more efficient alternative to grid search for handling the upper level problem, while solving the lower level problem using a cutting plane method for mixed integer linear programming problems. As a second method, we also investigate the use of nonsmooth nonlinear programming. For both approaches, sensitivity analysis and warm-starts are implemented to exploit efficiencies.

4.3.1 Pattern Search

In this paper, we are considering a problem instance with two types of biofuel, ester and ethanol, so it is convenient to perform a pattern search to handle the upper level problem. The pattern observed in the government objective function value and the feasible region after solving the lower level problem for a few different pairs of τ values indicates potential for success of the pattern search [54]. We have implemented the algorithm as follows:

Step 1: (Initialization)

(a) $\zeta =$ the minimum change of objective value

(b) $\epsilon^{lb} =$ the lower bound for the step length

(c) $\delta =$ the factor to change the search length, usually $\delta \leq 1$

(d) Set up the initial starting point as τ^i and the search length ϵ as ϵ_0

Step 2: (Check the feasibility of the initial point τ^i)

(a) Solve the lower level problem with p determined by τ^i in the constraint (4.2.4)

(b) Substitute the optimal solutions of the lower level problem into the upper level problem

(c) Check feasibility in the constraints (4.2.2) and (4.2.3) and calculate the objective value $U(\tau^i)$;

If feasible, go to Step 3;

Otherwise, randomly reselect a new start point τ^i and go back to Step 1;

Step 3: (Start the search)

Search the neighborhood of τ^i in all eight directions with step length as ϵ, thus we have the $\tilde{\tau}^i$ as

$$\left(\tilde{\tau}^i_{ethanol}, \tilde{\tau}^i_{ester}\right) = \begin{pmatrix} \tau^i_{ethanol} + \epsilon & \tau^i_{ester} \\ \tau^i_{ethanol} - \epsilon & \tau^i_{ester} \\ \tau^i_{ethanol} & \tau^i_{ester} + \epsilon \\ \tau^i_{ethanol} & \tau^i_{ester} - \epsilon \\ \tau^i_{ethanol} + \epsilon & \tau^i_{ester} + \epsilon \\ \tau^i_{ethanol} - \epsilon & \tau^i_{ester} + \epsilon \\ \tau^i_{ethanol} + \epsilon & \tau^i_{ester} - \epsilon \\ \tau^i_{ethanol} - \epsilon & \tau^i_{ester} - \epsilon \end{pmatrix}$$

Step 4: (Check the feasibility of the neighborhood):

(a) For each pair of $\tilde{\tau}^i$, calculate the corresponding price p respectively ;

(b) Substitute into the lower level problem and solve the lower level problem

(c) Substitute the optimal solution of the lower level problem into the upper level problem

(d) Check the feasibility in the constraints (4.2.2) and (4.2.3).

If none of $\tilde{\tau}^i$ can satisfy the constraints in the upper level problem,

If $\epsilon \leq \epsilon^{lb}$ conclude that the current solution cannot be improved;

Otherwise, let $\epsilon = \epsilon/\delta$ and go back to the Step 3 to continue iterations

Otherwise, go to Step 5

Step 5: (Adjust step length at the original τ^i as necessary):

(a) Get the upper level objective value $F(\tilde{\tau}^i_j)$ corresponding to each $\tilde{\tau}^i_j$;

(b) Set $F(\tilde{\tau}^i) = \min_j(F(\tilde{\tau}^i_j))$

(c) If $F(\tilde{\tau}^i) \geq F(\tau)$,

If $\epsilon \leq \epsilon^{lb}$ conclude that the current solution cannot be improved;

Otherwise, let $\epsilon = \epsilon/\delta$ and go back to the Step 2 to continue iterations

Otherwise, go to Step 6

Step 6: (Move to the new τ^i along the pattern as necessary):

(a) Stop and conclude optimal solution if $F(\tau^i) - F(\tilde{\tau}^i) \leq \zeta$

(b) Otherwise, let $\tau^i = \tilde{\tau}^i_j$, where $j \in \operatorname*{argmin}_j F(\tilde{\tau}^i_j)$, $\epsilon = \epsilon_0$, and go back to the Step 3 to continue iterations

This pattern search method greatly improves the computational efficiency over the traditional grid search. Instead of checking every pair of τ values in the possible area, pattern search makes good use of the related information from the previous iterations and follows the direction to further improve the objective value within the feasible region. Notice, however, that the method would be less effective if a large number of decision variables are included in the upper level problem.

The farms model

Because of the consideration of setup costs, the farms model given by (4.2.6)-(4.2.12) is a mixed-integer linear programming problem. We use a mixed-integer cutting plane method to solve this problem. Unlike branch-and-bound, which solves multiple problems with different dictionary, cutting plane algorithm can get the optimal solution by using a single dictionary, when the price is change, we can still use sensitivity analysis on the single dictionary.At each iteration of the algorithm, we relax the binary constraints and apply parametric self-dual simplex method [84].

Following the notation of the general bilevel problem with $m_2 = 0$ and letting $\mathcal{J}$ denote the index set of the integer variables, the lower level problem has the generalized form

$$\begin{aligned} \max_{y'} \quad & f(x, y') \\ \text{subject to} \quad & Ay' = h \\ & y'_j \geq 0 \text{ for } j \notin \mathcal{J} \\ & y'_j \in \{0, 1\} \text{ for } j \in \mathcal{J}. \end{aligned} \tag{4.3.1}$$

Note that in (4.3.1), the objective function is linear.

We partition the decision variables and the constraint coefficients as $y' = \begin{bmatrix} y'_{\mathcal{B}} \\ y'_{\mathcal{N}} \end{bmatrix}$ and $A = [\ A_{\mathcal{B}} \quad A_{\mathcal{N}}\ ]$ according to the optimal basis using the index sets $\mathcal{B}$ and $\mathcal{N}$. Then, the constraint rows of the final dictionary can be written as:

$$y'_{\mathcal{B}} = A_{\mathcal{B}}^{-1}h - A_{\mathcal{B}}^{-1}A_{\mathcal{N}}y'_{\mathcal{N}}. \tag{4.3.2}$$

For $j \in \mathcal{J}$, if $y'_j \notin \{0,1\}$, then we can add a Gomory mixed-integer cut to (4.3.2):

$$\begin{aligned} &\sum_{j \in N \cap \mathcal{J}: \overline{a}_{ij} - \lfloor \overline{a}_{ij} \rfloor \leq \overline{h}_i - \lfloor \overline{h}_i \rfloor} (\overline{a}_{ij} - \lfloor \overline{a}_{ij} \rfloor) y'_j \\ + &\sum_{j \in N \cap \mathcal{J}: \overline{a}_{ij} - \lfloor \overline{a}_{ij} \rfloor > \overline{h}_i - \lfloor \overline{h}_i \rfloor} \frac{(\overline{h}_i - \lfloor \overline{h}_i \rfloor)(1 - \overline{a}_{ij} + \lfloor \overline{a}_{ij} \rfloor)}{1 - \overline{h}_i + \lfloor \overline{h}_i \rfloor} y'_j \\ + &\sum_{j \in N / \mathcal{J}: \overline{a}_{ij} > 0} \overline{a}_{ij} y'_j + \sum_{j \in N / \mathcal{J}: \overline{a}_{ij} < 0} \frac{\overline{h}_i - \lfloor \overline{h}_i \rfloor}{1 - \overline{h}_i + \lfloor \overline{h}_i \rfloor} \overline{a}_{ij} y'_j \geq 0 \end{aligned} \tag{4.3.3}$$

where $\overline{a}_{ij}$ is the (i,j)th element of $A_{\mathcal{B}}^{-1}A_{\mathcal{N}}$ and $\overline{h}_i$ is the ith element of the vector $A_{\mathcal{B}}^{-1}h$ in the final dictionary. Thus, the resulting cutting plane algorithm starts by relaxing all the binary constraints and alternately solves the linear programming problem and adds cuts to obtain a mixed integer solution.

Sensitivity analysis and warm-starts

If we investigate the lower level problem, we find that only the objective function coefficients of the lower level problem depend on the decision variables τ from the upper level problem, since p is the only component affected by the government's choice. We know that changes in the coefficients of the objective function may not change the optimal solution and dictionary of the new problem, which means that in the pattern search framework we might be able to skip solving some of the lower level problems. Even if the solution changes, the previous optimum still remains feasible for the current problem, and starting the search from the final dictionary of the previous problem may require fewer iterations to reach the new optimum. Therefore, after solving each lower level problem, we save the optimal basis and reuse it as the starting basis for the solution of the next lower level problem. This simple strategy seems to have a significant impact on the efficiency of the solution method, as indicated by the numerical results presented in the next section.

4.3.2 Nonsmooth Nonlinear Optimization

Even though numerical results of the next section indicate good performance for the pattern search method, it cannot be easily extended to larger problems due to the significant computational effort required to perform a search in multiple dimensions. Another possible approach investigated in [11], albeit without much success, is nonsmooth nonlinear programming. With this method, the government's problem can be solved in one shot with the lower level problem treated as a nonsmooth, nonlinear constraint. A gradient-based nonlinear programming algorithm is used to solve the resulting problem, and the derivatives associated with the lower-level problem are computed numerically after a smoothing term is applied. In our numerical studies, we used the function fmincon in MATLAB which implements a sequential quadratic programming algorithm and has built-in capabilities to apply the finite differences method to numerically approximate derivatives by measuring the average change in the objective function with respect to small changes in the decision variables.

The government model.

For the upper level problem, the objective function (4.2.1) is an implicit function of the decision variable τ since the optimal land assignments xn are determined as functions of p, which is a function of τ as defined by (4.2.4). In other words, the general problem (1.2.1) can also be written as

$$\begin{aligned} \min_{x \in X} \quad & F(x, y(x)) \\ \text{s.t.} \quad & G(x, y(x)) \geq 0, \end{aligned} \tag{4.3.4}$$

where y is now expressed as a function of x, and the definition of y as the solution of the lower level problem is now treated as one of the nonlinear constraints G. Expressed as above, (4.3.4) is now a single-level nonlinear programming problem. In order to use a gradient-based approach to solve it, we need to evaluate the gradients of the objective function and the constraints with respect to the decision variables x. We now outline the finite differences approach used for the objective function, and the details of the constraint gradients are identical.

Using the chain rule, the gradient of F in (4.3.4) can be expressed as

$$\nabla_x F = \frac{\partial F(x,y)}{\partial x} + \frac{\partial F(x,y)}{\partial y}\frac{dy}{dx}$$

While we can explicitly formulate the partial derivatives of F with respect to x and y, y is a nonsmooth function of x defined as the solution of the lower level problem. Because of the definition, we can only evaluate $\frac{dy}{dx}$ numerically, and we use the smoothing technique of [11] to alleviate the nondifferentiability. This approach is outlined below.

If the derivative information can be obtained, (4.3.4) can be solved using any nonlinear solver. As mentioned, we used fmincon in MATLAB as our solver. We will discuss the numerical performance of this implementation in the next section.

The farms model.

In order to evaluate the function $y(x)$ for the upper level problem, we need to solve the lower-level problem once in each iteration of the nonlinear programming method. To do so, we use the mixed-integer cutting plane approach outlined for the pattern search method.

In order to evaluate the derivative $\frac{dy}{dx}$ for the upper level problem, we need to solve the lower-level problem again after a small perturbation to x. However, if the optimal solution of the lower-level problem does not change, the resulting numerical derivative will be zero, or if the solution does change, it can be a significant change. This jump in the value of the derivative is undesirable and may cause the overall algorithm to make erratic progress and/or stall. In order to alleviate the effects of the jump, we introduce a smoothing term to the objective function of the lower level problem:

$$\max_{y'} f(\hat{x}, y') - \psi \|y' - \tilde{y}\|, \tag{4.3.5}$$

where $\hat{x}$ is the new, perturbed vector x, $\psi > 0$ is a smoothing parameter, and $\tilde{y}$ is the solution of the problem at the original value of x. The quadratic penalty term smooths and regularizes the objective function.

In our implementation, we introduce an intermediate variable $\xi \in \mathbb{R}$ and rewrite the objective function as

$$\max_{y', \xi} \xi. \tag{4.3.6}$$

We also introduce a new nonlinear constraint

$$\xi - (f(\hat{x}, y') - \psi \|y' - \tilde{y}\|) \leq 0 \tag{4.3.7}$$

The reason for reformulating the nonlinear problem with a nonlinear objective function as a nonlinear problem with a nonlinear constraints is to implement a cutting plane method to solve the lower level problem. This allows us to reuse information gained by solving the lower-level problem to evaluate the function $y(x)$ for the current value of x and build on the cutting plane method used to solve that mixed-integer linear programming problem. As a result, in order to solve this mixed-integer nonlinear problem within the cutting-plane framework, we apply an extended cutting plane method introduced by Westerlund et al. [87].

Starting with the optimal basis obtained by solving the mixed-integer linear programming problem, we add the following linear cuts as necessary to obtain an optimal solution to the smoothed-lower level problem:

$$\sum_{i \in \mathcal{J}} (\frac{\partial f}{\partial y'_i}|_{y'^k}) y'_i + \sum_{i \notin \mathcal{J}} (\frac{\partial f}{\partial y'_i}|_{y'^k}) y'_i \leq f(y'^k) - \sum_{i \in \mathcal{J}} (\frac{\partial f}{\partial y'_i}|_{y'^k}) y'^k_i - \sum_{i \notin \mathcal{J}} (\frac{\partial f}{\partial y'_i}|_{y'^k}) y'^k_i \tag{4.3.8}$$

Westerlund et al. ([87]) have proved convergence of this approach.

