Developing an Optimal Dispatching System for Natural Gas pipeline Network

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Abstract
This paper consists of three main parts which aims at developing an optimal dispatching system for natural gas pipeline network. First of all, an expert model is proposed to determine the pressure level and the horsepower requirement in the network. This proficient system constructs input-output mapping based on human knowledge and input-output data pairs (in the form of fuzzy if-then rules). Moreover, the problem of minimizing the fuel cost consumed by compressor stations in a specified transmission network domain is addressed that is able to find an optimal solution using fuzzy linear programming method. Last but not the least, this paper focuses on the modeling of a gas distribution pipeline system; after deriving a simplified model from the set of PDE's governing the dynamic of the system, an analytical scheme is proposed to solve the system dynamics.

Keywords: Dispatching System, Natural gas pipeline network, Fuzzy Proficient system, Fuel cost minimization, Fuzzy linear programming, Dynamic modeling, Analytical scheme

1. Introduction

Pipeline networks are the most effective and common ways for transporting natural gas. According to the National Iranian Gas Company (NIGC), a huge amount of 5 milliard m^3 of natural gas are transported daily in Iran over 30,000 km of pipeline system. The main purpose of this project is to construct an autonomous natural gas dispatching system to satisfy customer demand with minimal operating cost, and document the operating knowledge of senior dispatchers as well as using a proficient system as a training tool for training new dispatchers. A brief literature history on the object of the presented research is proposed in section 2. Section 3 describes the main two tasks of the proficient system and its role in the presented problem. Sections 4 and 5 describe how this proficient system performs these two tasks using historical data as well as heuristic knowledge from human experts.

As the gas flows through the network, energy and pressure are lost due to both friction between the gas and the pipes' inner wall, and heat transfer between the gas and its environment. The lost energy of the gas is periodically restored at the compressor stations which are installed in the network. These compressor stations typically consume about 3-5% of the transported gas that leads to a significant amount of fuel cost. These facts make the problem of how to optimally operate the compressors driving the gas in a pipeline network important. Valves also play an important role in the operation of the piping system, and it is important to calculate the head loss and energy costs associated with them. The head loss from valves can be converted into an energy cost related to the compressor power needed to overcome the additional head loss from the valve. Section 7 of this paper addresses the problem of minimizing the fuel cost incurred by the compressor stations driving the gas in a transmission network under unsteady conditions.

This paper continues in Section 8 with a review of the set of PDEs (continuity and momentum equations) governing the dynamic of the gas flow through a pipeline. A simplified model is also derived from the original one. Section 9 shows a brief description of the analytical scheme proposed for the first time in this paper in order to solve the dynamic equations derived in section 8.

2. Relevant Literature

Different researchers have worked on issues relating to the problems in the dispatching system of natural gas pipeline network. C.K. Sun et al. (2000) have developed an integrated decision support system combining proficient systems and mathematical modeling in order to optimize natural gas pipeline operations. In this integrated approach, both proficient systems and operations research techniques are used to model the operations of the gas pipelines, [1]. A. Martin et al. (2005) developed a mixed integer model for the solution of the stationary gas optimization problem. They described techniques for a piece-wise linear approximation of the nonlinearities in this model resulting in a large mixed integer linear program, [2]. A. Chebouba et al. (2008) proposed an ant colony optimization (ACO) algorithm for operations of steady flow gas pipeline. Their system is composed of compressing stations linked by pipe legs. The decisions variables are chosen to be the operating turbo compressor number and the discharge pressure for each compressing station and the objective function is the power consumed in the system by these stations, [3]. S. Wu et al. (1999) addressed the problem of minimizing the fuel cost incurred by the compressor stations driving the gas in a transmission network under steady state assumptions. In particular, the decision variables include pressure drops at each node of the network, mass flow rate at each pipeline leg, and the number of units to be operating within each compressor station' [4]. Rios-Mercado *et al.*(2001) presented a reduction technique for natural gas transmission network optimization problems. The decision variables are the mass flow rate through each arc (pipeline segment), and the gas pressure level at each pipeline node, [5]. A. Herran-Gonzalez et al. (2008) have worked on the dynamic modeling and simulation of a gas distribution pipeline with a special emphasis on gas ducts. The set of PDE's and two simplified dynamic models have been derived. To solve the system dynamics through the proposed numerical schemes they've built a based MATLAB-Simulink library that is able to simulate the behavior of a gas distribution network, [6]. M. Herty et al. (2008) derived a model for gas dynamics in pipe networks by asymptotic analysis. The model is a strong improvement of the quasi static model and the SIMONE–model commonly used in the engineering community, [7]. Ke and Ti (2000) analyzed isothermal transient gas flow in the pipeline networks using the electrical analogy. The transient behavior of the pipe, conventionally taking the form of a set of a second-order partial differential equations, can be expressed by a set of first-order ordinary differential equations using the new model. This new method shows a promising feature in saving computational efforts and may be employed as a powerful means for pipe network design and control, [8]. Martinez-Romero et al. (2002) described steady-state compressible flow through a pipeline using software package "Gas Net". They presented a sensibility analysis for the most important flow equations defining the key parameters in the optimization process, [9]. Although there are lots of literatures in this field, the mentioned paper in this section were the most relevant ones.

