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Continuous Optimization

Natural gas cash-out problem: Bilevel stochastic optimization approach

Vyacheslav V. Kalashnikov[a,*,1], Gerardo A. Pérez-Valdés[a], Asgeir Tomasgard[b], Nataliya I. Kalashnykova[c]

[a] Department of Systems and Industrial Engineering, ITESM, Campus Monterrey (Tecnológico de Monterrey), av. Eugenio Garza Sada 2501 Sur, Monterrey, Nuevo León 64849, Mexico
[b] Department of Industrial Economics and Technology Management, NTNU, Trondheim NO-7491, Norway
[c] Physics and Mathematics Department, University of Nuevo León, San Nicolás de los Garza, Nuevo León 66450, Mexico

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ABSTRACT

A stochastic formulation of the natural gas cash-out problem is given in a form of a bilevel multi-stage stochastic programming model with recourse. After reducing the original formulation to a bilevel linear problem, a stochastic scenario tree is defined by its node events, and time series forecasting is used to produce stochastic values for data of natural gas price and demand. Numerical experiments were run to compare the stochastic solution with the perfect information solution and the expected value solutions.

1. Introduction

In 1992 in the United States (Energy Information Administration, 1992, 2008; Soto, 2008), and in 1995 in the European Union (IHS Engineering, 2008), a number of regulation acts were issued by the respective governmental institutions to effectively separate several of the processes that formed the natural gas supply chain. The resulting market configuration demanded the independence of the transportation and commercialization processes. As a result of this paradigm shift – and the accompanying re-structurization of the market, – a systematic analysis of several new features becomes indispensable.

Of particular interest to us is a problem that arises in the natural gas supply chain, namely that of *balancing* the fuel volumes over a distribution network. Such a balancing procedure directly concerns the Pipeline Operating Company (POC), since the correct operation of the pipeline means the well controlled volumes of the transported gas. Moreover, any Natural Gas Shipping Company (NGSC) is also concerned with the balancing the volumes because it is often impossible to avoid an *imbalance* justified by certain economic reasons. A natural gas shipping company's business is to sell the gas by moving it through the pipeline to its clients: it has to fulfill signed contracts first, and then market excesses of the gas to achieve the maximum profits. In order to do that, the NGSC has to manage the volumes at each selling point (so-called *pipeline meters*) taking into account the balance, the selling prices, and the total revenue. The basic mathematical framework of this problem's modeling is found in Ríos-Mercado et al. (1999).

We must emphasize that, while natural gas pipeline networks have been thoroughly studied, most of the existent models focus on aspects of this part of the supply chain other than the NGSC–POC interaction in the system balancing, such as network operation optimization (Borraz-Sánchor and Rís Mercado, 2005; Chebouba et al., 2009), or deployment of facilities (Kabirian and Hemmati, 2007). There are also papers considering the natural gas supply chain in a multilevel scheme, in which both the NGSC and the POC are present and accounted for, such as the related Gabriel et al. (2005) and Egging et al. (2008). These works are remarkable in the sense that they span the whole supply chain with much emphasis on the traders (financial front-ends of the natural gas producers,) so that there is little to no mention of imbalances in the system resulting from the dealings of the NGSCs and the POCs, even though both actors are present in the models.

Many authors do acknowledge (Hawdon, 2003; Arano and Blair, 2008) the existence of a problematic situation in the NGSC–POC system following the paradigm shift, yet we have found very few sources that explain plausible ways in which this problem is nowadays solved. Esnault (2003), for example, shows how storage is required by the NGSC when no flexibility exists in the network volume management, either because it is not allowed, or because it is not technically possible. Nevertheless, balancing is an important part of the modern natural

* Corresponding author. Tel.: +52 81 8358 2000x5548; fax: +52 81 8358 0771.
E-mail addresses: kalash@itesm.mx, slavkamx@mail.ru (V.V. Kalashnikov), gerardo.perez@ttu.edu (G.A. Pérez-Valdés), Asgeir.Tomasgard@sintef.no (A. Tomasgard), nkalash@einstein.fcfm.uanl.mx (N.I. Kalashnykova).
[1] On leave from the Central Economics and Mathematics Institute (CEMI) of the Russian Academy of Sciences (RAS), Nakhimovsky pr. 17, Moscow 117418, Russia.

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gas supply chain management, and to date, no policy has been accepted as optimal regarding the way, in which the imbalances produced by the NGSC, are physically and economically handled (Keyaerts et al., 2008). Among the most important tools that aid the POC in its task of restoring the balance of the system are the *arbitrage penalization policies*, in which the POC performs a maintenance redistribution of the imbalances in the system and charges the NGSC(s) for the cost of this operation.

In Kalashnikov and Ríos-Mercado (2001), one finds a modeling framework (which we are going to follow) of the penalization part of this problem. This penalization refers only to the cash-out that occurs between the NGSC and the POC: it leaves outside any reference to actual market conditions, which are obviously important to the NGSC. The paper presents a solution method through a modification of the original problem, as well as the analysis of how this modification affects the objective function and the obtained solutions. In Kalashnikov and Ríos-Mercado (2006), the authors compare two algorithms that solve the problem making use of certain numerical procedures. In the present paper, we adapt these algorithms to our extended model. We also make use of the idea proposed in Dempe et al. (2005) to divide our problem into several *generalized transportation problems* when finding its numerical solution.

In Dempe et al. (2006) and Kalashnikov et al. (2007), we study a modified version of the above-described problem, in which the upper level objective function includes certain new terms based upon the net profit of the leader – the natural gas shipping company. This formulation assumes, however, the complete knowledge (perfect information) about the changes in the prices of natural gas during the process, which is somewhat non-realistic and not quite useful, as the resulting function does not clearly reflect the reasons behind the actions of the NGSC.

Therefore, we propose here a stochastic reformulation of the problem, so that the NGSC is now able to forecast the next several values of the natural gas demand, and then to plan the extraction of natural gas from the pipeline. The resulting model is a stochastic variation of the original mixed-integer bilevel optimization problem, for which we propose and compare two different solution methods.

To our best knowledge, there is no literature, beyond the works listed in the paragraphs above and their derivatives, that explicitly deals with the NGSC–POC subsystem in the same way we propose, formulating a bilevel optimization problem out of the balancing operations. We attribute this to the relatively recent nature of the problem we are dealing with, as well as the difficulty of its accurate formulation for specific instances.

The rest of the paper is organized as follows: after recalling the original problem in Section 2, we formulate the new problem we deal with, while Section 3 describes the corresponding mathematical models. This section also contains details concerning the generation of the stochastic scenario tree. Section 4 presents the experimental results obtained when solving our problems with the proposed numerical algorithms, as well as the discussion thereof. Finally, Section 5 presents the conclusions and the directions of future research related to the model in question.

2. Problem specification

Following the scheme constructed in Kalashnikov and Ríos-Mercado (2001), we will consider a leader–follower system, in which the first agent (the leader), namely, the Natural Gas Shipping Company (NGSC), buys the gas at the wells, arranges for its injection into an (interstate) pipeline at its starting point, and extracts some amount of gas – ideally, equal to the deposited amount, from pipeline meters in several *pool zones* across the country. The follower here is the administrator of the pipeline, which we call the Pipeline Operating Company (POC), who permits the NGSC to extract amounts of natural gas that may differ from the originally injected volumes, thus creating positive or negative imbalances. The latter is a kind of usual market practice that allows for a dynamic flow of the fuel within the natural gas supply chain.

However, since disrupting the system in this way implies extra costs for the NGSC, the company attempts to do it only when its predictions of future market conditions show that the total revenues overcome the losses incurred by the penalization policy applied to the NGSC. It is clear that the NGSC needs tools that provide it with the best possible information and help it make advantage of the latter.

The NGSC–POC system operates in the following way:

1. The NGSC makes a forecast of the demand it is likely to have during the next period (month, year, etc.) and considers different scenarios, in which this can occur.
2. The NGSC books certain capacity D^c for every pool zone and stage (day, week, month, etc.)
3. For each subsequent stage, the NGSC determines the amount of gas to extract and sell, which possibly creates positive and negative imbalances in the process; this continues until the period is over.
4. The POC studies the resulting last day imbalances and rearranges them according to certain business rules.
5. The POC charges the NGSC with certain penalty for the final (rearranged) imbalances. The latter may occur to be negative, i.e. the POC may pay to the NGSC.
6. The NGSC calculates the net profits as its sales revenue minus the penalty.

The resulting model is a bilevel multi-stage stochastic optimization problem (Kall and Wallace, 1994), in which the upper level decision maker (the leader) is the NGSC who has the objective of maximizing its net profit as the revenue from the sales of its gas in the pipeline minus the penalty imposed by the POC. The lower level decision maker (the follower) is the POC who aims at minimizing the absolute value of the penalization cash-out flow, either from the POC to the NGSC or vice versa. The first stage of the stochastic problem corresponds to the capacity booking made by the NGSC, and these capacity values remain unchanged throughout the whole process. At the next stages, the decision variables are: the daily extraction amounts (and hence, the imbalances), unsatisfied demand, and the penalty cash-outs imposed by the POC.

Note that, while the POC may appear as the party with more influence in the system, the NGSC is the leader of the bilevel problem. The only reason why the NGSC is the upper level (leader) is because of the timing of the decision process. Indeed, it would seem logical that the POC, enjoying stronger control over its own facilities, has to abide to the decisions (regarding final day imbalances) that the NGSC has already made. This is because of the relative freedom that has been awarded (in the current business' practice) to the NGSC in creating imbalances to maintain healthy business in favor of its customers.

Table 1
Notation.