4.4 Numerical Results

In this section, we provide a numerical example to illustrate the effect of fixed costs on the model and its solution. Three farms of different size, $\sigma_{E1} = 3300$ acres, $\sigma_{E2} = 14600$ acres and $\sigma_{E3} = 6300$ acres respectively, need to allocate their lands to plant 7 food crops. Some of them can be used as nonfood crops to produce biofuels. Wheat, corn and sugar beet could be converted to ethanol by industry. Rapeseed and sunflower can produce ester. We used data consistent with [11] and filled any gaps in the data with further research into the agricultural conditions in Europe. We use $\gamma = 193.2$ \$/acre, $\theta = 0.35$, $\delta = 0.1$, $u_v = 100000$ acres and $\rho = 0.7674$, and the rest of the data are shown in Tables (B.3, B.4 and B.5) .

4.4.1 Solutions

We coded the solution procedures in Matlab, Version 7.3.0.267(R2006b), then ran them on a PC with Intel(R) Pentium(R)4 CPU at 3.20GHz, with 1.99 GB of RAM. Using the pattern search method, we get the optimal land assignment for each food crop and each nonfood crop in each farm listed in Table (B.6) and Table (B.7). We compare the results with fixed costs to those without fixed costs in the same table to gauge the effect of fixed costs on the optimal solution. When fixed costs are considered, the tax exemption level for ethanol is reduced from 45.77(\$/ton) to 43.50(\$/ton), while the level for ester increases a little bit from 3.93 (\$/ton) to 3.95 (\$/ton). With the change in the tax exemption levels, there is a 16.42% decrease in the total cost for the government. Although the farms are faced with an increased amount of costs, we observe that they do not produce less of the nonfood crops, but instead change their allocation in response to the fixed costs. As a result, the total profits of the farms decrease by 62.23%. It is obvious that fixed costs influence the expenses of the government and the profits of the farms as well, which is consistent with our intuition that the farmers would adjust their strategies, however, the total profit will be reduced according to the extra costs since the farms would not like to take efforts to switch from the food crops to nonfood crops. Simutaneously, the government might instead save the expense with less land to be used to produce crops for biofuel.

The same solutions were obtained by using the nonsmooth nonlinear programming approach. However, the nonsmooth nonlinear optimization is sensitive to initial points. Some initial points drive the system to become stuck at the initial point without any improvement or at an infeasible solution. For large enough values of the smoothing parameter and the use of bounds to define tax exemption levels that produce feasible solutions for the upper level problem, we were able to replicate the solution given in Table (B.6) and Table (B.7) using the nonlinear programming approach.

4.4.2 Computational Efficiency

To evaluate the computational effort required by each solution approach, we compare the points skipped and total iterations for each level of the pattern search in Table (B.8).

From the results of Table (B.8), we can observe our new method with sensitivity analysis and warm-starts outperforms the one without sensitivity analysis and warm-starts. For example, considering a searching step size of 25 at starting point $(250, 250)$ in the case of without fixed cost, 170 points have been skipped due to the use of sensitivity analysis and warm-starts. If we treat the traditional method without warm-starts and sensitivity analysis as the base situation, we find that the proposed method could reduce the number of iterations by 86.14%. Although there is no constant pattern for the amount of reduction, the proposed method finds the optimal solution efificiently. In summary, we can solve the discrete bilevel problem easily using our proposed new method for pattern search. The total number of iterations it takes is just 27159, much less than 41031, which is for the approach without sensitivity analysis and warm-starts. The CPU time also reduces significantly.

4.5 Conclusions

From the numerical example, we can see that there is a significant impact of the fixed cost on the final decision and the profits which farms and the government share. In order to encourage farms to produce nonfood crops in the presence of fixed costs, the government needs to provide more incentives. At the same time, farms also make their strategic plans to assign land so that they can reach their maximum profit. The consideration of fixed cost could reduce their profitability, however the loss could be controlled by changing the final assignment of lands directly and easily.

From a computational point of view, we have demonstrated that efficient solution methods can be developed to solve this problem. By incorporating sensitivity analysis and warm-starts into both of our solutions approaches, we have dramatically improved their performance. While the practical applicability of the pattern search approach is limited to problems with only a few variables in the upper-level problem, it can nevertheless be viable with increasing computational power and memory. On the other hand, the nonsmooth nonlinear programming approach allows us to get solutions quickly and with a good level of accuracy if a good initial solution and reasonable variable bounds can be provided.

5. Conclusions and Future Extensions

5.1 General Conclusions

Hierarchical decision making involves multiple decision makers who make decisions in a hierarchy of levels. This framework has wide application in technology development, public policy, and business. Due to the complexity of the current marketplace and intensive competition, more and more decision makers at different levels are getting actually involved. The decisions at each level present profit opportunities for the decision maker at this level. This framework also handles the interactions among different levels because the parameters at any level may be determined by the reaction of another level. Therefore, it is important to explore the nature of hierarchical decision making systems.

In this dissertation, applications of hierarchical decision making are provided in the context of supply chain management, specifically in a remanufacturing setting and in a biofuel production case. The relationship between suppliers and buyers, for instance, as the remanufacturer and the retailer or the feedstock provider and the government/ the industry, can be expressed by hierarchical decision making. Those decision makers pursue different objectives and make decisions from their own perspectives. With more and more problem aspects taken into consideration, the current solution methods for simple environments cannot satisfy the need for examining more complex business problems. Particularly when discrete decisions are involved, new solution procedures are often wanted. This work advances the current literature by proposing algorithms for solving complex hierarchical programming problems. This has been the motivation for conducting this research.

As the concepts of environment sustainability, lean production, and green manufacturing have been widely accepted by the public, remanufacturing supply chain management has become an importation research concern. This dissertation represents an early attempt to consider acquisition management and inventory management in an integrated manner. We have found explicit solutions for decentralized, as well as centralized scenarios and these solutions indicate that centralized decision making in remanufacturing leads to higher supply chain profits. Subsequent numerical results and sensitivity analysis confirmed our theoretical findings. Under our assumed deterministic scenario, sensitivity analysis indicates that the relative market share and the elasticity of the demand significantly influence the optimal decisions and the total profits.

The second application presented a case of biofuel production, representing the study of a more complicated hierarchical decision making model. Recently, due to the pressures of oil price increases and the potential shortage of food crops, intense debate has surrounded government tax exemptions, which serves as an incentive to encourage biofuel production. With data collected from real-world sources, we find that the government can set up the appropriate policy to provide appropriate incentives for biofuel production and ensure enough food crop output, as well. Unlike previous work in this area, we include fixed costs. Fixed costs are a significant part of biofuel production costs for farms, and therefore our models make a very important contribution to the field by incorporating them into the model. With recent advances in nonlinear programming and integer programming methods, we develop solution methods for solving these types of problems. In this study, search methods for derivative-free optimization are used to solve discrete bilevel problems, and smart implementation techniques, such as sensitivity analysis and warm-starts, greatly improve the performance of such solution procedures.

5.2 Suggested Future Research

The remanufacturing problem presented assumes a deterministic operating. In most situations, industries such as electronic products OEM, such an assumption is unrealistic environment. For example, the return rate is likely to depend on the price of new products, a stochastic model can assume that the return rate would follow some distribution with a mean value as a function of the collection price which the retailer pays to the customers. In addition, the quality of the returns may also be uncertain, which leads to an uncertain remanufacturing variable cost. This multilevel decision making problem in a stochastic environment needs further consideration and analysis.

Additionally, the simplifying assumptions made here on the model parameters and the decision making scenario present further opportunities for extension of this research. For instance, relaxation of the lot-for-lot assumption, incorporation of more realistic and complex demand and product return functions, and the considerations of the multiple products and manufacturers are likely to be more realistic and more complex to analysis.

Although the dissertation explores the solution algorithms and the applications of bilevel programming, some extensions are expected to make it even more practical and convenient to solve. We will continue to work on further improving the algorithm for the discrete bilevel problem by focusing on computational efficiency. Our attempt to solve the discrete linear bilevel problems in this paper will be further extended to solve other types of problems such as discrete nonlinear bilevel problems and stochastic nonlinear problem. Once the time and efforts to solve our discrete linear bilevel programming problems are further reduced, it will provide the possibility to improve the computational efficiency to solve nonlinear discrete bilevel programming problem.

There are possible extensions of the biofuel production model, as well. For example, if the influence of food supply on food price is investigated, a Nash game can be integrated in the farms' level to form a mathematical program with equilibrium constraints. If some of the farms switch from the food crops production to the nonfood crops production, the decrease in the production level of food crops leads to the increase in the price of food crops. Comparing the new price of food crops with the price of nonfood crops, the production level will be re-allocated among the total lands until final equilibrium is reached. If this perspective has been considered, the market could adjust itself. The only concern is the time lag.

Additionally, the biofuel models in this dissertation are deterministic models. In some settings, some parameters which have been assumed to be known in this paper are not certain. For example, weather is one of the uncertain factors. The change of the weather results in different yield levels of the lands, which cannot be predicted by the farms before they organize the land assignments. However, if the farms take these stochastic factors into consideration, their rational decisions showed focus on avoiding large variability in profits.

Finally, a third generation biofuel, produced from the biomass such as algae, will expand our research area. Related topics such as considerations of transportation, logistics, and inventory related factors can further extend and estabilish our biofuel models.

The adequacy of food supply is not only important to feed the world population, but also essential to the stability of society. As the result, the social consideration of ensuring enough food supply could be treated as an integral part of the goal of the government. In future extensions of the model, we will include constraints in the governments model to ensure the adequate and stable food supply. We will then assess whether the government can provide an incentive structure to the farms to achieve this goal naturally or whether legislation needs to be introduced to regulate individual farms obligations for growing food crops. In the latter case, we will need to also add constraints to farms' model.

Also called "green manufacturing", remanufacturing considers reusing products instead of merely disposing of them. Similarly, nonrenewable resources such as crude oil. Both ideas play important roles in the study of Sustainability and should be emphasized in courses such as production management, operations management, and supply chain management. I would integrate those contents into my teaching and enforce students understanding of these concepts, which will be essential in product design and operational strategy in our current stages of production management. Furthermore, the bilevel program presented in the dissertation also can be applied to illustrate to students the basic idea of game theory, which will help them solve lots of practical problems in real business since hierarchical structure has been widely used in contemporary management.

As we stressed in this dissertation, discrete bilevel programming problems have wide applications in real business problem. We are looking forward to extending them to many other problems. One of the potential applications is group decision making proble([49], where each group member needs to solve his individual problem in the lower level problem while the final selection has to be made with respect to the group member's importance weight in the upper level problem. The multilevel problem is inter-connected because the objective function values in the lower level problem are coefficients in the upper level problem. Also, since the deciison variables in the upper level problem do not affect the lower level problem, this type problem is easy to solve.

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Appendix A. Proof of Propositions in Remanufacturing Models

Note: All the proofs outlined are based on the following concepts: f is concave on D if and only $\nabla^2 f(x)$ is a negative semi-definite matrix for all $x \in D$. An $n \times n$ matrix A is negative semi-definite if and only if $(-1)^k |A_k| \geq 0$ for all $k \in \{1, \ldots, n\}$ where A_k is the upper left k-by-k corner of A.