3. Proficient System

Proficient system is used in problems and process control engineering where no mathematical models can be formulated, the working knowledge of the system is nonlinear and incomplete, and the knowledge of an experienced human expert can give a satisfactory solution. The main tasks of the proficient system in our model are first to determine the level of the pressure in the pipeline and recommend the control commands to be issued, and second to evaluate the associated horsepower requirement. An overview of the role that the proficient system plays in the operation of the pipeline network is shown in Fig. 1, [1].

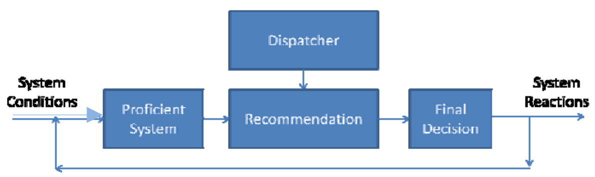

Fig. 1: Proficient system role in the operation of pipeline network

The knowledge base of the proficient system is based on an analysis of historical data, heuristic knowledge from human experts in the natural gas transportation company, and a computer simulation program that belonged to the gas company.

4. Determining the Level of the Pressure in a Pipeline and Recommend the Control Commands

Determining the level of the pressure in a pipeline is realized with an if-then decision algorithm. Both the conditional variables and the decision variable of this algorithm are listed in Table 1.

Table 1: Decision algorithm for natural gas pipeline operation

Kind of Variables	Variables	Variable Region
Conditional Variables IF	Change of pressure at the end points	€ {rapidly decrease, decrease, no change, increase, rapidly increase}
	Rate of change of pressure at the end	€ {+,-}
	Flow	€ {very high, high, medium, low, very low}
	Current Pressure level	€ {CLP1, CLP2, CLP3, CLP4, CLP5}
Decision Variables THEN	State of the line pack	€ { low, high, enough} No Action No Extra Compression Extra Compression

Total of 250 Rules are generated using the above if-then algorithm that are presented in 10 decision table. Table 2 is a sample of these tables.

Table 2: Sample Decision Table

Current Line Pack = CLP1					
+ Rate of Change of Pressure at the End Point					
ΔP at the end / Flow	Rapidly Decrease	Decrease	No change	Increase	Rapidly Increase
Very high	Enough	Enough	Enough	High	High
High	Enough	Enough	Enough	High	High
Medium	Enough	Enough	Enough	Enough	High
Low	Low	Enough	Enough	Enough	High
Very Low	Low	Low	Enough	Enough	Enough

5. Evaluating the Horsepower Requirement

The horsepower requirement can be derived from heuristic data points. x and y components of each point represent the volume of natural gas (Load) and the amount of HP requirement respectively. After clustering the data points, Takagi-Sugeno Kang Fuzzy Model (TSK), [10], of the following form is considered to be used in order to develop the linear rule fitting the historical data points. We assume the fuzzy inference system under consideration has one input x and one output y. Suppose that the rule base contains two fuzzy if-then rules of Takagi and Sugeno's type:

$R_i: IF\ x\ is\ A_i\ \ THEN\ f_i = p_i x + r_i$; i=1,2

Calculating the firing strength of the rule by equation (1);