Sets	
N	number of time periods at each node in the process; $N \in Z_{++}$
P	number of pool zones; $P \in Z_{++}$
K	number of event nodes in the process; $K \in Z_{++}$
S	number of stages in the process; $S \in Z_{++}$
$\mathbf{T}$	set of time periods at any given node; $\mathbf{T} = \{1, 2, \dots, N\}$
$\mathbf{J}$	set of pool zones; $\mathbf{J} = \{1, 2, \dots, P\}$
$\mathbf{K}$	set of event nodes; $\mathbf{K} = \{1, 2, \dots, K\}$
$\mathbf{K}^l$	set of nodes at stage l, $l = 1, \dots, S$
Upper level parameters	
x^L_{kti}, x^U_{kti}	lower and upper bounds for the daily imbalances on day t at node k, in pool zone i; $i \in \mathbf{J},\ t \in \mathbf{T},\ k \in \mathbf{K}$
x^L_{kt}, x^U_{kt}	lower and upper bounds for the sum of the daily imbalances on day t at node k; $t \in \mathbf{T},\ k \in \mathbf{K}$
sw^L_{kti}, sw^U_{kti}	bounds on balance swing before day t at node k starts, in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
x_{0i}	imbalance at the beginning of day 1 at node 1, in pool zone i, $i \in \mathbf{J}$
D_{kti}	expected demand on day t, at node k, in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
Π_{kti}	unit price for each unit of gas extracted/sold (contracted gas) on day t at node k in zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
C^I_{kti}, C^R_{kti}	recourse cost and booking capacity cost per gas unit on day t at node k, in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J}$
p_k	probability of node k to occur in any scenario; $k \in \mathbf{K}$
Lower level parameters	
e_{ij}	fraction of gas used as the fuel when moving one unit from pool zone i to pool zone j; $i,j \in \mathbf{J},\ i < j$
f_{ij}	forward haul cost for moving a unit of gas from pool zone i to pool zone j; $i,j \in \mathbf{J},\ i < j$
b_{ij}	backward credit for "returning" a unit of gas from pool zone j to pool zone i; $i,j \in \mathbf{J},\ i < j$
r_i	cash-out penalization coefficient in pool zone i; $i \in \mathbf{J}$

Table 2
Notation, cont.

Upper level decision variables	
x_{kti}	imbalance on day t at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
sw_{kti}	imbalance swing before day t starts at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
EA_{kti}	amount of gas actually extracted on day t at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
EP_{kti}	amount of gas planned to be extracted (i.e. booked pipeline capacity) on day t at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
xa_{ktj}	amount of gas actually extracted and sold during day t at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
xd_{ktj}	amount of demand D_{kti} unmet on day t at node k in pool zone i; $t \in \mathbf{T},\ i \in \mathbf{J},\ k \in \mathbf{K}$
Lower level decision variables	
y_i	final imbalance in pool zone i, $i \in \mathbf{J}$
u_{ij}	volume of gas moved from pool zone i to pool zone j; $i,j \in \mathbf{J},\ i < j$
v_{ij}	volume of gas credited from pool zone j to pool zone i; $i,j \in \mathbf{J},\ i < j$
z	total cash-out charge on the Natural Gas Shipping Company
Auxiliary variables	
q	binary variable equal to 1(0) if final imbalances y_i are all non-negative (non-positive). In the special case when $y_i = 0, \forall i \in \mathbf{J}$, we accept $q = 1$

2.1. Notation

Tables 1 and 2 show the notation used throughout this article.

In Fig. 1, one can see a three-staged scenario tree, which generates nine scenarios. Scenario 1 "passes through" nodes 1, 2, and 5; scenario 2 through nodes 1, 2, and 6, and so on. Each node has the probability to occur equal to the sum of probabilities of all the scenarios passing through it; the leaf (termination) nodes have the same probabilities to happen that the corresponding scenarios ending at them.

Since the probabilities across all the scenarios sum 1, we have that $\sum_{k \in \mathbf{K}^S} p_k = 1$, where $k \in \mathbf{K}^S$ represents the set of all leaf nodes (as they all belong to the last stage S.) It goes without saying that the root has the probability of 1. Then, for each subsequent stage, the sum of probabilities of all nodes at that stage is equal to 1. The probabilities p_k depend upon the scenario tree forecast scheme described in Section 3.3.

The function $a : \mathbf{K} \to \mathbf{K}$ identifies the *predecessor* of any given node k (except for the root, whose predecessor is void); that is, $a(k) = k'$ if k and k' share an arc in the scenario tree, and $k \in \mathbf{K}^l \Rightarrow k' \in \mathbf{K}^{l-1}, l = 2, \dots, S$. In the particular case of $k = 1$, we accept $x_{a(1),N,i} := x_{0i}, i \in \mathbf{J}$.

Notice that, since costs C^I, C^R are deterministic, they are the same for every node at a given stage l, that is, $C^I_{kti} = C^I_{k'ti}, C^R_{kti} = C^R_{k'ti}, k, k' \in \mathbf{K}^l, l = 1, \dots, S$. The similar is true for the variable EP, as explained below in (1i).

Finally, we will use the node imbalance matrix x_k, which is defined by fixing one node k and selecting the imbalance matrix entries as follows: $x_k = (x_{kti})_{t=1,i=1}^{N,\ P}$, as well as the last day imbalance vector x_{kN} introduced by $(x_{kNi})_{i \in \mathbf{J}}$.

3. Model description

Using the model from Kalashnikov and Ríos-Mercado (2001) as a basis, we form a multi-stage stochastic bilevel optimization problem described below by (1a)–(1j) and (4a)–(4l). This problem will be theoretically manipulated to reduce its complexity, transforming it into a bilevel linear optimization problem, where the lower level resembles both a generalized transportation problem, and a quadratic assignment problem. Bilevel problems over networks (Cruz et al., 1999; Chiou, 2005) often arise when dealing with transportation problems (Yang and Bell, 2001), and even examples of bilevel linear problems in networks exist (Ben-Ayed et al., 1988).

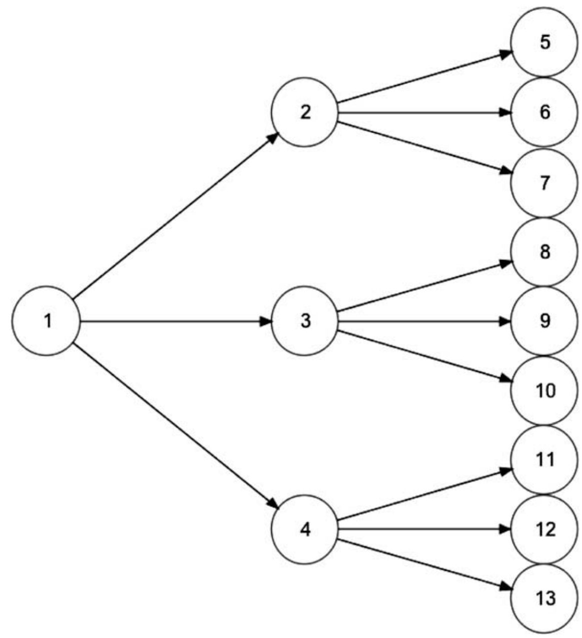


Fig. 1. A 3-staged, ternary scenario tree.

3.1. Upper level model

Lines (1b)–(1i) and (1j) represent the upper level of the bilevel problem; it is stochastic and based upon a ternary scenario tree similar to that depicted in Fig. 1.

$$\text{Minimize}: \quad H_1(x, sw, EA, EP, xd; y, u, v, z, d, q) \tag{1a}$$

$$= \sum_{k\in\mathbf{K}} p_k \left[\sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{J}} \left(C^I_{kti} xd_{kti} - \Pi_{kti} \min\{EA_{kti}, D_{kti}\} + C^R_{kti} EP_{kti} \right) \right] + \sum_{k\in\mathbf{K}^S} p_k h(x_{kN}; y, u, v, z, d, q) \tag{1b}$$

$$\text{subject to}: \quad x^L_{kti} \leqslant x_{kti} \leqslant x^U_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{1c}$$

$$sw^L_{kti} \leqslant sw_{kti} \leqslant sw^U_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{1d}$$

$$x^L_{kt} \leqslant \sum_{i\in\mathbf{J}} x_{kti} \leqslant x^U_{kt}, \quad k \in \mathbf{K},\ t \in \mathbf{T}; \tag{1e}$$

$$x_{kti} = \begin{cases} x_{a(k),N,i} + sw_{kti} & \text{if } t = 1 \\ x_{k,t-1,i} + sw_{kti} & \text{if } t \neq 1 \end{cases}, \quad k \in \mathbf{K}, t \in \mathbf{T}, i \in \mathbf{J}; \tag{1f}$$

$$x_{kti} = EP_{kti} - EA_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{1g}$$

$$xd_{kti} = \max\{0, D_{kti} - EA_{kti}\}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{1h}$$

$$EP_{kti} = EP_{k'ti}, \quad k, k' \in \mathbf{K}^l,\ l = 1, \ldots, S; t \in \mathbf{T},\ i \in \mathbf{J}. \tag{1i}$$

$$EA_{kti} \geqslant 0, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{1j}$$

In the objective function, the terms $h(x_{kN}; y, u, v, z, d, q) = z$ depend on both levels and are determined by the optimal solutions of the corresponding lower level problems (4a)–(4l), or, in other words, represent the penalty cash-out imposed on the NGSC by the POC. The term $\Pi_{kti} \min\{E^a_{kti}, D_{kti}\}$ is the revenue from the gas extracted and sold in pool zone i on day t at node k. Further, $C^I_{kti} xd_{kti}$ is the cost of the unsatisfied demand, which could be interpreted either as a penalization on part of the clients, or the cost of a supply purchased from a third party in the amount just enough to meet the demand completely.

The term $C^R_{kti} E^p_{kti}$ represents the cost the NGSC incurs in when allocating capacity in the pipeline on day t at node k in pool zone i; this term is particularly important for the non-triviality of solutions: were the capacity booking free for the NGSC, the latter would respond by maximizing the values of variables EP and selling all the available gas amount at the prices Π.

Notice that if we have more than one stage, then $\sum_{k\in\mathbf{K}} p_k > 1$. This means that the first term in the objective function is an expected value not *over the nodes* but rather *over the scenarios*. Here, the non-anticipativity constraints (Kall and Wallace, 1994) have been implicitly imposed by using the node formulation instead of a scenario formulation.

Constraints 1c, 1d and 1e describe the technological limits on the creation of imbalances at each node, on their daily totals, and the daily imbalance swings.

Relationships (1f) show the nature of the day-to-day imbalance swings. At every node, time period and pool zone, the imbalance on one day must not differ too much from the imbalance on the previous day; this is expressed in distinct ways depending upon whether we are at an in-node (i.e. when $t = 2, \ldots, N$,) or at a cross-node (when $t = 1$), although the essence is the same. If we are at an in-node, then the swing from the imbalance $x_{k,t-1,i}$ to the imbalance x_{kti} is determined by the value of the swing variable sw_{kti}, and the latter is bounded from above and from below by parameters sw^U_{kti}, sw^L_{kti}. This also holds for the cross-node case, but the day before the first day at such a node is the last day at the predecessor node, thus the notation $x_{kti} = x_{a(k),N,i} + sw_{kti}$ is used.

A further note on the imbalance swing variables sw is due: their values are completely determined by the imbalance variables x, along with the predefined initial imbalances x_0. The purpose of adding them here is merely illustrative, but they are not actually used in the solution process. The modified upper level problem that is actually solved as described later by Eqs. (2b)–(1i) and (2j), drops these variables in favour of equivalent bounds on imbalances x.