Proof. Proposition (3.4.1)

If Q^*,r_c^*,p_s^* obtained from (3.4.2),(3.4.3) and (3.4.4) are globally optimal, the sufficient condition is that the objective function (3.3.2) should be jointly concave in these three variables. The Hessian matrix for (3.3.2) is:

$$D^2\Pi_r(Q, r_c, p_s) = \begin{vmatrix} \frac{\partial^2\Pi_r}{\partial Q^2} & \frac{\partial^2\Pi_r}{\partial Q\partial r_c} & \frac{\partial^2\Pi_r}{\partial Q\partial p_s} \\ \frac{\partial^2\Pi_r}{\partial Q\partial r_c} & \frac{\partial^2\Pi_r}{\partial r_c^2} & \frac{\partial^2\Pi_r}{\partial r_c\partial p_s} \\ \frac{\partial^2\Pi_r}{\partial Q\partial p_s} & \frac{\partial^2\Pi_r}{\partial r_c\partial p_s} & \frac{\partial^2\Pi_r}{\partial p_s^2} \end{vmatrix}$$

$$= \begin{vmatrix} -2S_r\frac{d}{Q^3} & -\frac{ah_{rr}}{2d} & -\frac{BS_r}{Q^2} + \frac{bh_{rr}}{2d} - \frac{Bh_{rr}x}{2d^2} \\ -\frac{ah_{rr}}{2d} & -2a & b - \frac{aBQh_{rr}}{2d^2} \\ -\frac{BS_r}{Q^2} + \frac{bh_{rr}}{2d} - \frac{Bh_{rr}x}{2d^2} & b - \frac{aBQh_{rr}}{2d^2} & -2B + \frac{bBh_{rr}Q}{d^2} - \frac{h_{rr}B^2Q}{d^3} \end{vmatrix}$$

Thus,

$$|A_1| = \frac{\partial^2\Pi_r}{\partial Q^2} = -2S_r\frac{d}{Q^3} \leq 0$$

Since all the parameters are strictly positive, $|A_1| \leq 0$ always holds. Also,

$$|A_2| = \begin{vmatrix} \frac{\partial^2 \Pi_r}{\partial Q^2} & \frac{\partial^2 \Pi_r}{\partial Q \partial r_c} \\ \frac{\partial^2 \Pi_r}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_r}{\partial r_c^2} \end{vmatrix} = \begin{vmatrix} \frac{-2S_r d}{Q^3} & \frac{-ah_{rr}}{2d} \\ \frac{-ah_{rr}}{2d} & -2a \end{vmatrix} = (\sqrt{\frac{4aS_r d}{Q^3}} + \frac{ah_{rr}}{2d})(\sqrt{\frac{4aS_r d}{Q^3}} - \frac{ah_{rr}}{2d}) \geq 0$$

Simplification of this expression leads to condition (a) under Proposition (3.4.1).

$$\begin{aligned}
|A_3| = & \begin{vmatrix} \frac{\partial^2 \Pi_r}{\partial Q^2} & \frac{\partial^2 \Pi_r}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_r}{\partial Q \partial p_s} \\ \frac{\partial^2 \Pi_r}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_r}{\partial r_c^2} & \frac{\partial^2 \Pi_r}{\partial r_c \partial p_s} \\ \frac{\partial^2 \Pi_r}{\partial Q \partial p_s} & \frac{\partial^2 \Pi_r}{\partial r_c \partial p_s} & \frac{\partial^2 \Pi_r}{\partial p_s^2} \end{vmatrix} \\
= & -2S_r \frac{d}{Q} \left[(-2a)(-2B + \frac{bBh_{rr}Q}{d^2} - \frac{h_{rr}B^2Q}{d^3}) - (b - \frac{aBQh_{rr}}{2d^2})^2 \right] \\
& - (\frac{ah_{rr}}{2d})^2(-2B + \frac{bBh_{rr}Q}{d^2} - \frac{B^2h_{rr}Q}{d^3}) \\
& - (\frac{-ah_{rr}}{2d})(\frac{-BS_r}{Q^2} + \frac{bh_{rr}}{2d} - \frac{Bh_{rr}x}{2d^2})(b - \frac{aBQh_{rr}}{2d^2}) \\
& + (\frac{-BS_r}{Q^2} + \frac{bh_{rr}}{2d} - \frac{Bh_{rr}x}{2d^2}) \left[(\frac{-ah_{rr}}{2d})(b - \frac{aBQh_{rr}}{2d^2}) - (-2a)(\frac{-BS_r}{Q^2} + \frac{bh_{rr}}{2d} - \frac{Bh_{rr}x}{2d^2}) \right] \\
= & \frac{2aBh_{rr}S_r(Ab - aBr_c)}{2Q^2d^3} + \frac{2DS_rb^2}{Q^3} + \frac{a^2Bh_{rr}^2}{2d^2} - \frac{8aBDS_r}{Q^3} \\
& + \frac{2abh_{rr}S_r}{Q^2d} - \frac{abh_{rr}^2(Ab - aBr_c)}{2d^3} + \frac{2DB^2S_r^2}{Q^4} + \frac{ah_{rr}^2(Ab - aBr_c^2)}{2d^4} \leq 0
\end{aligned}$$

This implies condition (b) under proposition (3.4.1).

□

Proof. (Proposition 3.4.2)

Only when all the remanufacturing cost parameters are smaller than the corresponding cost parameters associated with producing the product afresh, the producer would benefit from a remanufacturing strategy. In addition, we check whether it is profitable for the producer to resort to such a strategy. The profit per time unit for the manufacturer without remanufacturing is

$$\Pi_m^{'} = (p_w - c_s)d - \frac{d}{Q}(S_m + S_{rm} + S_i) - \frac{h_m Q d}{2m} - \frac{h_m Q d}{2m} - \frac{h_i Q d}{2m} - c_m d$$

Comparing this with equation (3.3.4) it is clear that the manufacturer can benefit from remanufacturing if $\Pi_m - \Pi_m^{'} \geq 0$, i.e.

$$-(r_m + c_r)x - \frac{h_{ir} Q x^2}{2md} - \frac{h_i Q}{2m}(\frac{x^2}{d} - 2x) + c_m x - c_{rm} x \geq 0$$

This directly yields condition (e) under proposition (3.4.2).

□

Proof. (Proposition 3.4.3)

If Q^{***},$r_c^{***}$and p_s^{***} obtained from (3.4.11), (3.4.13) and (3.4.14) are to be globally optimal, the sufficient condition is that the objective function (3.3.5) should be jointly concave in these three variables. The Hessian for the manufacturer's profit function (3.3.4) is

$$D^2\Pi_m(Q, r_c, p_s) = \begin{vmatrix} \frac{\partial^2 \Pi_m}{\partial Q^2} & \frac{\partial^2 \Pi_m}{\partial Q \partial p_s} & \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} \\ \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_m}{\partial p_s^2} & \frac{\partial^2 \Pi_m}{\partial r_c \partial p_s} \\ \frac{\partial^2 \Pi_m}{\partial Q \partial p_s} & \frac{\partial^2 \Pi_m}{\partial r_c \partial p_s} & \frac{\partial^2 \Pi_m}{\partial r_c^2} \end{vmatrix} = \begin{vmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{vmatrix}$$

where

$$h_{11} = \frac{-2d(S_m + S_{rm} + S_i)}{Q^2}$$

$$h_{22} = \frac{-(h_{ir} - h_i)Q(db - xB)^2}{md^3}$$

$$h_{33} = -\frac{-a^2 Q(h_{ir} - h_i)}{md}$$

$$h_{12} = h_{21} = \frac{-B(S_m + S_{rm} + S_i)}{Q^2} + \frac{Bh_m - h_i(2b - B)}{2m} + \frac{(2db - Bx)x(h_{ir} - h_i)}{2md^2}$$

$$h_{13} = h_{31} = \frac{ah_i}{2m} - \frac{ax(h_{ir} - h_i)}{md}$$

$$h_{23} = h_{32} = \frac{aQ(h_{ir} - h_i)(db - xB)}{md^2}$$

Set $H = D^2\Pi_m(Q, r_c, p_s)$, then if H is negative semi-definite, the following must hold:

$$|H_1| = \frac{\partial^2 \Pi_m}{\partial Q^2} = h_{11} = \frac{-2d(S_m + S_{rm} + S_i)}{Q^2} \leq 0$$

Since all variables and parameters are nonnegative, this requirement is always satisfied.

$$\Big|H_{12}\Big| = \begin{vmatrix} \frac{\partial^2 \Pi_m}{\partial Q^2} & \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} \\ \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_m}{\partial r_c^2} \end{vmatrix} = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} \geq 0$$

It can be easily verified that the above inequality results in condition (c) in Proposition (3.4.3).

$$\begin{aligned} |H_3| = &\begin{vmatrix} \frac{\partial^2 \Pi_m}{\partial Q^2} & \frac{\partial^2 \Pi_m}{\partial Q \partial p_s} & \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} \\ \frac{\partial^2 \Pi_m}{\partial Q \partial p_s} & \frac{\partial^2 \Pi_m}{\partial p_s^2} & \frac{\partial^2 \Pi_m}{\partial r_c \partial p_s} \\ \frac{\partial^2 \Pi_m}{\partial Q \partial r_c} & \frac{\partial^2 \Pi_m}{\partial r_c \partial p_s} & \frac{\partial^2 \Pi_m}{\partial r_c^2} \end{vmatrix} \\ = &\frac{a^2B^2Q(h_{ir} - h_i)(h_i + h_m)^2}{4m^3d} - \frac{a^2B^2Qxh_i(h_{ir} - h_i)(h_i - h_m)}{m^3d^2} \\ - &\frac{a^2BQx^2(h_{ir} - h_i)(Bh_{ir}h_i - 6bh_{ir}h_i + Bh_{ir}h_m - 5Bh_i^2 + 6bh_i^2 - Bh_ih_m)}{4m^3d^3} \\ - &\frac{a^2B^2Qx^3h_i(h_{ir} - h_i)^2}{m^3d^4} + \frac{a^2B^2Qx^4(h_{ir} - h_i)^3}{4m^3d^5} \\ + &\frac{a^2B^2(h_{ir} - h_i)(S_m + S_{rm} + S_i)^2}{Q^3md} - \frac{a^2B^2(h_{ir} - h_i)(h_i - h_m)(S_m + S_{rm} + S_i)}{Qm^2d} \\ + &\frac{2a^2B^2xh_i(h_{ir} - h_i)(S_m + S_{rm} + S_i)}{Qm^2d^2} - \frac{a^2B^2x^2(h_{ir} - h_i)^2(S_m + S_{rm} + S_i)}{Qm^2d^3} \leq 0 \end{aligned}$$

Again, this inequality reduces to condition (d) in Proposition (3.4.3).

Conditions (c) and (d) are necessary to ensure that the profit function (3.3.4) of the manufacturer is jointly concave with respect to Q^*, r_c^* and p_s^*. Also, as shown earlier, conditions (a) and (b) are sufficient for joint concavity of the retailer's profit function (3.3.2). Since the total supply chain profit (3.3.5) is the sum of (3.3.4) and (3.3.2), all four of these conditions are necessary for it to be jointly concave in Q^*, r_c^*, and p_s^*.

$\square$

Appendix B. Tables

Table B.1: Summary of Results(1)

		Q (units)	p_s ($/unit)	r_c ($/unit)	p_w ($/unit)
Retailer's optimal policy	given p_w	428.5	30.032	1.485	20
	variable p_w	388.4	31.680	1.498	23.286
Manufacturer's optimal policy	given p_w	6083.1	29.900	1.400	20
	variable p_w	12146.0	32	1	24.138
Centralized optimal policy		1562.4	23.3	0.5	-

Table B.2: Summary of Results(2)

		d (units/day)	X (units/day)	$\prod_r$ ($/day)	$\prod_m$ ($/day)	$\prod_s$ ($/day)
Retailer's optimal policy	given p_w	29.904	19.391	318.361	299.918	618.279
	variable p_w	24.960	19.299	228.215	358.616	585.830
Manufacturer's optimal policy	given p_w	30.200	18.000	275.698	377.596	653.293
	variable p_w	24.000	16.000	113.753	399.039	512.792
Centralized optimal policy		50.2	5.5	427.781*	402.999*	830.780

Table B.3: Parameters Table (1)

		π	α(ton/acre)			χ		
biofuel	crops	(\$/ton)	$E1$	$E2$	$E3$	$E1$	$E2$	$E3$
ethanol	wheat	-42.3	26.46	24.955	24.5	0.07	0.1	0.13
	corn	-49.3	30.4	30.4	34.2	0.12	0.11	0.08
	sugar	33	28	28	28	0.14	0.07	0.1
ester	rapeseed	-4	50	50	50	0.08	0.13	0.11
	sunflower	11	80	80	80	0.1	0.09	0.12

Table B.4: Parameters Table (2)

	m(\$/acre)		
crops	$E1$	$E2$	$E3$
wheat	609.04	608.04	607.04
barley	121.2	123.2	125.2
corn	1813	1708	1713
sugar	200	200	200
rapeseed	472.8	272	272
sunflower	479.8	279	279
peas	234.3	231.3	230.3

Table B.5: Parameters Table (3)

	t(\$)			c(\$/acre)		
crops	$E1$	$E2$	$E3$	$E1$	$E2$	$E3$
wheat	1956000	299000	212000	1262.53	1262.53	1262.53
corn	200000	200000	200000	1164.25	1764.25	1764.25
sugar	200000	200000	200000	164.25	164.25	164.25
rapeseed	800000	200000	600000	162.52	162.52	162.52
sunflower	273000	270000	290000	236.42	232.52	252.52

Table B.6: Solution of the Numerical Example Without Fixed Cost

x(acres)	F1	F2	F3
wheat	0	0	0
barley	0	0	0
corn	264	1606	504
sugar	0	0	0
rapeseed	264	730	567
sunflower	330	1314	756
peas	1287	5840	2268
xn(acres)	F1	F2	F3
wheat	231	1460	819
corn	132	0	0
sugar	462	1022	630
rapeseed	0	1168	126
sunflower	0	0	0
xf(acre)	330	1460	630
τ_{ester}($/ton)	3.93	$\tau_{ethanol}$($/ton)	45.77
gov_{obj} ($)	5747435	$farm_{obj}$($)	11219413

Table B.7: Solution of the Numerical Example with Fixed Cost

x(acres)	F1	F2	F3
wheat	0	0	0
barley	0	0	0
corn	264	1606	504
sugar	0	0	0
rapeseed	264	0	567
sunflower	330	1314	756
peas	1287	6570	2268
xn(acres)	F1	F2	F3
wheat	231	1460	819
corn	132	0	0
sugar	462	292	630
rapeseed	0	1898	126
sunflower	0	0	0
xf(acres)	330	1460	630
τ_{ester}($/ton)	3.95	$\tau_{ethanol}$($/ton)	43.50
gov_{obj}($)	4828156	$farm_{obj}$($)	6915626