$w_i = \mu_{A_i}(x)$; i=1,2 (1)

The overall output can be expressed as linear combinations of the consequent parameters as

$\bar{w}_i = \frac{w_i}{w_1+w_2}$; i=1,2 (2)

$f = \bar{w}_1 f_1 + \bar{w}_2 f_2$ (3)

The simplified fuzzy if-then rules become of the following form in which the output is described by a crisp value (or equivalently, a singular membership function)

$R_i: IF\ x\ is\ A_i\ \ THEN\ z\ is\ f$

Most of all, with this simplified fuzzy if-then rule, it is possible to prove that under certain circumstance, the resulting fuzzy inference system has unlimited approximation power to match any nonlinear functions arbitrarily well on a compact set [11]. We will precede this in a descriptive way by applying the Stone-Weierstrass theorem [12] stated below.

Weierstrass theorem: let domain D be a compact space of N dimensions, and let F be a set of continuous real-valued functions on D, satisfying the following criteria:

1. Identity function: The constant f(x)=1 is in F.
2. Separability: For any two points $x_1 \neq x_2$ in D, there is an f in F such that $f(x_1) \neq f(x_2)$.
3. Algebraic closure: If f and g are any two functions in F, then fg and af + bg are in F for any two real numbers a and b.

Then F is dense in C(D), the set of continuous real-valued functions on D. In other words, for any $\epsilon > 0$, and any function g in C(D), there is a function f in F such that $|g(x) - f(x)| < \epsilon$ for all $x \in D$.

In application of fuzzy inference systems, the domain in which we operate is almost always closed and bounded and therefore it is compact. For the first and second criteria, it is trivial to find simplified fuzzy inference systems that satisfy them. Now all we need to do is examine the algebraic closure under addition and multiplication. Suppose we have two fuzzy inference systems S and $\tilde{S}$; each has two rules and the output of each system can be expressed by equations (4) and (5).

$S: z = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2}$ (4)

$\tilde{S}: z = \frac{\tilde{w}_1 \tilde{f}_1 + \tilde{w}_2 \tilde{f}_2}{\tilde{w}_1 + \tilde{w}_2}$ (5)

where $f_1, f_2, \tilde{f}_1, and\ \tilde{f}_2$ are constant outputs of each rule.

$$az + b\tilde{z} = a\frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} + b\frac{\tilde{w}_1 \tilde{f}_1 + \tilde{w}_2 \tilde{f}_2}{\tilde{w}_1 + \tilde{w}_2} = \frac{w_1\tilde{w}_1(af_1 + b\tilde{f}_1) + w_1\tilde{w}_2(af_1 + b\tilde{f}_2)}{w_1\tilde{w}_1 + w_1\tilde{w}_2 + w_2\tilde{w}_1 + w_2\tilde{w}_2} + \frac{w_2\tilde{w}_1(af_2 + b\tilde{f}_1) + w_2\tilde{w}_2(af_2 + b\tilde{f}_2)}{w_1\tilde{w}_1 + w_1\tilde{w}_2 + w_2\tilde{w}_1 + w_2\tilde{w}_2} \quad (6)$$

$$z\tilde{z} = \frac{w_1\tilde{w}_1 f_1\tilde{f}_1 + w_1\tilde{w}_2 f_1\tilde{f}_2 + w_2\tilde{w}_1 f_2\tilde{f}_1 + w_2\tilde{w}_2 f_2\tilde{f}_2}{w_1\tilde{w}_1 + w_1\tilde{w}_2 + w_2\tilde{w}_1 + w_2\tilde{w}_2} \quad (7)$$

which are of the same form as equation (4) and (5). Apparently the model architectures that compute $az + b\tilde{z}$ and $z\tilde{z}$ are of the same class of S and $\tilde{S}$ if and only if the class of membership functions is invariant under multiplication. This is loosely true if the class of membership functions is the set of all bell-shaped and scaled Gaussian membership functions, as pointed out by Wang [10].