Constraint (1g) shows the relationship between the imbalance, the booked capacity, and the extraction in every pool zone at each time period, whereas (1h) gives the definition of the unmet demand variables. Finally, constraint (1i) represents the one-stage nature of variables EP, in the sense that every node at a given stage must share the same values of those variables.

While model (1b)–(1i) and (1j) satisfactorily abstracts the NGSC–POC subsystem for our purposes, the difficulty of solving a nonlinear problem with *max* and *min* operators is considerable when compared to, say, a linear programming problem. We can reduce the complexity of this problem by adding certain variables carefully chosen so that the objective function remains the same. This equivalent problem, while no longer having *max, min* operators, remains nonlinear in the sense that the objective function contains the variable z. This variable is controlled by the lower level in response to the upper level's actions. However, if we omit z (as the only variable not controlled by the upper level) from the objective function, we have a completely linear model, and, once the lower level has been reduced in turn to a linear equivalent, the resulting scheme (a lower level linear problem plus an upper level linear problem except for a set of variables linearly represented and controlled at the lower level) is called a *bilevel linear optimization problem.*

We will deem the optimization problem (2b)–(1i) and (2j) as an"almost" linear programming problem, one that is equivalent to the original upper level formulation as is detailed in Lemma 1 in Appendix A.

$$\text{Minimize}: \quad H_2(x, EA, xa; y, u, v, z, d, q) \tag{2a}$$

$$= \sum_{k\in\mathbf{K}} p_k \left[\sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{J}} C^I_{kti}(D_{kti} - xa_{kti}) - \Pi_{kti} xa_{kti} + C^R_{kti}(x_{kti} + EA_{kti}) \right] + \sum_{k\in\mathbf{K}^S} p_k h(x_k; y, u, v, z, d, q) \tag{2b}$$

$$\text{subject to}: \quad x^L_{kti} \leqslant x_{kti} \leqslant x^U_{kti}, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}; \tag{2c}$$

$$x^L_{kt} \leqslant \sum_{i\in\mathbf{J}} x_{kti} \leqslant x^U_{kt}, \quad k\in\mathbf{K},\ t\in\mathbf{T}; \tag{2d}$$

$$sw^L_{k1i} \leqslant x_{k1i} - x_{a(k),N,i} \leqslant sw^U_{k1i}, \quad k\in\mathbf{K},\ i\in\mathbf{J}; \tag{2e}$$

$$sw^L_{kti} \leqslant x_{kti} - x_{k,t-1,i} \leqslant sw^U_{kti}, \quad \begin{array}{l} t = 2,\dots,N, k\in\mathbf{K}, \\ i\in\mathbf{J}; \end{array} \tag{2f}$$

$$xa_{kti} \leqslant D_{kti}, \quad k\in\mathbf{K}, t\in\mathbf{T},\ i\in\mathbf{J}; \tag{2g}$$

$$xa_{kti} \leqslant EA_{kti}, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}; \tag{2h}$$

$$x_{kti} + EA_{kti} = x_{k'ti} + EA_{k'ti}, \quad k,k'\in\mathbf{K}^l,\ l = 1,\dots,S;\ t\in\mathbf{T},\ i\in\mathbf{J}. \tag{2i}$$

$$EA_{kti} \geqslant 0, xa_{kti} \geqslant 0, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}, \tag{2j}$$

where $h(x_k; y, u, v, z, d, q) = z$ is the lower level's response to this upper level problem, as defined in (3a)–(3k) and (3l).

3.2. Lower level model

This lower level is exactly the same as the one found in Kalashnikov and Ríos-Mercado (2006), using the linear objective function d to substitute an absolute-value objective function, with the aid of constraints (3i):

$$\text{Minimize}: \quad h_1(x_{kN}; y, u, v, q, d, z) = d \tag{3a}$$

$$\text{subject to}: \quad y_j = x_{kNj} + \sum_{i\in\mathbf{J}:i<j} [(1 - e_{ij})u_{ij} - v_{ij}] + \sum_{m\in\mathbf{J}:m>j} (v_{jm} - u_{jm}),\ j\in\mathbf{J}; \tag{3b}$$

$$\sum_{j\in\mathbf{J}:i<j} u_{ij} + \sum_{m\in\mathbf{J}:m<i} v_{mi} \leqslant \max\{0, x_{kNi}\},\ i\in\mathbf{J}; \tag{3c}$$

$$u_{ij} \leqslant \begin{cases} x_{kNi} & \text{if } x_{kNi} > 0 \text{ and } x_{kNj} < 0, \\ 0 & \text{otherwise}, \end{cases} \quad i,j\in\mathbf{J},\ i<j; \tag{3d}$$

$$v_{ij} \leqslant \begin{cases} x_{kNj} & \text{if } x_{kNj} > 0 \text{ and } x_{kNi} < 0, \\ 0 & \text{otherwise}, \end{cases} \quad i,j\in\mathbf{J},\ i<j; \tag{3e}$$

$$\min\{0, x_{kNi}\} \leqslant y_i \leqslant \max\{0, x_{kNi}\}, \quad i\in\mathbf{J}; \tag{3f}$$

$$-\mathbf{M_1}(1 - q) \leqslant y_i \leqslant \mathbf{M_1} q, \quad i\in\mathbf{J}; \tag{3g}$$

$$z = -\sum_{i\in\mathbf{J}} r_i y_i - \sum_{(i,j):i<j} v_{ij} b_{ij} + \sum_{(i,j):i<j} f_{ij}(1 - e_{ij})u_{ij}; \tag{3h}$$

$$-d \leqslant z \leqslant d; \tag{3i}$$

$$y_i, z\in\Re, \quad i\in\mathbf{J}; \tag{3j}$$

$$u_{ij}, v_{ij} \geqslant 0, \quad i,j\in\mathbf{J},\ i<j; \tag{3k}$$

$$q\in\{0,1\}; \tag{3l}$$

here, $\mathbf{M_1} > 0$ is a large fixed scalar parameter.

Constraint (3b) shows the relationship between the last day imbalances x_{kN}, the final imbalances y, and the forward and backward hauls of gas u, v performed by the POC in order to minimize the absolute value of the resulting cash-out payments. Constraints (3c)–(3e) and (3f) taken together say that the maximum amount of gas hauled forward [backward] from zone i to zone j [from zone j back to zone i] cannot exceed either the positive part of the last day imbalance existing in the donor pool zone i [j] or the negative part of the imbalance present in the recipient pool zone j [i] on the last day N.

Inequalities (3f) also imply that by the business rules, the final imbalance y_i in zone i must have the same sign as the last day imbalance x_{kNi} in this zone. Next, inequalities (3g) force the final imbalances to be either all non-negative ($q = 1$) or all non-positive ($q = 0$), which is a usual business demand as well. Constraint (3h) describes how the penalty cash-out imposed on the NGSC is calculated, while 3j, 3k and 3l define the bounds of the variables.

Note that while the upper level problem (2b)–(2i) and (2j) is an "almost" linear program, this is not the case with the lower level one, that is, not even when removing the upper level controlled variables this problem becomes linear. Just as we did with the upper level, instead of solving this problem, we will solve an equivalent mixed-integer programming problem with linear constraints, so that the iterative process of solving the lower level program be faster. This lower level mixed-integer programming problem with linear objective function and constraints, is equivalent (under the conditions given in Dempe et al., 2005) to problem (3a)–(3k) and (3l). The equivalence is guaranteed by Lemma 2 in Appendix A.

$$\text{Minimize}: \quad h_2(x_{kN}; y, u, v, q, d, z, \xi, \zeta) = d + \mathbf{M_2} \sum_{i \in \mathbf{J}} (\xi_i + \zeta_i) \tag{4a}$$

$$\text{subject to}: \quad u_{ij} \leqslant \xi_i, \quad i,j \in \mathbf{J},\ i < j; \tag{4b}$$

$$u_{ij} \leqslant \zeta_j, \quad i,j \in \mathbf{J},\ i < j; \tag{4c}$$

$$v_{ij} \leqslant \xi_j, \quad i,j \in \mathbf{J},\ i < j; \tag{4d}$$

$$v_{ij} \leqslant \zeta_i, \quad i,j \in \mathbf{J},\ i < j; \tag{4e}$$

$$-\zeta_i \leqslant y_i \leqslant \xi_i, \quad i \in \mathbf{J}; \tag{4f}$$

$$\xi_i \geqslant x_{kNi}, \quad i \in J; \tag{4g}$$

$$\xi_i \geqslant 0, \quad i \in J; \tag{4h}$$

$$\zeta_j \geqslant -x_{kNj}, \quad j \in J; \tag{4i}$$

$$\zeta_j \geqslant 0, \quad j \in J, \tag{4j}$$

$$\text{and } 3\text{b},\ 3\text{g},\ 3\text{h},\ 3\text{i},\ 3\text{j},\ 3\text{k},\ 3\text{l}. \tag{4k}$$

The value of $\mathbf{M_2}$ in the linear lower level problem's objective function is loosely bounded from below by the expression

$$\max\left\{ \left| -\sum_{i \in \mathbf{J}} r_i x^U_{kti} - \sum_{(i,j):i<j} (x^U_{ktj})_+ \cdot b_{ij} \right|, \quad \left| -\sum_{i \in \mathbf{J}} r_i x^L_{kti} + \sum_{(i,j):i<j} (x^U_{kti})_+ \cdot f_{ij} \right| \right\}, \tag{5}$$

where $(\eta)_+ = \max\{0, \eta\}$. In order to determine a suitable value for parameter M_2, one can calculate (5) and multiply it by, say, one thousand. The result, when assigned to parameter M_2, should be enough to guarantee the forceful maximization of the values of ξ and ζ, and therefore the equivalence of the linear lower level and the original one.

3.3. Scenario tree construction

The upper level problem (and hence, the lower level one, too) is subject to a scenario scheme: the decisions taken by the NGSC will vary according to a scenario assumed and/or met by the company. This section shows the scheme of creating the scenario tree that underlies the problem.