Table B.8: Computational Efficiency of the Proposed Solution Methods

$(\tau^0_{ester}, \tau^0_{ethanol}) = (250, 250)$, $\epsilon_0 = 25$			
	CPU time	iterations	skipped points
without sensitivity analysis and warm-starts without fixed cost	21.32	24371	0
with sensitivity analysis and warm-starts without fixed cost	8.05	3379	170
without sensitivity analysis and warm-starts with fixed cost	105.40	46961	0
with sensitivity analysis and warm-starts with fixed cost	75.92	27483	13

Appendix C. Figures and Graphics

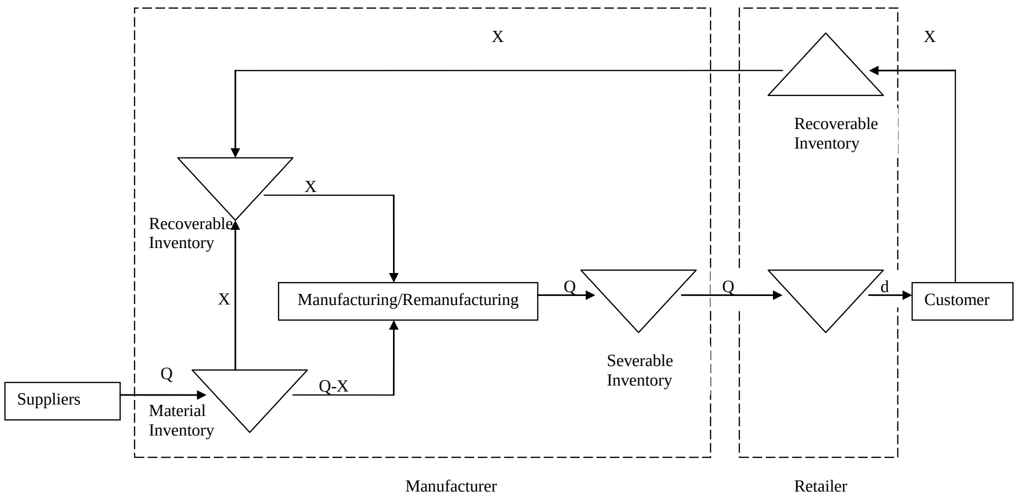

Figure C.1: The Recovery and Remanufacturing Process

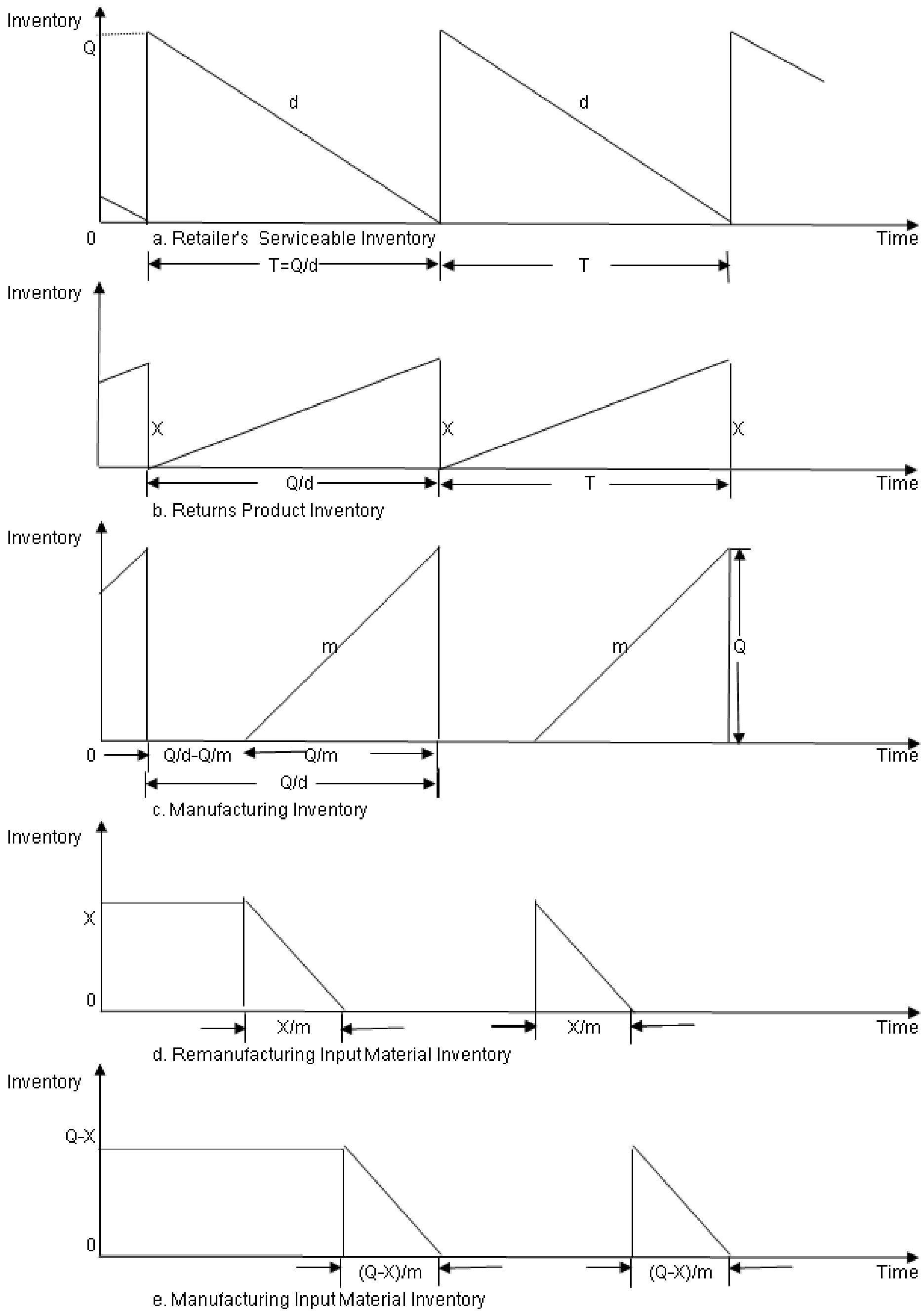

Figure C.2: Inventory Time Plot

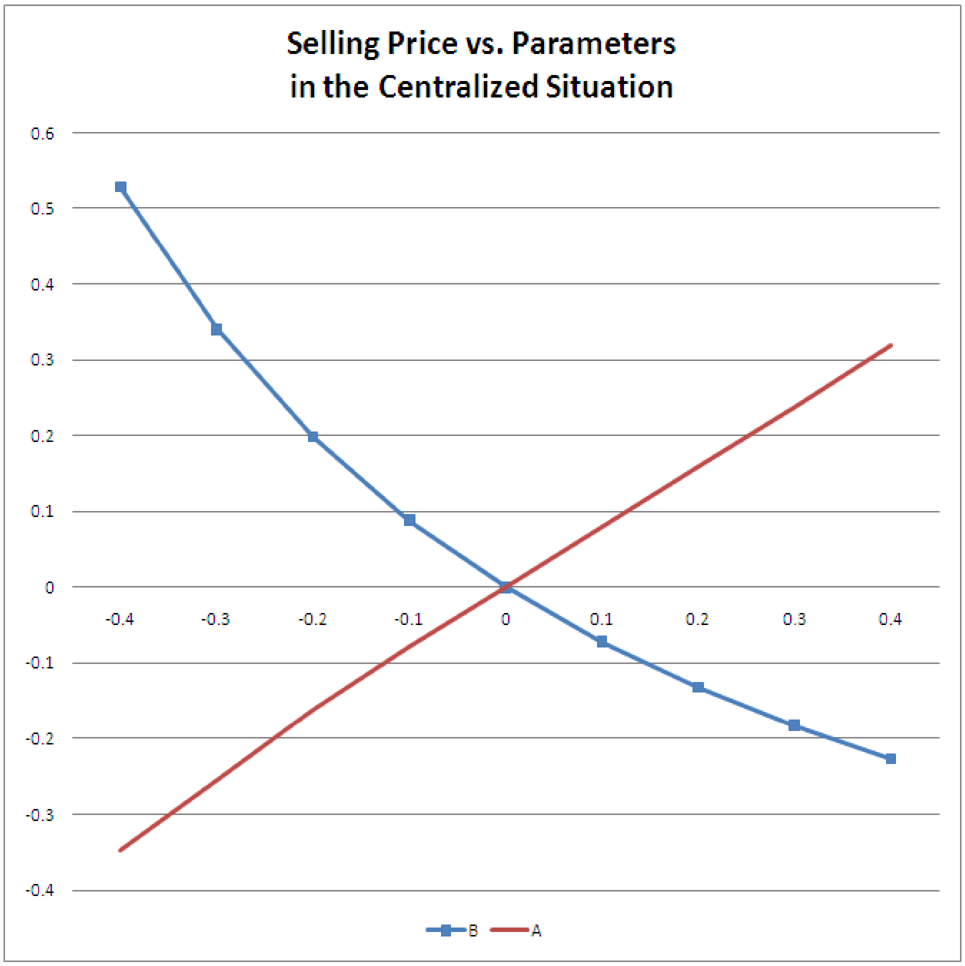