Therefore by choosing an appropriate class of membership functions, we can conclude that the TSK model with simplified fuzzy if-then rules satisfy the four criteria of the Stone-Weierstrass theorem. Consequently, for any given $\epsilon > 0$, and any real-valued function g, there is a fuzzy inference system S such that $|g(x) - S(x)| < \epsilon$ for all x in the underlying compact set. Moreover, we can draw the conclusion that all the TSK model that is used to evaluate the horsepower requirement, have unlimited approximation power to match any given data set. However, caution has to be taken in accepting this claim since there is no mention about how to construct the model according to the given data set.

6. Problem Domain

Fig 2, [13], shows the 3rd operational region of the Natural Iranian Gas Transportation Company (NIGTC) operating region. It has 3 inputs, 4 compressors and 15 customers.

Fig. 2: 3rd operational region of NIGTC

In the first step a small part of the above complicated region is considered as it is shown in Fig. 3. It consists of two compressor station (C. Qum1 and C. Qum 2) and four cities that involve both domestic and industrial customers. Industrial customers consume a specific amount of natural gas while domestic customer demands change over time.

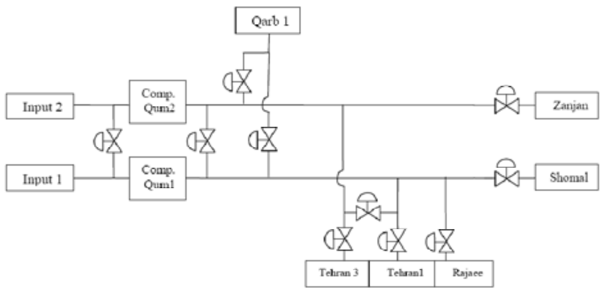

Fig. 3: Problem Domain

7. Mathematical Model for the Fuel Cost Minimization

Once the pipeline, compressor stations, and ancillary facilities are constructed and the pipeline is put into operation, there will be annual operating costs over the useful life of the pipeline, which might be 30 to 40 years or more. These annual costs consist of fuel, energy and utility costs, operating and maintenance costs, rental, permitting, and annual right of way costs.

Pipeline maintenance costs, such as pipe repair, relocation, aerial patrol, and monitoring plus compressor station costs that include periodic equipment maintenance and overhaul costs are not modeled here, since the time horizon in the presented model is assumed to be about 8 month. For example, a gas turbine–driven compressor unit may have to be overhauled every 18 to 24 months [14].

The problem domain in this paper that is a part of the 3rd operational region (see Fig.3) has two compression stations, Qum 1 and Qum 2. A compression station is used to increase the pressure for a particular section in a given area, in order to meet customer demand for natural gas in that area. All areas are monitored by a dispatcher who is stationed at a central location. The dispatcher makes the decision as to which compressor in any given station should be turned on/off, and when to do so. Each compressor can provide only a specified amount of horsepower, which is different for different compressors. In addition each compressor associated with a start-up cost. The operating cost of each type of compressor is also different, depending on its horsepower. If there is not enough pressure in the system, there will be the opportunity cost of lost revenue. If there is too much pressure, the system would produce something for no purpose, and waste energy. There are three compressor units in compression station Qum 1, and five compressor units in compression station Qum 2.

Assuming that the network consists of pipes, compressor stations and valves, the objective function of the problem is the sum of the fuel costs over all compressor stations in the network.

The horsepower for compression varies as the cube of the speed change [14]. As a simplification here we consider a piecewise linear relation between HP and N in the working speed range of the compressor in order to have a linear programming problem.

$$HP \sim B' N^3 \sim BN \qquad (8)$$

According to pipeline rules of Thumb Hand Book [15],

$$cost\ of\ natural\ gas\ per\ MCF = 0.008\,(HP)\,(cost\ of\ gas\ per\ MCF) \qquad (9)$$

$$cost \sim ANF \qquad (10)$$

where A, B, and B' are known coefficients.

An initial linear programming formulation is derived from Sun, Uraikul, and Chan (2000) [1]. An assumption has been made that the parameters of the problem are known with certainty.

Objective Function:

$$\min z = \sum_{i=1}^{8}\sum_{p=1}^{8} (AN_{i,p}F_i + S_i) \qquad (11)$$

The second term on the right hand side of the above equation is added if and only if $N_{i,p-1} = 0$.