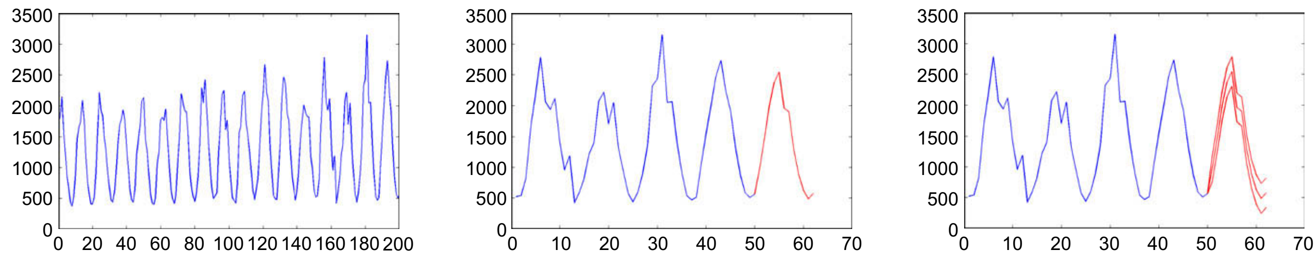
The Scenario Tree, as shown in Fig. 1, consists of several event nodes (Midthun, 2007), each of which contains N periods, and also of several branches connecting it with other nodes. In our case, each node represents one week (five working days), although this is not mandatory. For every node k, the data concerning prices/demands for all scenarios passing through k are calculated as certain α prediction percentile based on a forecast made with the corresponding data up to the last node in the tree. The possible outcome of prices higher (or lower) than the mean forecast corresponds to α taking a value higher (or lower) than 0.5, whereas the outcome of prices following the expected mean forecast will have $\alpha = 0.5$. Every weekend, however, we will allow for the high, mean, or low expected outcome to change for the upcoming week.

Consider the time series for the monthly consumption of natural gas in the state of Alaska, USA, from January 1989 to September 2005 (200 observations,) shown in Fig. 2(a). This is a rather well-behaving time series with a noticeable 12-period seasonality. The expected 12-observations forecast using the Seasonal Holt-Winters (SHW) (Brockwell and Davis, 2002) method is shown next in Fig. 2(b), cropped to the last observations and with the forecast values in red. These forecast data correspond to the first node of the scenario tree (Fig. 1).

For nodes 2, 3, and 4, we will obtain 12 new observations per each node by taking the historic time series used in the forecast at node 1, *adding* the forecasts obtained in that node as new historic observations, then forecasting the 25th, 50th, and 75th percentiles ‰, corresponding to low, mean and high predictions and calculated by obtaining the nth prediction error parameters for an assumed normal distribution. The resulting three time series can be seen in Fig. 2(c), sharing the blue section among them. We have now a three-leaf scenario tree, formed by nodes 1, 2, 3, and 4, and underlying the shown time series.

For nodes 5 through 13, we make this process iterative, by taking the three time series shown in Fig. 2(c) (formed by a historic part, a mean forecast, and the three percentiles, one for each series) as historic and branching out at their endpoints as done with nodes 2 through 4, thus obtaining nine time series and the corresponding nine-leaved scenario tree in Fig. 1 underlying the series in Fig. 3.

Each scenario is then assigned the probability to occur. If trends are likely to be higher (lower) in the near future, then one would increase the chances of the middle and upper (lower) branches of each node. Assigning equal probabilities to both upper and lower branches will reduce the impact of the scenario stochasticity in the optimization problem. Every three nodes branching from a single node k will



(a) Alaska's historic consumption

(b) Alaska's mean forecast consumption

(c) Alaska's comsumptions 0.25, 0.5 and 0.75 prediction percentiles

Fig. 2. Scenario generation through forecasting.

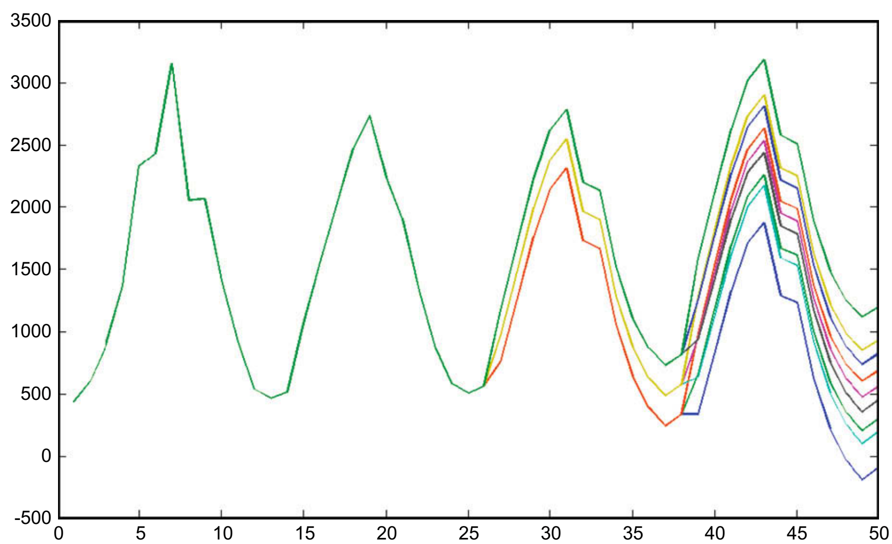


Fig. 3. Nine (partially overlapping) time series resulting from a 3-staged scenario tree: All share the initial observation, whereas only groups of tree share the second section (first forecast).

have probabilities of being chosen whose sum equals p_k. Hence, the three nodes branching from the root will have probabilities p_2, p_3, p_4 summing up to $p_1 = 1$; the three nodes branching from node 2 will have probabilities summing up to p_2, and so forth. As stated above, this process guarantees that the sum of probabilities of all leaf nodes totals one.

4. Results of the numerical experiments

The above-described models were tested computationally on a batch of 21 problems, labeled B001 to B020, in order to compare: (1) the perfect information solution value, (2) the obtained stochastic value, and (3) the expected value solution, as well as the implementation values for the latter two. Each problem instance was generated by an algorithm which guarantees the existence of a solution to the upper level by proposing a random solution based in input parameters and then setting the different values for the bounds so that the proposed solutions is feasible, using inverse uniform random distributions to determine the bounds' tightness. The problem instance generating algorithm then goes on assigning random suitable values for the parameters not involved in bounds (like matrices e_{ij}, f_{ij}, and b_{ij}, which are created based on a Poisson inverse distribution.) The consumption and price parameters in the upper level were obtained from a *Seasonal Holt-Winters* forecast performed over the time series for residential natural gas consumption and price found in the data by the United States Energy Information Administration (Energy Information Administration, 2008).

The consumption (forecast) figures from many of the states of the US that were used as parameters vary considerably. Maine's consumption, for example, its average being one of the lowest, ranges from 18×10^6 ft^3 in summer to 235×10^6 ft^3 in winter. On the contrary, Illinois consumption, one of the largest throughout the year, lies between 8400×10^6 ft^3 and 104083×10^6 ft^3 for the summer/winter periods. As for prices, at their minimum (in Utah,) they are around 8.98 dollars per ft^3 during winter and 12.29 dollars per ft^3 during summer. The range goes up to 31.42 and 41.94 dollars per ft^3 in winter/summer in the state of Hawaii, one of the most expensive states. The r_i values are usually set at around 120 dollars per million ft^3 (which is treated as the imbalance unit), and transportation costs at the lower level vary around 10–12 dollars per imbalance unit.

Table 3
Solution reports for problems B001–B020.

Instance	N	P	S	ULOV	mLLOV	RT (s)	q
B001	12	4	3	891,190.48	29.01	245.77	1
B002a	8	5	2	2,043,938.57	49.69	905.63	1
B002b	8	5	2	1,564,324.37	0	1,856.12	1
B003	12	5	3	8,413,667.18	68.34	929.99	1
B004	12	5	3	7,943,942.17	−53.73	3,319.53	0
B005	12	5	3	1,306,501.04	−7.86	421.23	0
B006	12	5	3	510,335.27	−2.13	120.43	0
B007	12	5	3	2,123,587.05	−87.48	281.89	0
B008	12	5	3	2,753,643.38	0	650.31	0
B009	12	5	3	1,853,811.99	−6	398.76	0
B010	12	5	3	3,973,184.60	47.30	706.78	1
B011	28	4	4	2,663,283.93	−56.74	7,893.23	0
B012	8	5	2	3,298,333.98	0.00	2,321.69	0
B013	8	5	2	3,417,055.13	0.00	2,078.14	0
B014	8	5	2	2,477,883.48	0.00	1,891.12	0
B015	8	5	2	3,667,407.35	−2.04	701.21	0
B016	10	7	2	4,742,822.11	−49.00	5,002.13	0
B017	10	7	2	2,862,635.34	38.00	2,548.44	1
B018	10	7	2	1,901,243.90	0.00	4,518.93	1
B019	10	7	2	2,413,298.05	37.34	4,950.69	0
B020	10	7	2	3,437,553.53	10.00	4,735.71	1

More detailed data is given in Appendix B, where a complete example for the largest problem instance, B011 is displayed. While individual values for each problem are different, they generally resemble the figures above.

The experiments were run on an Intel® Core 2 Quad™ 2.66 MHz running Windows Vista™ Home Premium, and using Matlab® R2008b. The upper level was solved using Matlab's Optimization Toolbox™ optimization function `fmincon`, while the lower level was processed with the linear programming tool `linprog`.

4.1. Solution reports

The report for each instance provides one upper level value (weighted for all scenarios,) one $N \times P$ matrix for the variable EP, plus 3^{S-1} blocks of solution variables (one per scenario) and one final running time, as well as the best value of variable q for the instance. Each scenario block consists of $P \times N$ -matrices for the upper level variables x, EA; one P-vector for the final imbalances y, and two $P \times P$-matrices for the backward/forward hauls of gas u, v. Scenarios that share one or more nodes have values of x, EA equal at the corresponding nodes. Appendix C contains charts comparing the some data presented throughout this section.

The figures in Table 3 are, for each problem: the optimal value for the upper level (ULOV), the mean optimal lower level value (mLLOV), the running time for the problem (RT), and the optimal value for the variable q.

Running times are varied, even among similarly sized problems. Overall, around half of the 21 instances tested delivered results in less than 1000 s. This, of course, is not overly meaningful, as different hardware and software configurations may deliver vastly different results. All running times should be evaluated only in the context of this experimentation.

An important feature to look at is the comparison of the stochastic solution (SS) shown above, the perfect information solution (PIS, obtained when knowing beforehand the values of the parameters previously considered stochastic,) and the expected value solution (EVS,) obtained using the expected values of the stochastic parameters in a single-staged optimization problem.

Both the PIS and the EVS are procured by solving a $SPR2$ problem with a single scenario. The PIS is the best possible solution: it can only be attained by having a perfect forecast method. The EVS is the result of solving a simply-designed stochastic problem using only mean prediction estimates. The results for these values are summarized in Table 4, in columns PIS, SS and EVS. Notice the considerably low revenues predicted by the EVS.

The solution values provided by the SS and the EVS are calculated using **estimates** of the realized (future) prices and demands the NGSC will face. If estimates are highly optimistic (pessimistic,) the revenue obtained may overestimate (underestimate) the actual attainable revenue, represented by the PIS. It is easy to see from the data in columns PIS and SS in Table 4 that the overestimation occurs in problems B002a, B003, B004, B010, B014, and B020, whereas the underestimation happens for the rest of the test problems.