Figure C.3: p_s vs. Parameters in the Centralized Situation

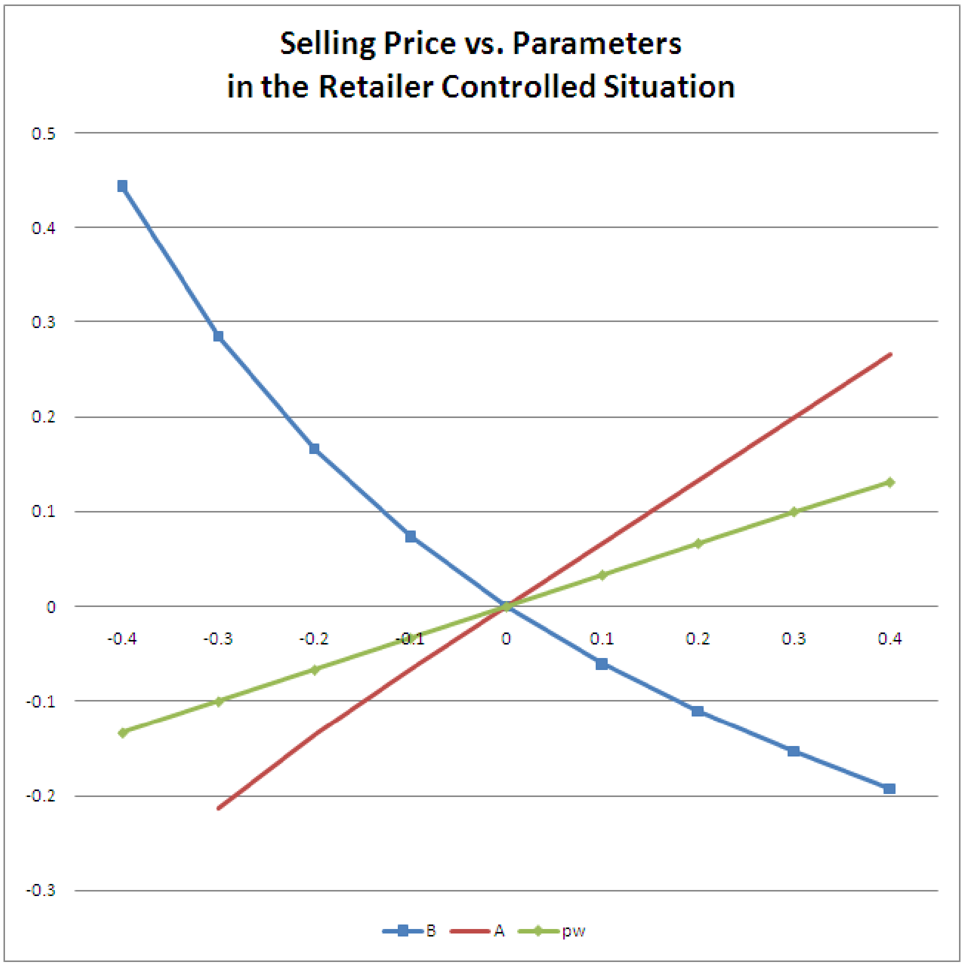

Figure C.4: p_s vs. Parameters in the Retailer Controlled Situation

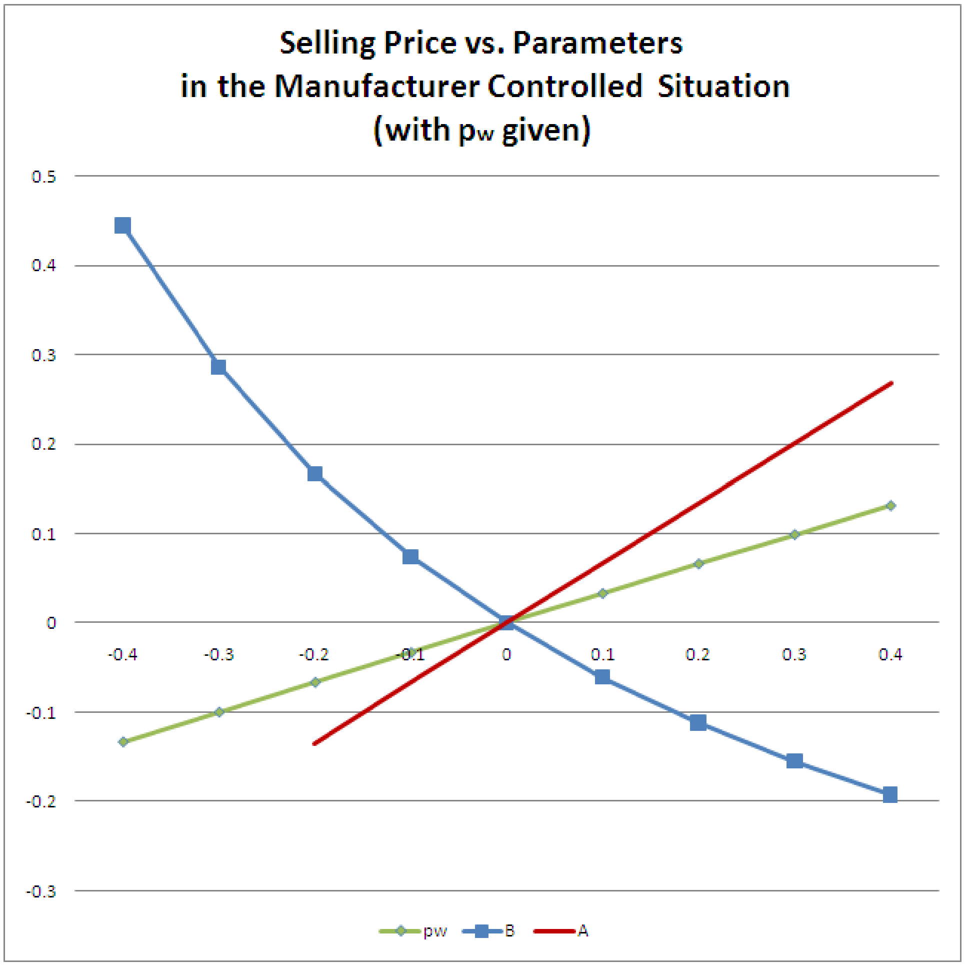

Figure C.5: p_s vs. Parameters in the Manufacturer Controlled Situation (p_w given)

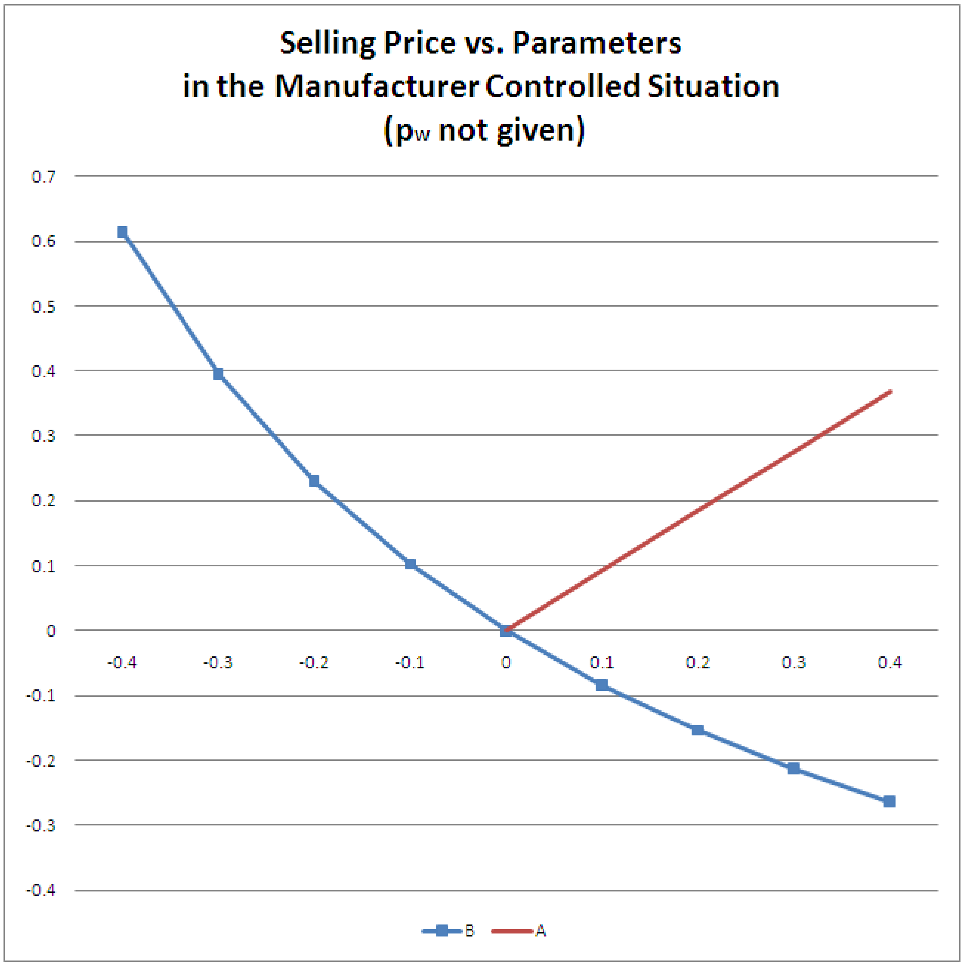

Figure C.6: p_s vs. Parameters in the Manufacturer Controlled Situation (p_w not given)

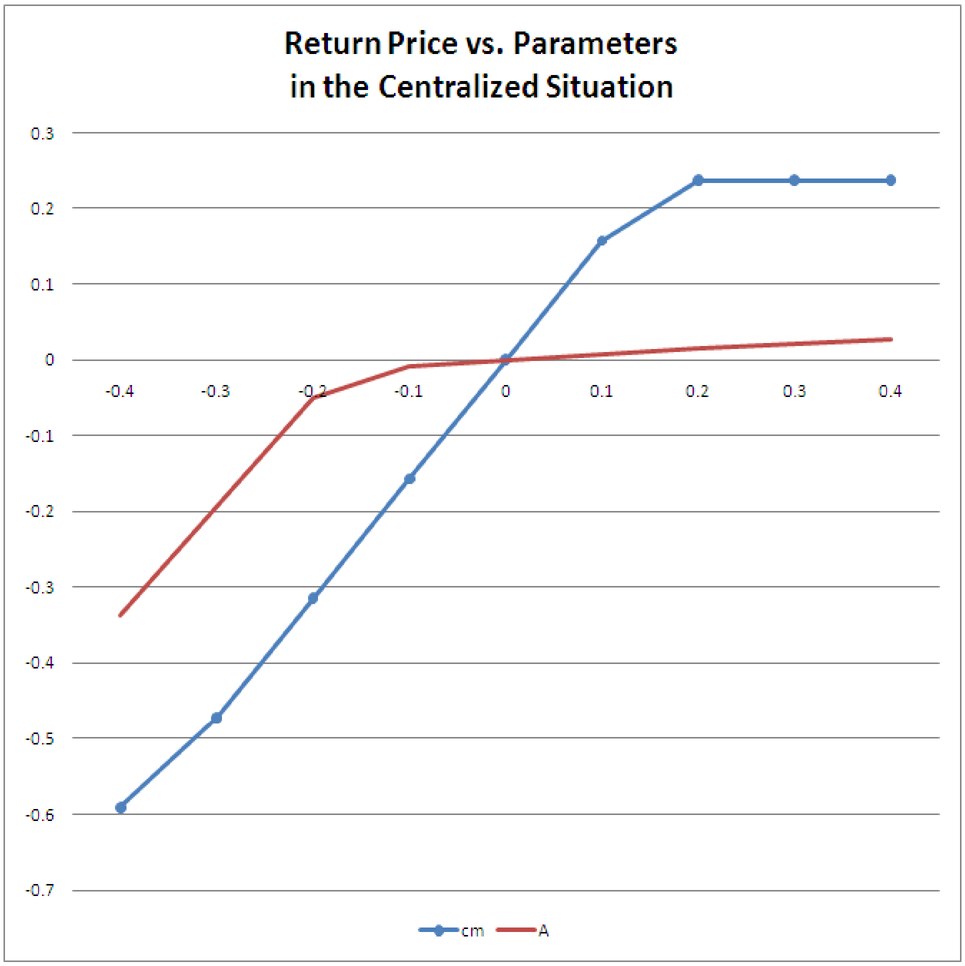

Figure C.7: r_c vs. Parameters in the Centralized Situation

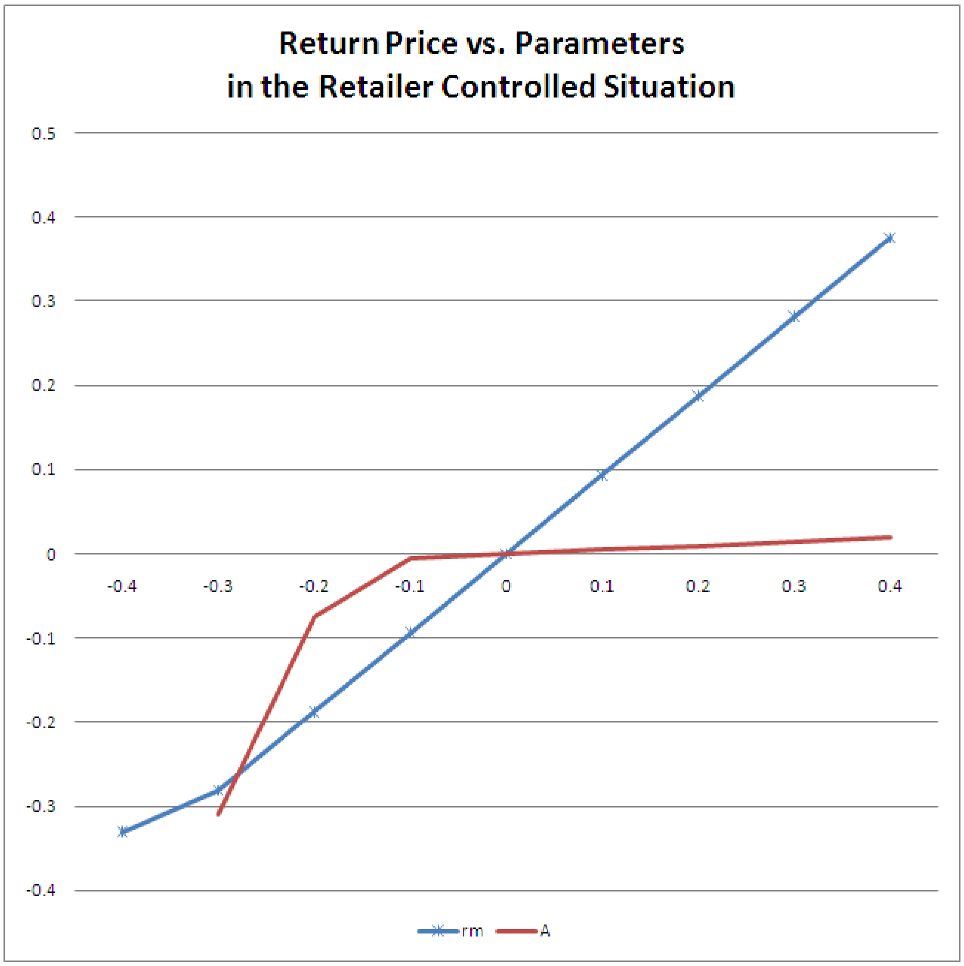

Figure C.8: r_c vs. Parameters in the Retailer Controlled Situation

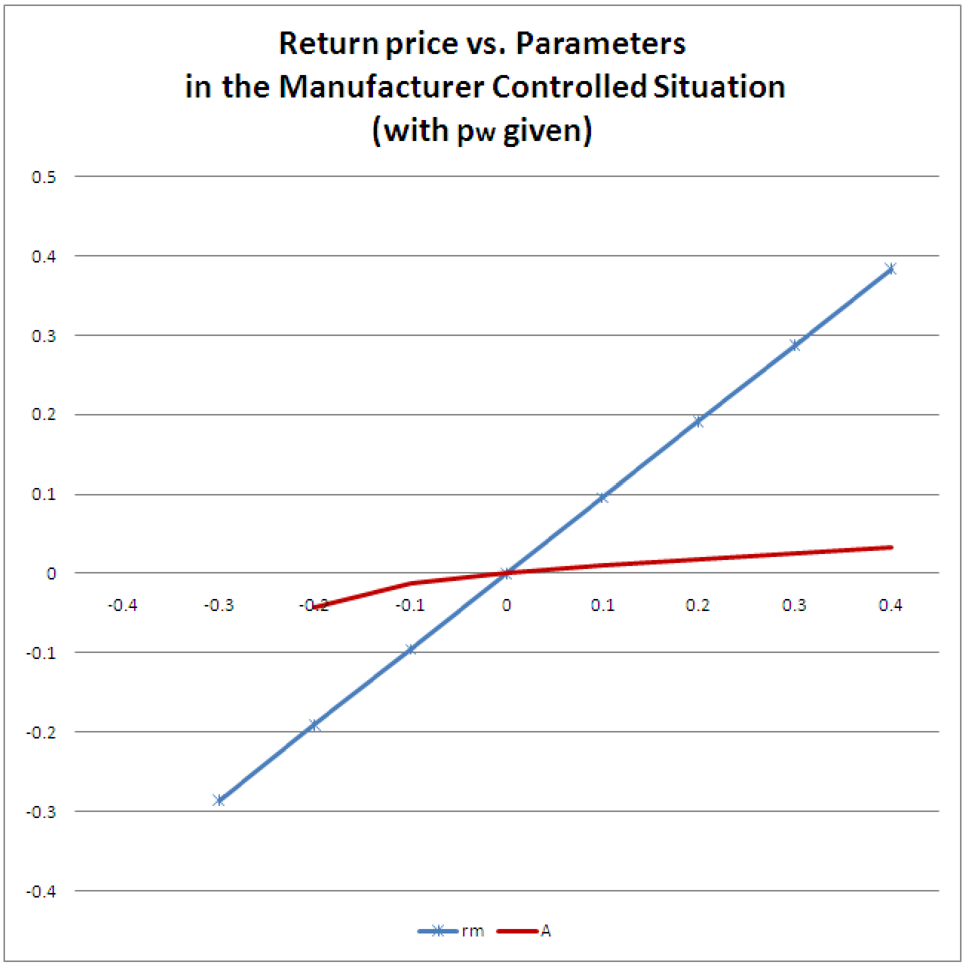

Figure C.9: r_c vs. Parameters in the Manufacturer Controlled Situation (p_w given)