If the NGSC uses the SS and EVS decisions to plan its operations, the realized parameters will likely differ from the estimates used in the stochastic problems, hence the actual revenue would be less than the estimated by the SS and EVS (because the SS and EVS are optimized for the estimates, and not for the unknown future realizations of the parameters.)

The values for the stochastic solution implementation (column SSI) and the expected value solution implementation (column EVSI,) represent the actual gain from implementing the SS and EVS solutions. They are obtained by evaluating the SS and EVS solutions with the realized parameters that the PIS uses. Both the SSI and the EVSI may be only as high as the PIS (which optimizes for perfect information,) and will likely be lower than their correspondent SS and EVS solutions: only in the case that realized prices/demands are advantageous and the SS or EVS approximate them well, will the SSI and EVSI be better than the SS and EVS, respectively.

To compare the quality of the SSI and EVSI values, we also present the relative error ratios, SSRE and the EVSRE. These relative errors are calculated as the difference between the PIS value and the SS (EVS) value, divided by the PIS value, that is,

Table 4
Revenue comparisons for various solutions.

Instance	PIS	SS	SSI	SSRE	EVS	EVSI	EVSRE
B001	892,046	891,190	690,430	0.226	321,597	225,820	0.747
B002a	2,003,464	2,043,938	1,391,160	0.305	480,584	397,752	0.801
B002b	1,705,313	1,564,324	1,425,829	0.163	389,726	16,356	0.990
B003	7,565,976	8,413,667	6,775,471	0.104	299,320	243,360	0.968
B004	7,314,748	7,943,942	6,679,739	0.087	277,613	254,197	0.965
B005	1,450,506	1,306,501	1,191,949	0.178	64,176	23,320	0.984
B006	687,882	510,335	440,987	0.359	33,709	8,356	0.988
B007	4,009,499	2,123,587	2,836,230	0.293	75,496	71,259	0.982
B008	4,753,643	4,464,049	3,439,549	0.276	107,444	103,326	0.978
B009	3,660,097	1,853,811	2,563,110	0.300	69,717	56,860	0.984
B010	3,973,184	5,732,921	3,636,830	0.085	148,418	142,463	0.964
B011	2,860,385	2,663,283	1,281,300	0.552	10,941	20,145	0.993
B012	3,339,391	3,298,333	3,248,556	0.027	594,800	410,164	0.877
B013	3,456,927	3,417,055	2,350,283	0.320	613,688	430,266	0.876
B014	2,346,027	2,477,883	1,885,270	0.196	404,106	223,830	0.905
B015	3,770,818	3,667,407	2,995,647	0.206	729,237	504,094	0.866
B016	5,544,639	4,742,822	3,928,716	0.291	1,009,750	466,874	0.916
B017	3,498,196	2,862,635	378,844	0.892	484,217	22,034	0.994
B018	3,620,877	1,901,243	3,190,119	0.119	364,856	394,682	0.891
B019	4,007,071	2,413,298	2,860,895	0.286	438,157	460,516	0.885
B020	2,714,532	3,437,553	2,355,637	0.132	484,318	297,257	0.890

$$\mathrm{SSRE} = \frac{|\mathrm{PIS} - \mathrm{SSI}|}{\mathrm{PIS}}, \quad \mathrm{EVSRE} = \frac{|\mathrm{PIS} - \mathrm{EVSI}|}{\mathrm{PIS}},$$

for each instance. A ratio near 0 means that the approximation of the PIS value by the SSI (EVSI) is good; on the contrary, a ratio near 1 implies that the SSI (EVSI) are considerably poor when implemented in a given situation.

Of the 21 SSREs, only two are above 0.5 (the SSI solution is less than half that of the PIS,) with the highest relative error (abnormally large) at 0.89 in problem B017, while still below the EVSRE corresponding to the problem. Only one of the EVSREs values displayed is below 0.75. Results employing the ARAR forecasting method instead of the Seasonal Holt-Winters method delivers similar results, heavily underestimating the PIS value.

It is clear that solving the problem with forecasting only the mean predictions heavily underestimates the possible future gains, and, since the solution vector is obtained optimizing under thus underrated gains scenario, the implementation also ends up producing profit values far below the optimal ones, at least for this data and model selection. Comparing the relative errors for the EVS and the SS, it is safe to state that the usage of a stochastic framework considerably improves over the simpler approach of solving a deterministic-like variant obtained by solely forecasting the mean predictions (as it is done with the EVS.)

In a final note, solving the original, nonlinear problem (1b)–(1i), (3a)–(3k) and (3l), proved to be very inefficient. As it was expected, solving nonlinear models took considerably longer time and, in most cases, their solutions obtained after interrupting very long runs were worse than those produced by solving their bilevel linear reformulations. Again, using other specialized (nonlinear) solvers such as IPORT, KNITRO could modify these findings, but our hardware-software configuration points to a clear superiority of the bilevel linear implementations.

5. Conclusions and future work

In this paper, we present a bilevel multi-stage stochastic optimization model, which is developed to deal with a certain subsystem of the natural gas supply chain. While former models were focused on the arbitrage policies in a deterministic setting, here we have expanded the problem to include such elements as gas sales and booking costs, and added a stochastic framework to model the uncertainty in demand and prices faced by the upper level decision maker (the leader).

The developed model was implemented numerically and compared to the perfect information solution (PIS) and the expected value solutions (EVS). Experimental findings show that 19 of the 21 instances deliver implementation values of over half of the PIS, whereas only one of the EVS implementation values has a relative error below 0.75. The Stochastic Solution Implementation values are better than those of the EVS values in all but one case – which corresponds to the simplest instance tested, – which testifies in favour of our approach. The performed linear reformulation also proved advantageous, as solving the original model with nonlinear levels takes considerably longer time and does not provide better solutions after up to 10 hours of running time in 20 of the 21 experiments.

Future work includes assessing the convenience of using heuristic approaches for solving the lower level (as opposed to using a specialized linear solver,) and reformulating the linear lower level in the form of its duality conditions, adding these to the upper level to solve a single-level problem instead of a bilevel one. We also intent to study these models under different time series not showing seasonality is also planned, as it is the implementation of a rolling horizon approach to remedy the lack of accuracy over long-period problems (such as problem B011 involving 28 periods).

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Appendix A. Linearization lemmas

Lemmas 1 and 2 present the way in which the nonlinear bilevel problem (A.1b)–(A.1j), (A.2a)–(A.2l) is reduced to the equivalent bilevel "almost" linear problem (A.3b)–(A.3i), (A.5a)–(A.5j) and (A.5k). Both lemmas rely on using artificial variables to eliminate the *max* and *min* operators.

A.1. Original bilevel problem

First we recall the original bilevel problem, which is nonlinear at both levels and has *max* and *min* operators.

Upper level:

$$\text{Minimize}: \quad H_1(x, sw, EA, EP, xd; y, u, v, z, d, q) \tag{A.1a}$$

$$= \sum_{k\in\mathbf{K}} p_k \left[\sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{J}} \left(C^I_{kti} xd_{kti} - \Pi_{kti} \min\{EA_{kti}, D_{kti}\} + C^R_{kti} EP_{kti} \right) \right] + \sum_{k\in\mathbf{K}^S} p_k h(x_{kN}; y, u, v, z, d, q) \tag{A.1b}$$

$$\text{subject to}: \quad x^L_{kti} \leqslant x_{kti} \leqslant x^U_{kti}, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}; \tag{A.1c}$$

$$sw^L_{kti} \leqslant sw_{kti} \leqslant sw^U_{kti}, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}; \tag{A.1d}$$

$$x^L_{kt} \leqslant \sum_{i\in\mathbf{J}} x_{kti} \leqslant x^U_{kt}, \quad k\in\mathbf{K},\ t\in\mathbf{T}; \tag{A.1e}$$

$$x_{kti} = \begin{cases} x_{a(k),N,i} + sw_{kti} & \text{if } t = 1 \\ x_{k,t-1,i} + sw_{kti} & \text{if } t \neq 1 \end{cases}, \quad \begin{matrix} k\in\mathbf{K},\ t\in\mathbf{T}, \\ i\in\mathbf{J}; \end{matrix} \tag{A.1f}$$

$$x_{kti} = EP_{kti} - EA_{kti}, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}; \tag{A.1g}$$

$$xd_{kti} = \max\{0, D_{kti} - EA_{kti}\}, \quad k\in\mathbf{K}, t\in\mathbf{T},\ i\in\mathbf{J}; \tag{A.1h}$$

$$EP_{kti} = EP_{k'ti}, \quad k,k'\in\mathbf{K}^l,\ l = 1,\dots,S;\ t\in\mathbf{T}, i\in\mathbf{J}. \tag{A.1i}$$

$$EA_{kti} \geqslant 0, \quad k\in\mathbf{K},\ t\in\mathbf{T},\ i\in\mathbf{J}, \tag{A.1j}$$

where $h(x_k; y, u, v, z, d, q) = z$ is the lower level's response to this upper level problem, as defined in (A.2a)–(A.2k) and (A.2l).

Lower level:

$$\text{Minimize}: \quad h_1(x_{kN}; y, u, v, q, d, z) = d \tag{A.2a}$$

$$\text{subject to}: \quad y_j = x_{kNj} + \sum_{i\in\mathbf{J}:i<j} [(1-e_{ij})u_{ij} - v_{ij}] + \sum_{m\in\mathbf{J}:m>j} (v_{jm} - u_{jm}),\ j\in\mathbf{J}; \tag{A.2b}$$

$$\sum_{j\in\mathbf{J}:i<j} u_{ij} + \sum_{m\in\mathbf{J}:m<i} v_{mi} \leqslant \max\{0, x_{kNi}\},\ i\in\mathbf{J}; \tag{A.2c}$$

$$u_{ij} \leqslant \begin{cases} x_{kNi} & \text{if } x_{kNi} > 0 \text{ and } x_{kNj} < 0, \\ 0 & \text{otherwise}, \end{cases} \quad \begin{matrix} i,j\in\mathbf{J}, \\ i<j; \end{matrix} \tag{A.2d}$$

$$v_{ij} \leqslant \begin{cases} x_{kNj} & \text{if} x_{kNj} > 0 \text{ and } x_{kNi} < 0, \\ 0 & \text{otherwise}, \end{cases} \quad \begin{matrix} i,j\in\mathbf{J}, \\ i<j; \end{matrix} \tag{A.2e}$$

$$\min\{0, x_{kNi}\} \leqslant y_i \leqslant \max\{0, x_{kNi}\}, \quad i\in\mathbf{J}; \tag{A.2f}$$

$$-\mathbf{M_1}(1-q) \leqslant y_i \leqslant \mathbf{M_1} q, \quad i\in\mathbf{J}; \tag{A.2g}$$

$$z = -\sum_{i\in\mathbf{J}} r_i y_i - \sum_{(i,j):i<j} v_{ij} b_{ij} + \sum_{(i,j):i<j} f_{ij}(1-e_{ij})u_{ij}; \tag{A.2h}$$

$$-d \leqslant z \leqslant d; \tag{A.2i}$$

$$y_i, z\in\Re, \quad i\in\mathbf{J}; \tag{A.2j}$$

$$u_{ij}, v_{ij} \geqslant 0, \quad i,j\in\mathbf{J},\ i<j; \tag{A.2k}$$

$$q\in\{0,1\}; \tag{A.2l}$$

here, $\mathbf{M_1} > 0$ is a large fixed scalar parameter.

A.2. Linearization lemmas

Lemmas 1 and 2 show the theoretical work needed to reduce the bilevel problem (A.1b)–(A.1i), (A.2a)–(A.2k) and (A.2l) to an equivalent bilevel "almost" linear model, provided that the condition on M_2 is fulfilled. Each of the levels are linearized in such a way that the only nonlinearities allowed are the lower level variables appearing in linear expressions at the upper level.

Lemma 1. *Consider the following "almost" linear programming problem:*

$$\text{Minimize}: \quad H_2(x, EA, xa; y, u, v, z, d, q) \tag{A.3a}$$

$$= \sum_{k\in\mathbf{K}} p_k \left[\sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{J}} C^I_{kti}(D_{kti} - xa_{kti}) - \Pi_{kti} xa_{kti} + C^R_{kti}(x_{kti} + EA_{kti}) \right] + \sum_{k\in\mathbf{K}^S} p_k h(x_k; y, u, v, z, d, q) \tag{A.3b}$$

$$\text{subject to}: \quad x^L_{kti} \leqslant x_{kti} \leqslant x^U_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{A.3c}$$

$$x^L_{kt} \leqslant \sum_{i\in\mathbf{J}} x_{kti} \leqslant x^U_{kt}, \quad k \in \mathbf{K},\ t \in \mathbf{T}; \tag{A.3d}$$

$$sw^L_{k1i} \leqslant x_{k1i} - x_{a(k),N,i} \leqslant sw^U_{k1i}, \quad k \in \mathbf{K},\ i \in \mathbf{J}; \tag{A.3e}$$

$$sw^L_{kti} \leqslant x_{kti} - x_{k,t-1,i} \leqslant sw^U_{kti}, \quad t = 2, \ldots, N,\ k \in \mathbf{K},\ i \in \mathbf{J}; \tag{A.3f}$$

$$xa_{kti} \leqslant D_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{A.3g}$$

$$xa_{kti} \leqslant EA_{kti}, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{A.3h}$$

$$x_{kti} + EA_{kti} = x_{k'ti} + EA_{k'ti},\ k, k' \in \mathbf{K}^l,\ l = 1, \ldots, S;\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{A.3i}$$

$$EA_{kti} \geqslant 0,\ xa_{kti} \geqslant 0, \quad k \in \mathbf{K},\ t \in \mathbf{T},\ i \in \mathbf{J}; \tag{A.3j}$$

The two assertions below imply that this problem is equivalent to the original nonlinear program (1c)–(1j):

1. Let $(x^*, sw^*, EA^*, EP^*, xd^*; y^*, u^*, v^*, z^*, d^*, q^*)$ be an optimal solution to (A.1b)–(A.1i) and (A.1j). Then, there exists an xa^* such that $(x^*, EA^*, xa^*; y^*, u^*, v^*, z^*, d^*, q^*)$ is an optimal solution of (A.3b)–(A.3i) and (A.3j) with the same optimal objective function value.
2. Let $(x^{**}, EA^{**}, xa^{**}; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**})$ be an optimal solution of (A.3b)–(A.3i) and (A.3j). Then there exist sw^{**}, EP^{**}, xd^{**} such that $(x^{**}, sw^{**}, EA^{**}, EP^{**}, xd^{**}; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**})$ solves (A.1b)–(A.1i) and (A.1j) with the same optimal objective function value.

Proof

(a) As the vector x^* from an optimal solution $(x^*, sw^*, EA^*, EP^*, xd^*; y^*, u^*, v^*, z^*, d^*, q^*)$ to (A.1b)–(A.1i) and (A.1j) satisfies (A.1c) and (A.1e), then (A.3c) and (A.3d) are trivially true for this x^*. Next, because (A.1g) and (A.1i) are valid for (x^*, EA^*, EP^*), then (A.3i) clearly holds for (x^*, EA^*). Further, if (x^*, sw^*) satisfies (A.1f), (A.1d), then (A.3e) and (A.3f) are evidently true for x^*.
Now define $xa^*_{kti} = \min\{D_{kti}, EA^*_{kti}\}, k \in \mathbf{K}, t \in \mathbf{T}, i \in \mathbf{J}$ and notice that xa^* yields (A.3g), (A.3h) and (A.3j). Furthermore, taking (A.1h) into account, it is easy to see that $xd^* = D - xa^*$. Plugging the latter equality and (A.1g) into the upper level objective function of the nonlinear problem and taking into account that $x^* = x^{**}$ implies $h_1(x^*_{kN}; y^*, u^*, v^*, z^*, d^*, q^*) = h_1(x^{**}_{kN}; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**})$ (i.e. the optimal value of the lower level objective function is completely determined by the imbalance term x), we come to the equality

$$H_1(x^*, sw^*, EA^*, EP^*, xd^*; y^*, u^*, v^*, z^*, d^*, q^*) = H_2(x^*, EA^*, xa^*; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**}). \tag{$*$}$$

(b) Let x^{**} from an optimal solution $(x^{**}, EA^{**}, xa^{**}; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**})$ to (A.3b)–(A.3i) and (A.3j) satisfy (A.3c) and (A.3d); then it is easy to see that (A.1c) and (A.1e) also hold for x^{**}. Next, if (A.3i) is valid for (x^{**}, EA^{**}) then we can define $EP^{**} = EA^{**} + x^{**}$ so that $(x^{**}, EA^{**}, EP^{**})$ satisfies (A.1g) and (A.1i).
Furthermore, if (A.3e) and (A.3f) are true for x^{**}, we can define sw^{**} as

$$\begin{aligned} &sw^{**}_{k1i} = x^{**}_{k1i} - x^{**}_{a(k)Ni}, \quad k \in \mathbf{K},\ i \in \mathbf{J},\ \text{and} \\ &sw^{**}_{kti} = x^{**}_{kti} - x^{**}_{k,t-1,i}, \quad t = 2, \ldots, N,\ k \in \mathbf{K},\ i \in \mathbf{J}; \end{aligned} \tag{A.4}$$

so that (A.1f) and (A.1d) are valid for sw^{**}.
Now take xa^{**} and consider the coefficient of the objective function H_2 for xa_{kti} in (A.3b), that is, $(-C^I_{kti} - \Pi_{kti})$. Since no variable other than EA^{**} – together with the parameter value D, – limits the growth of xa^{**}, then we can state that, in the minimization process, the variable xa_{kti} will naturally achieve the maximum value allowed by constraints (A.3g) and (A.3h), which is the minimum of the EA^{**}_{kti} and D_{kti}. This means that in the optimal solution of (A.3b)–(A.3i) and (A.3j), one has $xa^{**} = \min\{D_{kti}, EA^{**}_{kti}\}$. Defining $xd^{**}_{kti} = D_{kti} - xa^{**}_{kti}, k \in \mathbf{K}, t \in \mathbf{T}, i \in \mathbf{J}$, yields the xd^{**} satisfying (A.1h) and (A.1j). Therefore, if we substitute $xd^{**} = D - xa^{**}$ into the objective function of the nonlinear problem (A.1b) we obtain:

$$H_1(x^{**}, sw^{**}, EA^{**}, EP^{**}, xd^{**}; y^*, u^*, v^*, z^*, d^*, q^*) = H_2(x^{**}, EA^{**}, xa^{**}; y^{**}, u^{**}, v^{**}, z^{**}, d^{**}, q^{**}). \tag{$**$}$$

(c) We have proved so far that starting from an optimal solution of one problem, a feasible solution for the other problem can be constructed such that their corresponding objective function values will be equal. Hence, it is readily seen that any optimal solution for one problem will give a correspondent optimal solution for the other problem with the same optimal value. Indeed, if any problem had an optimal solution with the objective function value κ^* strictly less than the optimal value of the other problem, τ^*, then, by either (*) or (**), this latter problem would also have a feasible solution with the objective function value $\kappa^* < \tau^*$, thus denying the optimality of the value τ^* and bringing us to a contradiction. This establishes both assertions of the lemma and implies the equivalence of problems (A.1b)–(A.1i), (A.3b)–(A.3i) and (A.3j). □

Lemma 2. *Consider the (linearized) problem:*

$$\text{Minimize}: \quad h_2(x_{kN}; y, u, v, z, q, d, z, \xi, \zeta) = d + \mathbf{M_2} \sum_{i \in \mathbf{J}} (\xi_i + \zeta_i) \tag{A.5a}$$

$$\text{subject to}: \quad u_{ij} \leqslant \xi_i, \quad i,j \in \mathbf{J}, i < j; \tag{A.5b}$$

$$u_{ij} \leqslant \zeta_j, \quad i,j \in \mathbf{J}, i < j; \tag{A.5c}$$

$$v_{ij} \leqslant \xi_j, \quad i,j \in \mathbf{J}, i < j; \tag{A.5d}$$

$$v_{ij} \leqslant \zeta_i, \quad i,j \in \mathbf{J}, i < j; \tag{A.5e}$$

$$-\zeta_i \leqslant y_i \leqslant \xi_i, \quad i \in \mathbf{J}; \tag{A.5f}$$

$$\xi_i \geqslant x_{kNi}, \quad i \in J; \tag{A.5g}$$

$$\xi_i \geqslant 0, \quad i \in J; \tag{A.5h}$$

$$\zeta_j \geqslant -x_{kNj}, \quad j \in J; \tag{A.5i}$$

$$\zeta_j \geqslant 0, \quad j \in J. \tag{A.5j}$$

$$\text{and } (A.2b), (A.2g) - (A.2l). \tag{A.5k}$$

where $\mathbf{M_2}$ *is as described in the main body of the article.*

1. Let $(x_{kN}; y^*, u^*, v^*, q^*, z^*, d^*)$ solve the original problem (A.2a)–(A.2k) and (A.2l). Then there exists (ξ^*, ζ^*) such that $(x_{kN}; y^*, u^*, v^*, q^*, z^*, d^*, \xi^*, \zeta^*)$ is an optimal solution to (A.5a)–(A.5j) and (A.5k).
2. Let $(x_{kN}; y^{**}, u^{**}, v^{**}, q^{**}, z^{**}, d^{**}, \xi^{**}, \zeta^{**})$ solve the mixed-integer program (A.5a)–(A.5j) and (A.5k). Then $(x_{kN}; y^{**}, u^{**}, v^{**}, q^{**}, z^{**}, d^{**})$ is an optimal solution to problem (A.2a)–(A.2k) and (A.2l).