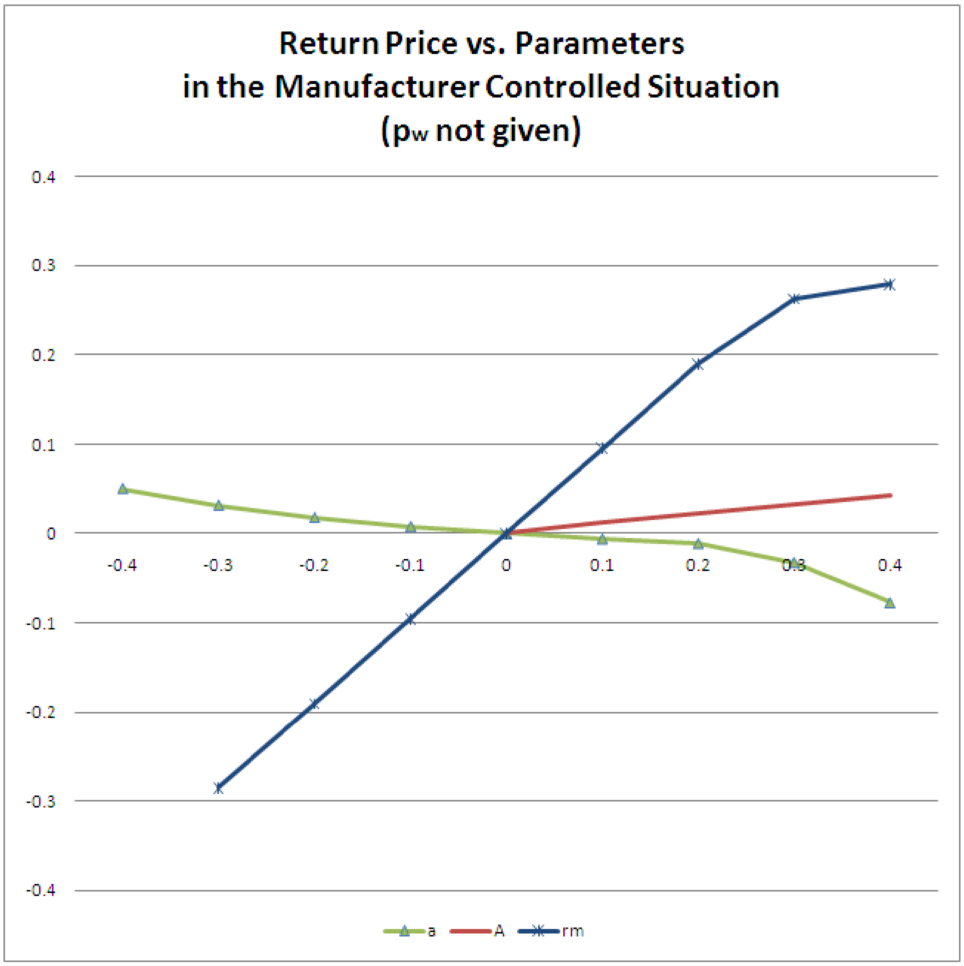

Figure C.10: r_c vs. Parameters in the Manufacturer Controlled Situation (p_w not given)

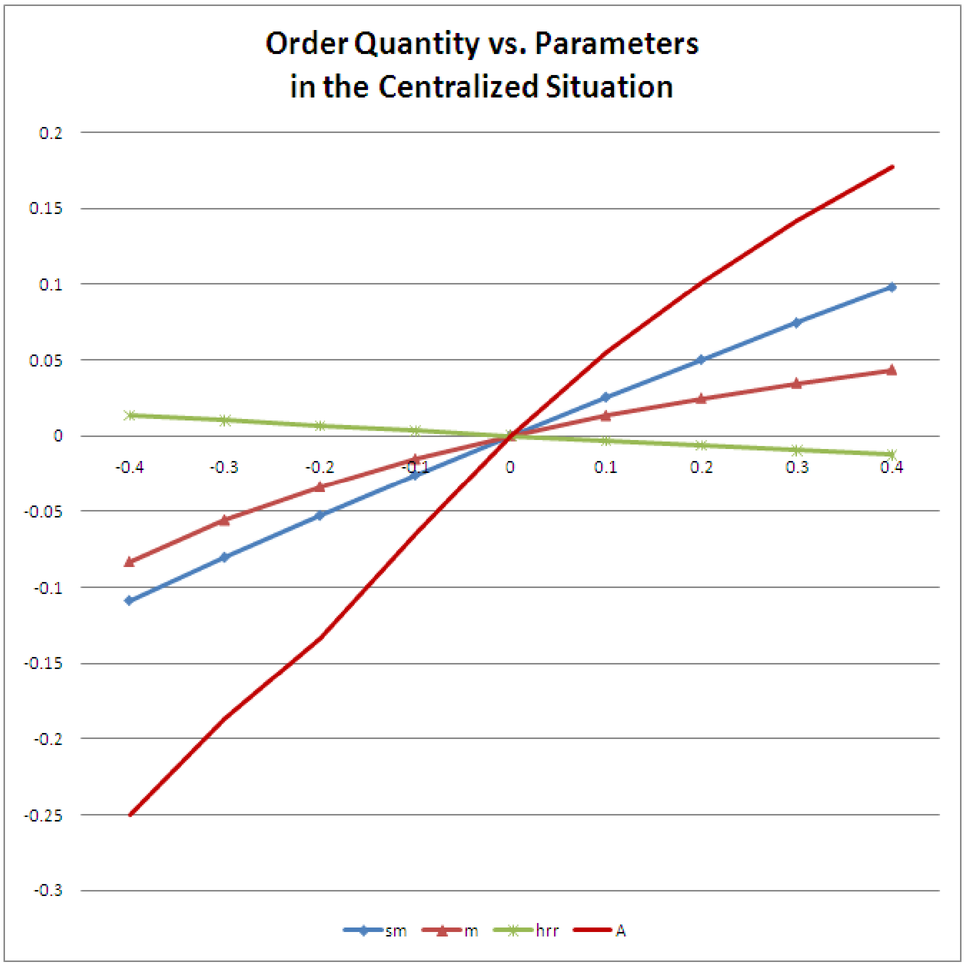

Figure C.11: Q vs. Parameters in the Centralized Situation

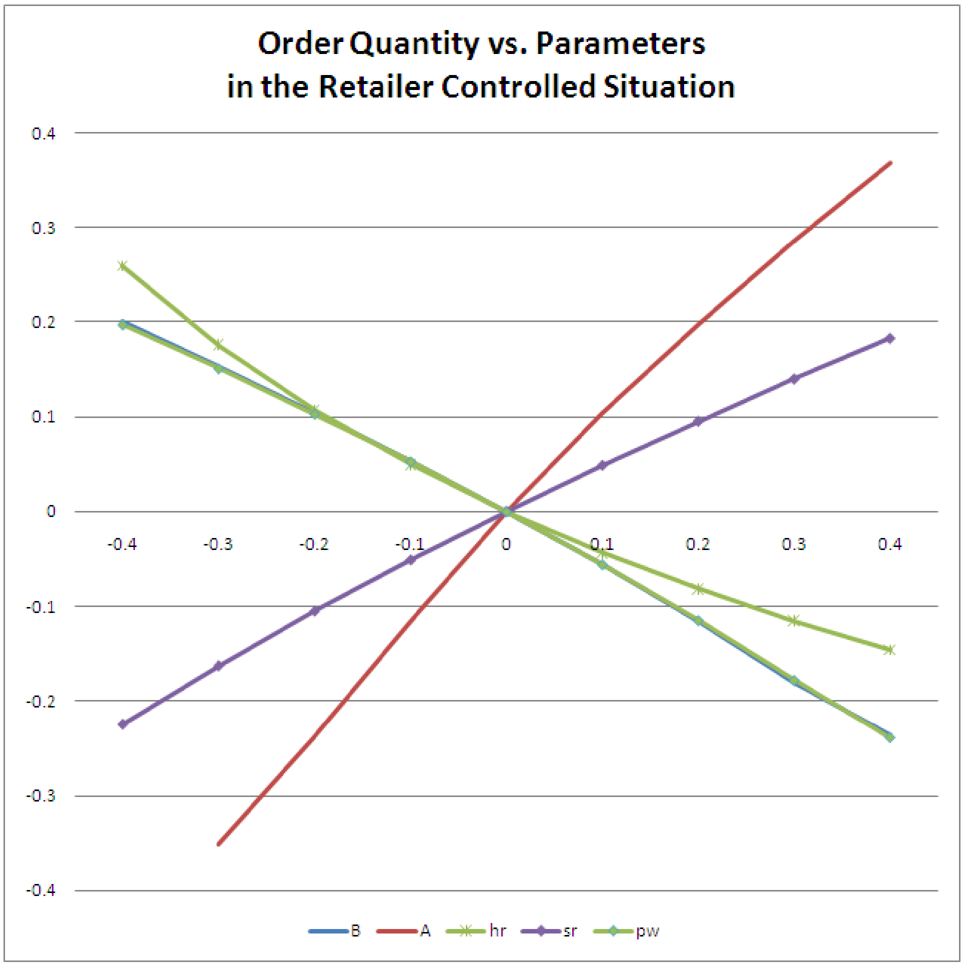

Figure C.12: Q vs. Parameters in the Retailer Controlled Situation

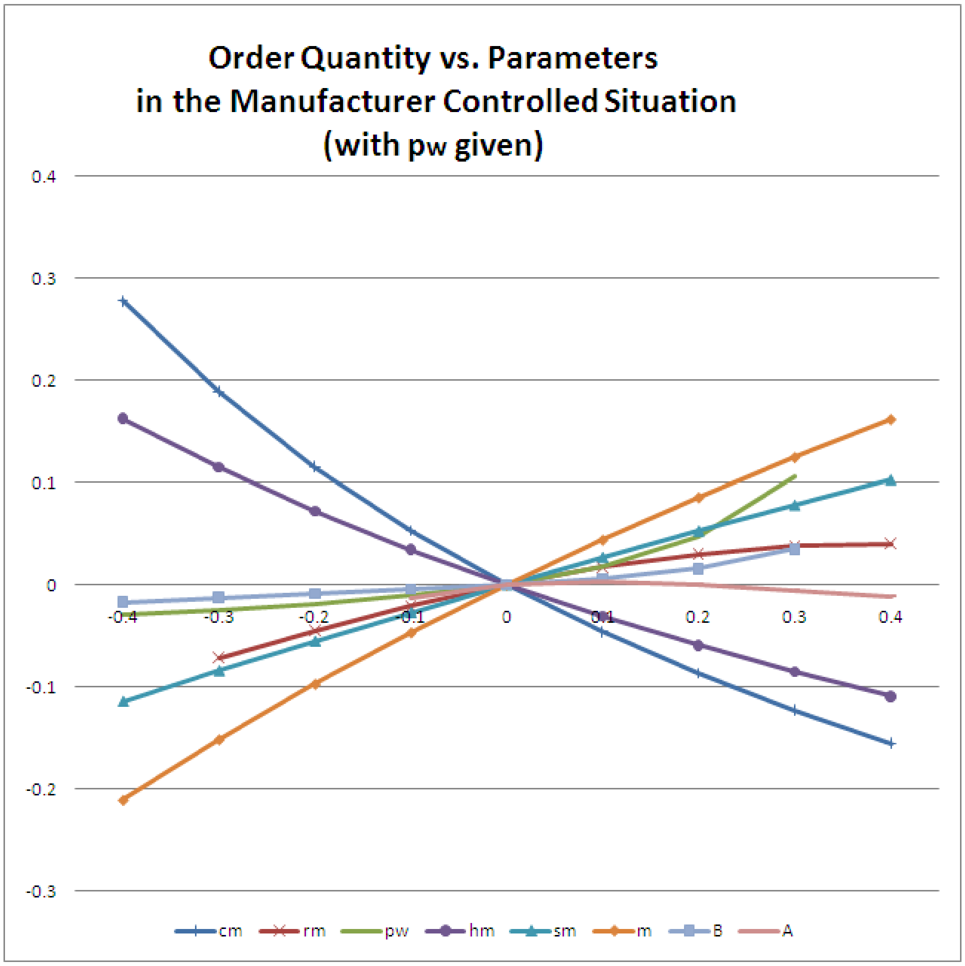

Figure C.13: Q vs. Parameters in the Manufacturer Controlled Situation (p_w given)