Proof

(a) Let $(x_{kN}; y^*, u^*, v^*, q^*, z^*, d^*)$ solve the original problem (A.2a)–(A.2k) and (A.2l). If we define $\xi_i^* = \max\{x_{kNi}, 0\}, \zeta_i^* = \max\{-x_{kNi}, 0\}, i \in \mathbf{J}$, then it is clear that ξ^* and ζ^* satisfy (A.5g)–(A.5i) and (A.5j). Variables $y^*, u^*, v^*, q^*, z^*, d^*$ trivially satisfy the constraints referenced in (A.5k). It is also clear that if restrictions A.2c, A.2d and A.2e hold for u^*, v^* , then the latter make (A.5b) and (A.5d) true, which, together with (A.2b) and (A.2f), imply (A.5c) and (A.5e) being valid. With ξ^*, ζ^* defined above, constraint (A.2f) can be rewritten as (A.5f), i.e. as y^* satisfies the former, then it yields the latter. Hence, the vector $(x_{kN}; y^*, u^*, v^*, q^*, z^*, d^*, \xi^*, \zeta^*)$ is feasible for problem (A.5a)–(A.5j) and (A.5k). The objective function value (5a) of this problem at that point coincides with

$$H_4(x_{kN}; y^*, u^*, v^*, q^*, z^*, d^*, \xi^*, \zeta^*) = h_2(x_{kN}; y^*, u^*, v^*, z^*, q^*, d^*, z^*) + \mathbf{M_2} \sum_{i \in \mathbf{J}} (\max\{x_{kNi}, 0\} + \max\{-x_{kNi}, 0\}) \tag{A.6}$$

(b) Consider now an optimal solution $(x_{kN}; y^{**}, u^{**}, v^{**}, q^{**}, z^{**}, d^{**}, \xi^{**}, \zeta^{**})$ to the linearized problem (5a)–(5k). If $\mathbf{M_2}$ is large enough, a minimization process will force the variables ξ^{**} and ζ^{**} to take their minimum feasible values in order to minimize their contribution to the objective function. Thus, we will have

$$\xi_i^{**} = \max\{x_{kNi}, 0\}, \zeta_i^{**} = \max\{-x_{kNi}, 0\}; i \in \mathbf{J}. \tag{A.7}$$

The variables ξ_i and ζ_j represent the *amount of gas that can be drawn from zone i* and the *amount of gas that can be deposited into zone j*, respectively. Now if either ξ_i^{**} or ζ_j^{**} is equal to zero, then u_{ij}^{**}, v_{ij}^{**} is also 0 because of (A.5b)–(A.5d) and (A.5e). Hence, u^{**}, v^{**} will satisfy (A.2d) and (A.2e). With ξ^{**}, ζ^{**} defined in (A.7), constraint (A.5f) can be rewritten as (A.2f), that is, if the former is true for y^{**} then the latter also holds. Let us establish that (A.2c) is valid for u^{**}, v^{**}. If $x_{kNi} \geqslant 0$ for an arbitrary $i \in \mathbf{J}$, then expression (A.5f) becomes

$$-\zeta_i^{**} = 0 \leqslant y_i \leqslant x_{kNi} = \xi_i^{**}.$$

Constraint (A.2b) can be rewritten as follows:

$$\sum_{j:j>i} u_{ij} + \sum_{m:m<i} v_{mi} = x_{kNi} + \sum_{j:j<i} (1 - e_{ji}) u_{ji} + \sum_{m:m>i} v_{im} - y_i, \quad i \in \mathbf{J}. \tag{A.8}$$

By (A.5c) and (A.5e), the sums in the right-hand side of the latter equation become equal to 0, which, together with (A.5f), yields:

$$\sum_{j:j>i} u_{ij} + \sum_{m:m<i} v_{mi} = x_{kNi} - y_i \leqslant x_{kNi} = \xi_i^{**} \tag{A.9}$$

Now, on the contrary, suppose that $x_{kNi} \leqslant 0$, for an arbitrary $i \in \mathbf{J}$. In this case, the left-hand side sums in (A.8), when combined with (A.5b) and (A.5d), become zero:

$$\sum_{j:j>i} u_{ij} + \sum_{m:m<i} v_{mi} = 0 = \xi_i^{**}. \tag{A.10}$$

Relationships (A.9) and (A.10) show that constraint (A.2c) is fulfilled, and hence the values $y^{**}, u^{**}, v^{**}, q^{**}, z^{**}, d^{**}$ are feasible for problem (A.2a)–(A.2k) and (A.2l). The objective function value of the (nonlinear) problem is then

$$h_2(x_{kN}; y^{**}, u^{**}, v^{**}, z^{**}, q^{**}, d^{**}, z^{**}) = H_4(x_{kN}; y^{**}, u^{**}, v^{**}, q^{**}, z^{**}, d^{**}, \xi^{**}, \zeta^{**}) - \mathbf{M_2} \sum_{i \in \mathbf{J}} (\max\{x_{kNi}, 0\} + \max\{-x_{kNi}, 0\}) \tag{A.11}$$

(c) We have shown that for any optimal solution of either of the two problems we can find a corresponding feasible solution of the other problem with an explicit link between the two problems' objective function values. Now it is clear that if a vector solves one problem, so does its counterpart to the other problem. Indeed, let

$$\boldsymbol{\kappa} = \mathbf{M_2} \sum_{i \in \mathbf{J}} (\max\{x_{kNi}, 0\} + \max\{-x_{kNi}, 0\});$$

then if the nonlinear problem has an optimal solution with the objective function value $\boldsymbol{\sigma}^*$ strictly less than $(\boldsymbol{\tau}^* - \boldsymbol{\kappa})$, where $\boldsymbol{\tau}^*$ is the optimal objective function value in the linearized problem, then part (a) of this proof would imply the linearized problem to have a feasible solution with its objective function value $\boldsymbol{\sigma}^* + \boldsymbol{\kappa} < \boldsymbol{\tau}^*$. That would deny the optimality of $\boldsymbol{\tau}^*$ and thus yield a contradiction. The similar argument can be easily applied to the reciprocal case. This verifies both assertions of the lemma and justifies the linearization of the original problem. □

Appendix B. Problem instance B011

Problem instance B011 has $N = 28$ days divided in $S = 4$ stages, and $P = 4$ pool zones. It is the largest instance tested – among problem instances B001 to B020, – in terms of days and stages, with a total of 28 nodes of 7 days.

Table B.1 shows the upper and lower bounds for the problem, $x^L_{\mathbf{t}i}$ and $x^L_{\mathbf{t}i}$, as well as the daily total bounds $x^L_{\mathbf{t}}$ and $x^U_{\mathbf{t}}$. Since all nodes in a given stage share the same bounds, these values are indexed by $\mathbf{t}$, which represent the chronological day rather than the nodal day. For the first stage, $\mathbf{t}$ corresponds to t as is given in the notation table. For all nodes k in stage 2, for example, $\mathbf{t} = t + 7$, and so on.

Table B.2 shows the mean forecasted consumptions $D_{\mathbf{t}i}$, and the mean forecasted prices $\Pi_{\mathbf{t}i}$. They correspond to the centermost of the 27 scenarios of this problem instance, that is, scenario 15. These numbers are also used to obtain the Expected Value Solution.

Likewise the values in the Table B.1, the rows here are indexed by $\mathbf{t}$ instead of by k and t.

Finally, Tables B.3 and B.4 display the most important parameters for the lower level problem. The forward haul cost charged to the NGSC in the final re-arrangement f_{ij}, the backward credit given to him, b_{ij}, and the imbalance penalization per pool zone, r_i, are all shown first, whereas the percentage of gas lost when moving it forward (mainly considered to be used as fuel for the pipeline pumps,) e_{ij}, appears in the final table.

Appendix C. Figures

In this Appendix C we show a graphical exposition of the findings in Tables 3 and 4 of the article's main body.

Fig. C.1 shows four graphs for the running times (RT), agains the instance number, days N, pool zones P, and stages S. The number of pool zones seems to be clearly affecting the time required to reach a solution, with the problems with 7 pool zones having a noticeable larger average running time than 4- and 5-staged instances. As for days, while the 28-day instance B011 takes longer than any other, 12-day are in average faster than 10-day instances. There is, therefore, no conclusive findings regarding days and running time for small-scale differences. Results regarding stages are also not so conclusive. Problems with 2 stages may take, in general, longer than problems with

Table B.1
Lower and upper bounds x^L_{kti}, x^U_{kti}, and total lower and upper bounds $x^L_{kt}, x^{U_{kt}}$.

$\mathbf{t}$	$x^L_{\mathbf{t}i}$				$x^U_{\mathbf{t}i}$				$x^L_{\mathbf{t}}$	$x^U_{\mathbf{t}}$
	i				i					
1	6	−190	−125	−79	78	−121	−59	−15	−307	−198
2	11	−193	−131	−84	78	−116	−58	−13	−304	−195
3	8	−189	−128	−81	76	−123	−58	−14	−308	−197
4	11	−193	−125	−82	78	−122	−55	−19	−305	−193
5	10	−189	−127	−88	86	−120	−61	−16	−302	−199
6	12	−190	−133	−88	79	−117	−56	−21	−309	−200
7	14	−182	−132	−89	85	−111	−64	−21	−303	−200
8	21	−184	−130	−93	83	−109	−61	−20	−304	−199
9	22	−178	−132	−94	93	−115	−62	−24	−304	−193
10	21	−181	−136	−91	92	−110	−64	−24	−301	−193
11	20	−181	−132	−95	90	−109	−63	−22	−305	−194
12	19	−172	−138	−93	91	−107	−64	−26	−299	−187
13	24	−171	−134	−97	89	−102	−70	−21	−297	−184
14	21	−169	−141	−98	96	−101	−71	−20	−297	−182
15	24	−172	−135	−92	91	−102	−62	−23	−291	−177
16	22	−173	−141	−97	92	−100	−71	−28	−287	−177
17	26	−167	−136	−97	97	−103	−74	−25	−290	−185
18	31	−169	−138	−98	95	−91	−69	−31	−288	−183
19	34	−167	−137	−103	103	−98	−67	−25	−287	−178
20	30	−159	−142	−98	105	−98	−70	−27	−287	−184
21	30	−164	−144	−101	106	−94	−75	−30	−293	−179
22	34	−159	−144	−101	100	−94	−75	−37	−289	−180
23	38	−164	−143	−111	105	−90	−74	−35	−295	−178
24	36	−153	−145	−109	104	−89	−69	−43	−289	−181
25	39	−155	−146	−110	110	−81	−78	−41	−293	−181
26	40	−154	−148	−112	109	−80	−72	−42	−287	−176
27	39	−152	−148	−112	111	−87	−75	−45	−291	−175
28	40	−153	−152	−115	110	−78	−79	−47	−289	−176