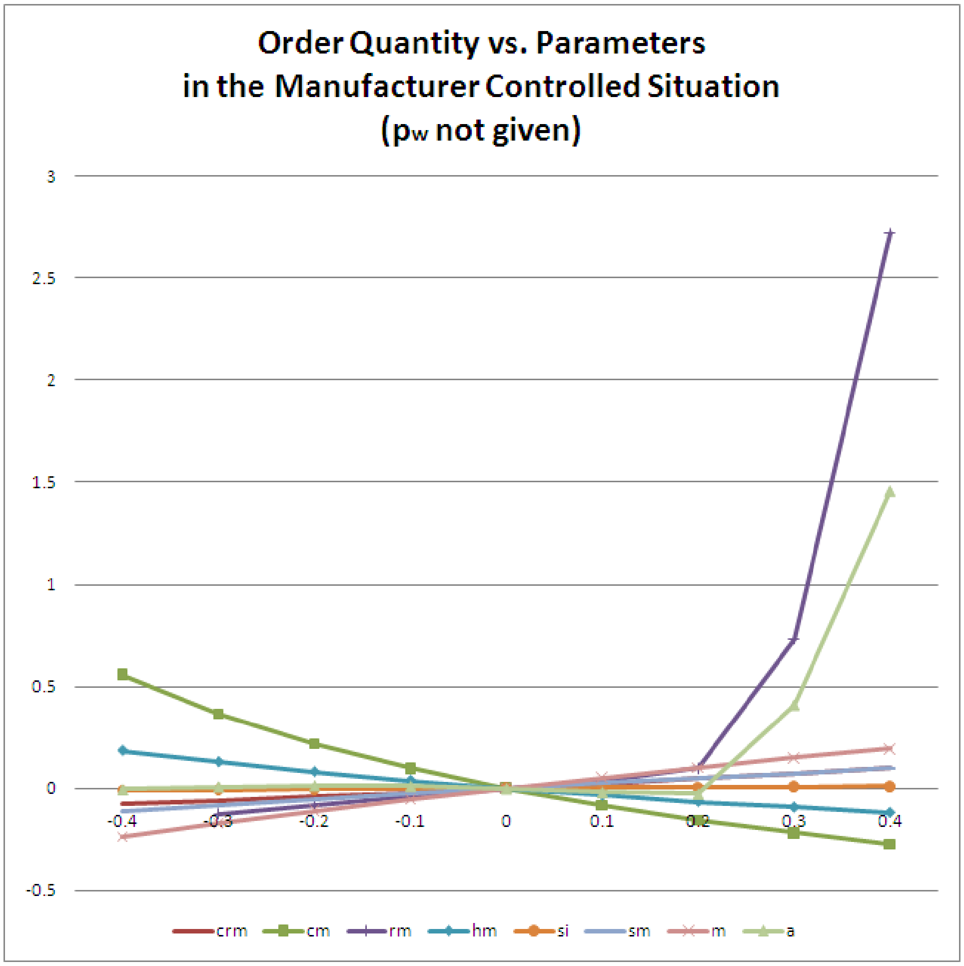

Figure C.14: Q vs. Parameters in the Manufacturer Controlled Situation (p_w not given)

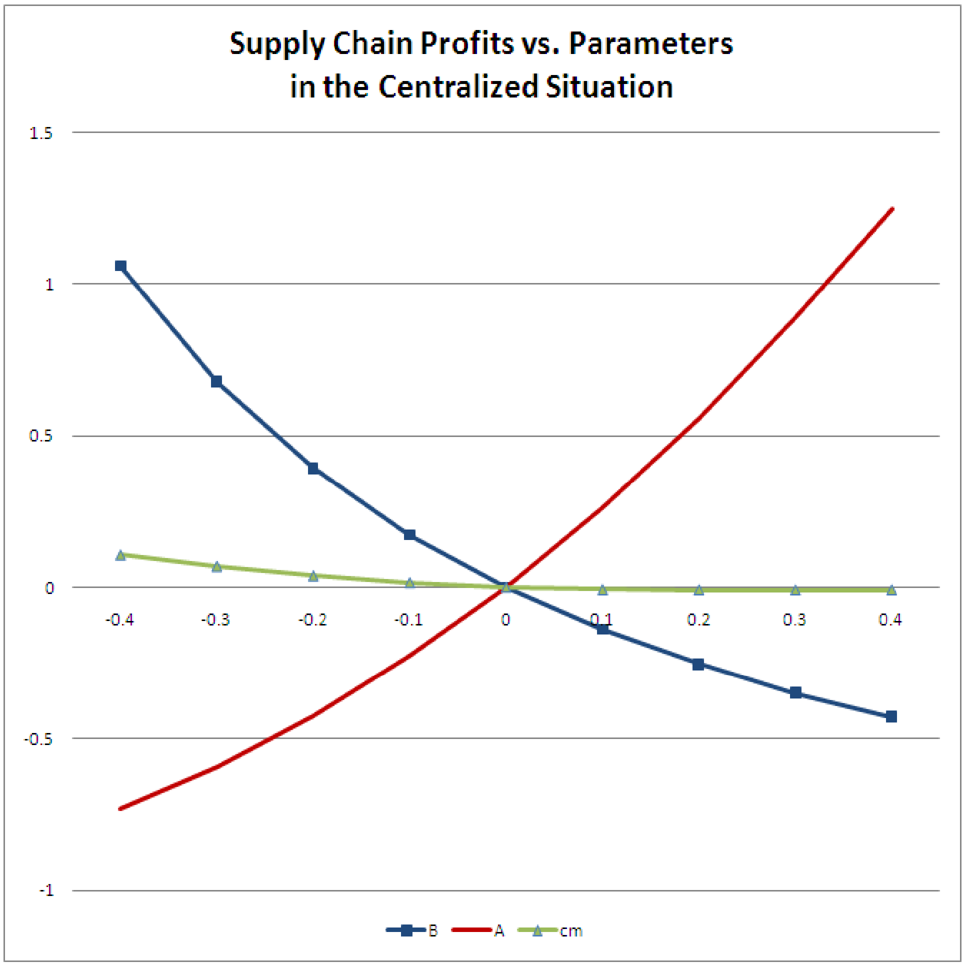

Figure C.15: Supply Chain Profits in the Retailer Controlled Situation

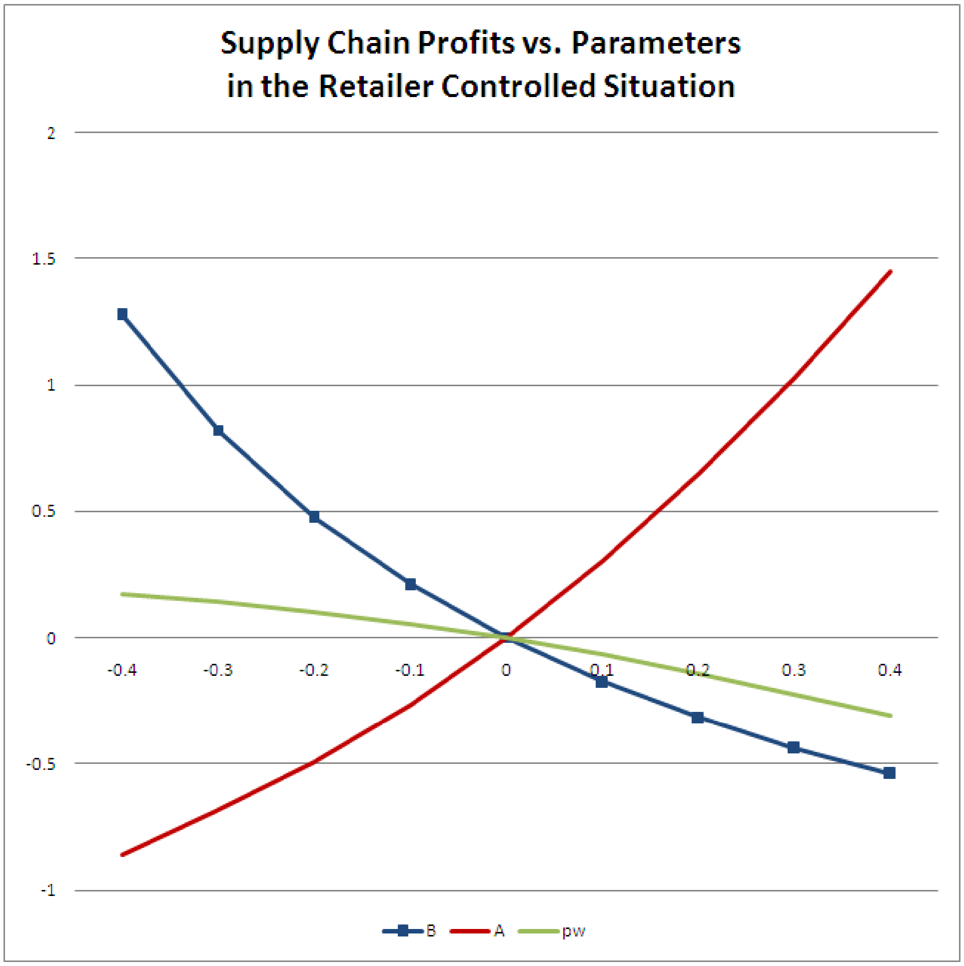

Figure C.16: Supply Chain Profits in the Manufacturer Controlled Situation (p_w given)

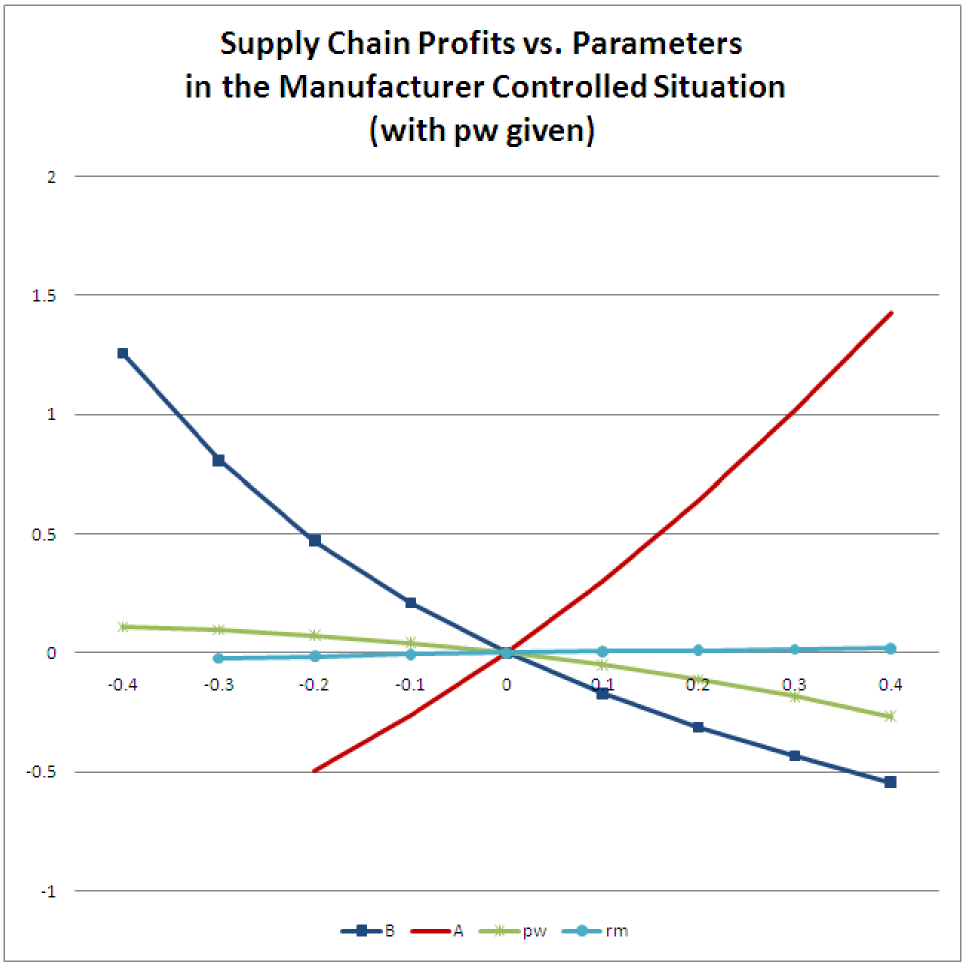

Figure C.17: Supply Chain Profits in the Manufacturer Controlled Situation (p_w given)

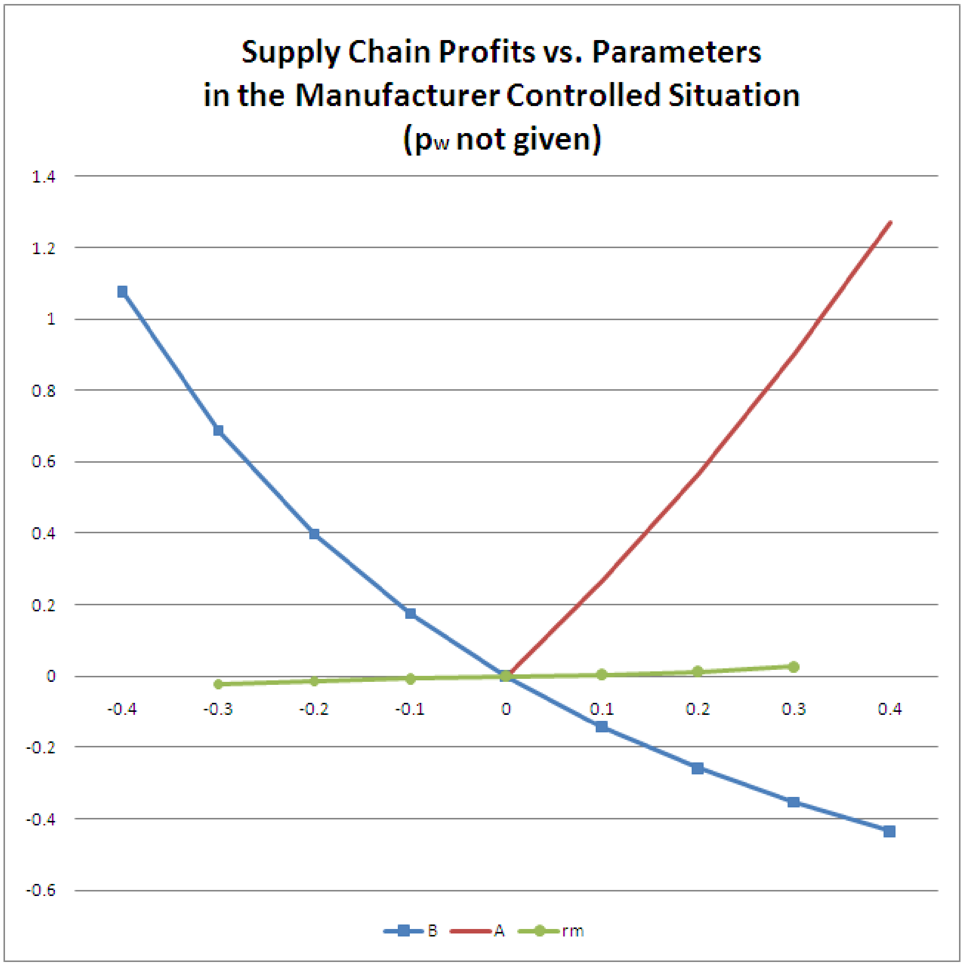

Figure C.18: Supply Chain Profits in the Manufacturer Controlled Situation (p_w not given)

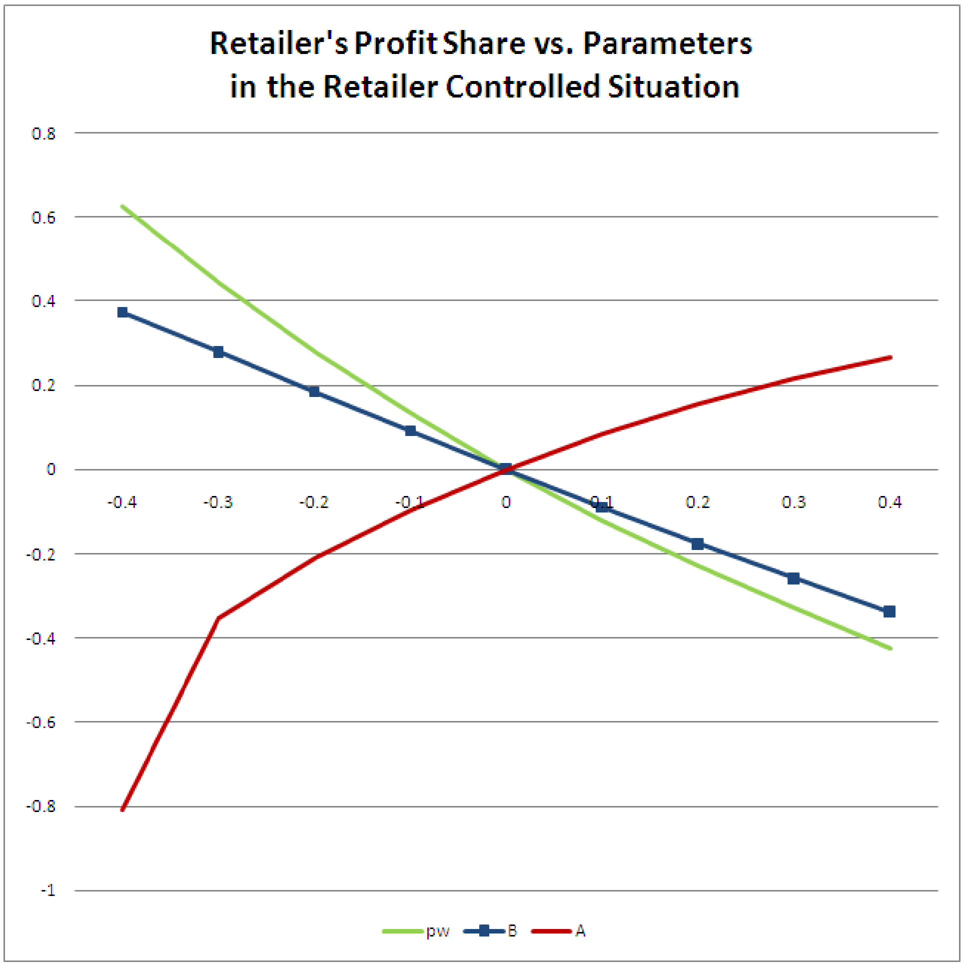

Figure C.19: Π_r/Π_s in the Retailer Controlled Situation

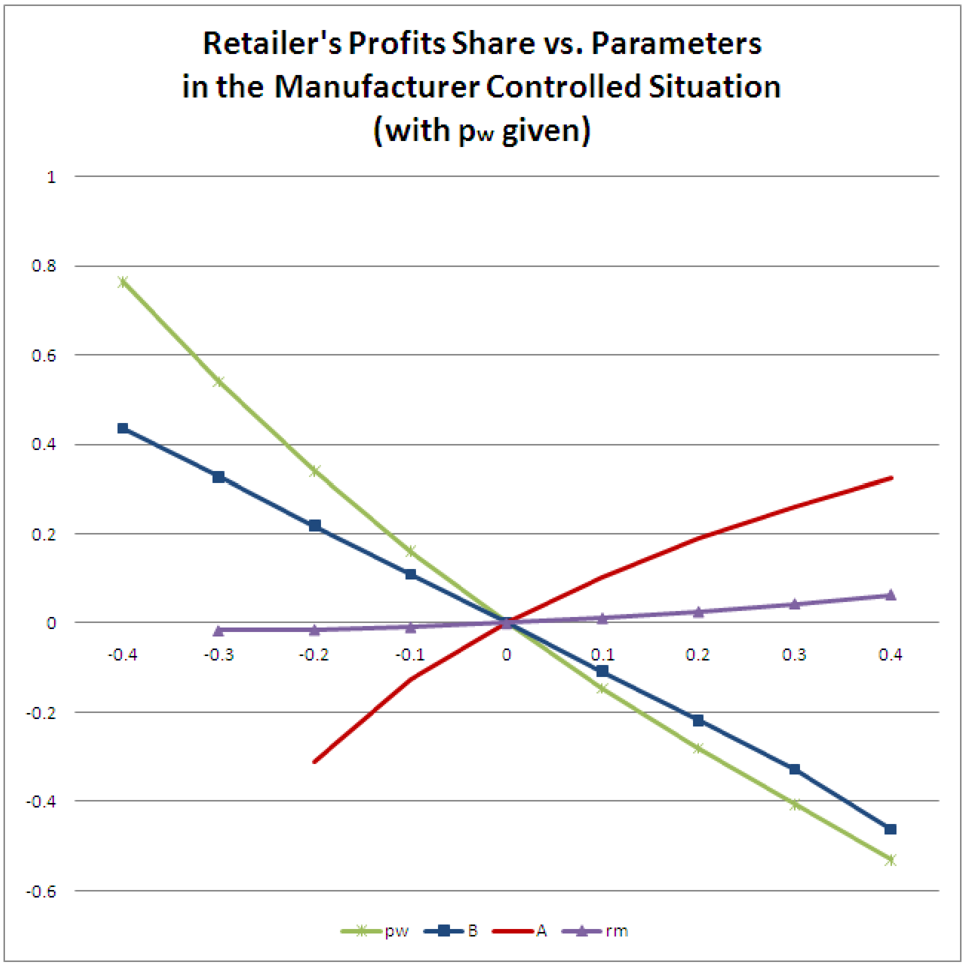

Figure C.20: Π_r/Π_s in the Manufacturer Controlled Situation (p_w given)

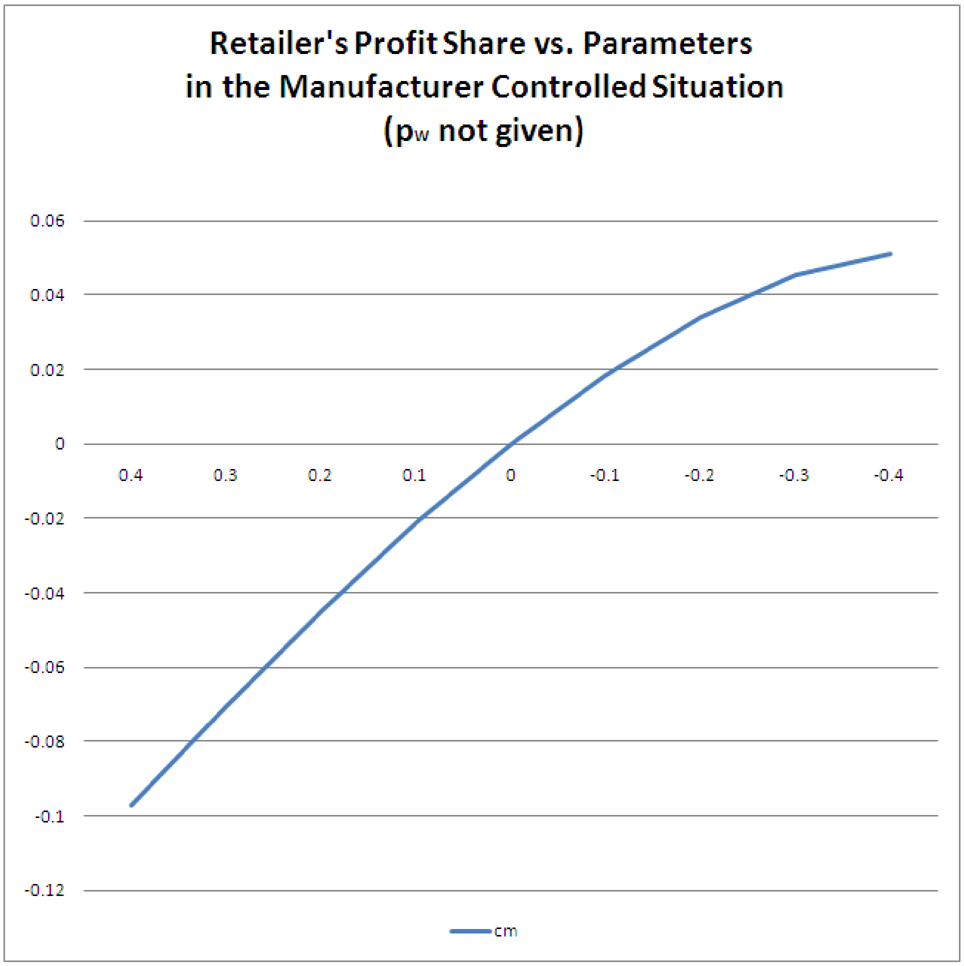

Figure C.21: Π_r/Π_s in the Manufacturer Controlled Situation (p_w not given)

Vita

Xiangrong Liu was born in Hefei, P. R. China. In 1996, she enrolled as a Technology Economics major at Wuhan University. While still in college, she took courses in Economic Law and got a second degree in Law. After she graduated from Wuhan University, she worked for an Electricity company for one year as an operations assistant. Then she joined the graduate program at Wuhan University with a major in Management Sciences. In 2004, she was admitted to the Ph.D Program at Drexel University. She is a member of Beta Gamma and Sigma, a national honorary society for business students.

Most of her research is at the interface of operations management and operations research. She has several proceedings publications, including those titled “A Collaborative Contract to Maintain Long-term Relationship and Quality Improvement in Supply Chain”, “Modified DEA Models for Supplier Selection to Coordinate between Supplier and Buyer,” and one journal publication, titled “Replicating Meier and Mueller’s 2006 JMCB Article”. On the basis of her dissertation she has written two papers “Integrated Inventory Models for Retail Pricing and Return Reimbursements in a JIT Environment for Remanufacturing a Product” and “A Discrete Bilevel Programming Problem Arising in Biofuel Production,” which are ready for submission as journal publications.

During her Ph.D program, Xiangrong Liu independently taught operations management multiple times. She also served as an instructor at Rider University for the course Statistical Methods in Fall, 2009. She won the Best Teaching Assistant Award in 2008.