Table B.2
Mean forecasted consumption D_{ti} and mean forecasted price Π_{ti}.

t	D_{ti}				Π_{ti}			
	i				i			
1.00	31155.79	13994.25	14411.06	12611.35	9.14	9.10	10.45	10.00
2.00	24691.38	12166.05	12114.09	9855.41	9.57	9.02	10.66	10.01
3.00	19816.90	10083.20	10291.52	7743.05	10.47	9.57	10.92	9.98
4.00	12061.31	5749.90	5582.72	3625.71	10.95	10.02	12.14	10.23
5.00	6660.73	3011.50	2679.29	1625.01	11.71	11.26	13.31	10.55
6.00	3752.60	1671.43	1623.71	1172.28	13.12	13.70	14.63	11.02
7.00	2577.78	1321.90	1329.66	1108.88	13.41	14.49	14.75	11.50
8.00	2585.35	1215.14	1234.47	1092.33	12.43	14.27	14.59	11.64
9.00	3182.31	1406.26	1312.38	1327.00	10.80	13.71	14.16	11.50
10.00	7450.13	3241.46	2148.22	2842.65	9.02	9.85	12.60	10.64
11.00	14039.66	6834.42	5431.59	6294.54	8.60	9.07	10.44	10.20
12.00	24677.59	11173.85	10730.31	11195.84	8.44	9.13	9.91	9.91
13.00	31282.64	13980.78	14379.48	12647.35	9.03	9.43	10.92	10.23
14.00	24818.23	12152.57	12082.50	9891.41	9.47	9.35	11.13	10.24
15.00	19943.75	10069.73	10259.93	7779.05	9.61	9.90	11.39	10.42
16.00	12188.15	5736.42	5551.13	3661.71	9.85	10.35	12.61	11.24
17.00	6787.57	2998.03	2647.70	1661.01	10.18	11.59	13.78	12.65
18.00	3879.44	1657.96	1592.13	1208.28	10.75	14.03	15.10	13.07
19.00	2704.62	1308.43	1298.07	1144.88	10.87	14.82	15.22	13.14
20.00	2712.19	1201.67	1202.88	1128.33	10.86	14.60	15.06	13.08
21.00	3309.14	1392.78	1280.79	1363.00	10.15	14.04	14.63	12.15
22.00	8433.53	3680.93	2609.77	3292.77	9.39	10.56	14.47	11.56
23.00	15069.22	7302.87	5933.64	6746.70	9.30	9.82	14.09	11.24
24.00	25717.85	11648.63	11241.35	11647.91	9.36	9.93	14.01	11.06
25.00	32338.20	14473.55	14909.41	13101.72	10.22	10.26	14.05	11.47
26.00	25875.63	12657.51	12624.80	10347.81	10.05	10.21	14.15	11.55
27.00	21000.68	10578.38	10808.82	8236.52	10.23	10.76	14.21	11.56
28.00	13255.74	6250.17	6103.85	4120.54	10.47	11.22	14.45	11.82

Table B.3
Forward haul costs f_{ij}, backward move credits b_{ij}, and pool zone imbalance penalizations r_i.

f_{ij}			j	
i	0	16	9	10
	0	0	9	13
	0	0	0	9
	0	0	0	0
b_{ij}			j	
i	0	16	9	10
	0	11	3	11
	0	0	15	11
	0	0	0	14
	0	0	0	0
r_i	120	100	120	140

Table B.4
Forward haul gas loss e_{ij}.

e_{ij}			j	
i	0	16	9	10
	0	0.003	0.085	0.021
	0	0	0.001	0.130
	0	0	0	0.034
	0	0	0	0

3 stages, though the 4-staged problem takes clearly longer than the smaller instances, while this may be attributed to the 28 days instance B011 has.

A comparison of the perfect information solution (PIS) values, stochastic solution implementation (SSI) values, and expected value solution implementation (EVSI) values, is displayed next in Fig. C.2. The red line, connecting all the observation points, represents the PIS value against the instance number, that is, the maximum revenue that can be obtained by the NGSC, only attainable with perfect forecasting abilities. The blue line is the SSI value. This value is obtained by solving the stochastic optimization problem (2 b)–(2j), (4a)–(4k), and then

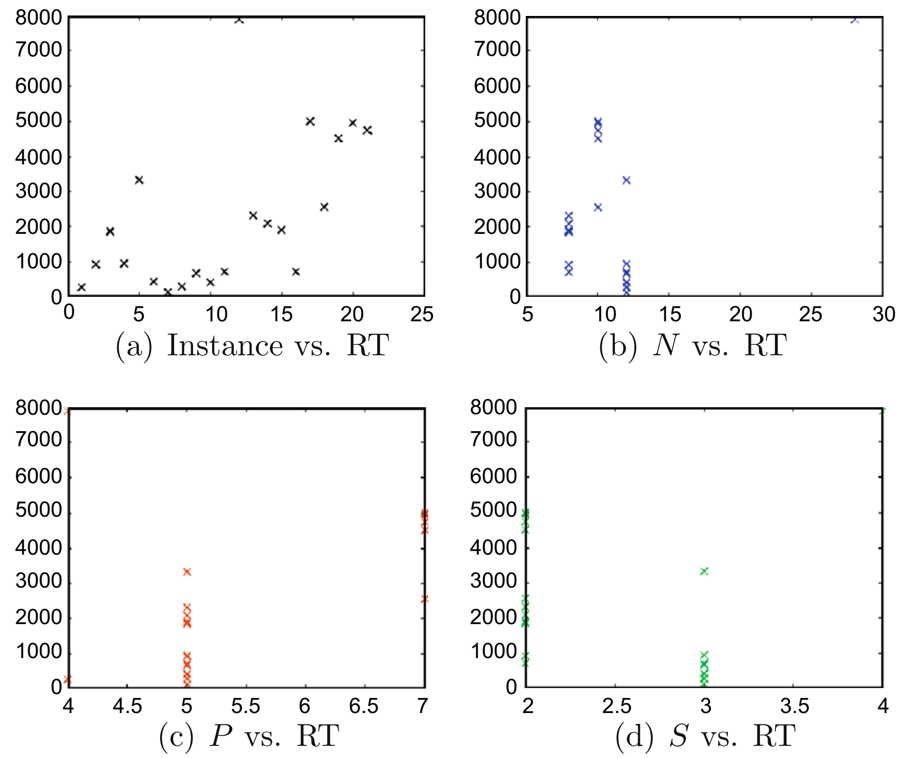

Fig. C.1. Plots for instances B001-B020 running times.

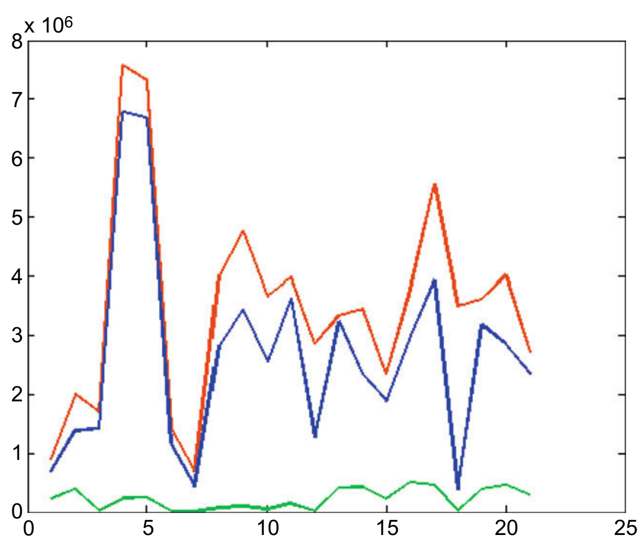

Fig. C.2. PIS, SSI, and EVSI comparison.

plugging the obtained values into the corresponding PIS problem. The blue line, of course, cannot surpass the red line; we can see that, except for instance B017, it follows the PIS values relatively well. On the contrary, the EVSI values, represented by the green line, stands quite low compared to the other two. Only a handful of instances have the green and blue lines relatively closer to one another, which speaks of the lack of efficiency of the EVS approach when compared to the SS approach we propose.

Finally, Fig. C.3 compares the relative errors for the SSI and the EVS, SSRE and EVSRE, when compared to the PIS, for every instance tested. The closest the points are to 1, the more different the EVS or the SS are from the PIS. The closest these latter are to 0, the more the SS and EVS resemble the PIS. The red line, corresponding to the SSREs, only raises above 0.5 (meaning a SSI value below half of the PIS value,) in two instances, B011 and B017. The first one, though, is the largest instance with four stages and 28 days, so forecasts become increasingly unreliable and as a consequence the SSI accurateness is downgraded. Instance B017, though, appears as a very problematic

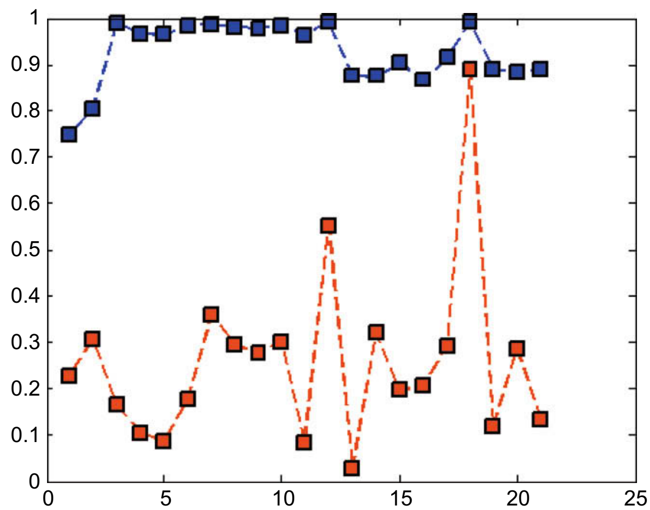

Fig. C.3. SSRE and EVSRE.

instance in spite of the resemblance of its design parameters (ranges for initial imbalances, tolerances for bounds, etc.) to problems B016–B020.

Every instance has a EVSRE over 0.7, above the red line (except for instance B017.)

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