Subject to:

$$\sum_{i=1}^{8} f_i(N_{i,p}) \geq D_p \qquad \forall p$$

$$N_{i,p} + N_{i,p-1} - x_{i,p} \leq 1 \qquad \forall i,p$$

$$-N_{i,p} - N_{i,p-1} + 2x_{i,p} \leq 0 \qquad \forall i,p$$

$$N_{i,p} + N_{i,p-1} - x_{i,p} \geq 0 \qquad \forall i,p$$

In reality, however, customer demand is not known with certainty, nor can it be precisely predicted. Using this information and assuming a linear representation of the problem, a fuzzy 0-1 linear programming formulation of the problem has been developed. This formulation is derived from Li-Xin Wang (1997) [10] and Sun, Uraikul, and Chan (2000) [1].

Objective Function:

$$\min \alpha \qquad (12)$$

$$\alpha \leq 1 - \left(\frac{\sum_{i=1}^{8}\sum_{p=1}^{8} (AN_{i,p}F_i + S_i) - z}{T_o} \right)$$

The second term in the parentheses is added if and only if $N_{i,p-1} = 0$.

Subject to:

$$\alpha \leq 1 + \left(\sum_{i=1}^{8} f_i(N_{i,p}) - D_p \right) \Big/ T_p \qquad \forall p$$

$$N_{i,p} + N_{i,p-1} - x_{i,p} \leq 1 \qquad \forall i,p$$

$$-N_{i,p} - N_{i,p-1} + 2x_{i,p} \leq 0 \qquad \forall i,p$$

$$N_{i,p} + N_{i,p-1} - x_{i,p} \geq 0 \qquad \forall i,p$$

$$\alpha \leq 1$$

Where:

$N_{i,p}$	speed of ith natural gas compressor in period p
F_1	fuel cost for the natural gas compressor
S_1	start-up cost for the natural gas compressor
D_p	customer demand in period p (1-8 months)
f_i	horsepower rating of ith natural gas compressor
T_o	tolerance for customer demand variation in period p
T_p	tolerance for deviation in objective function value for the non-fuzzy optimization
$x_{i,p}$	$N_{i,p}N_{i,p-1}$ Linearization variable
HP	horsepower for compressor
MCF	cubic feet per day (load of natural gas)

It is important to calculate the head loss and energy costs associated with valves. Although many other valve characteristics must be considered to guarantee the best selection of valve type for a given application, head loss is the only contributing factor to the total cost of our model. Other cost considerations are the purchase, installation, and maintenance costs that are not modeled here. The head loss from valves can be converted into an energy cost related to the pumping electrical power needed to overcome the additional head loss from the valve with the equation (13), [16].

$$A = (1.65\ Q\ \Delta H\ S_g\ C\ U) / E \qquad (13)$$

Where:

A	annual energy cost, dollars per year
Q	flow rate, gpm
ΔH	head loss, ft
S_g	specific gravity, dimensionless
C	cost of gas, $/MCF
U	usage, percent *100 (1.0 equals 24 hr per day)
E	efficiency of compressor
v	flow velocity, ft/sec
d	valve diameter, in

Head Loss is also derived from equation (14), [16].

$\Delta H = K_v v^2 / 2g$ (14)

where K_v is the dimensionless resistance coefficient of the valve. The flow factor K_v can also be related to the flow coefficient C_v by equation (15).

$K_v = 890\ d^4 / C_v^2$ (15)

8. Modeling of the Gas Pipeline Network

The set of Partial Differential Equations (PDEs) governing the one-dimensional gas flow dynamic through a gas pipeline, continuity, momentum and energy equations, are obtained from Herran-Gonzalez [1] and Anderson [17].

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \quad (16)$$

$$\frac{\partial}{\partial t}(\rho v A) + \frac{\partial}{\partial x}(pA + \rho v^2 A) + |\tau|\pi D + \rho A g \sin\theta = 0 \quad (17)$$

$$\frac{\partial}{\partial t}\left[\left(e + \frac{v^2}{2}\right)\rho A\right] + \frac{\partial}{\partial x}\left[\left(h + \frac{v^2}{2}\right)\rho v A\right] - \Omega + \rho A g v \sin\theta = 0 \quad (18)$$

And the equation of state for gas is

$$p = \rho \frac{Z R_u T}{M} = \rho Z R_g T \quad (19)$$

where

D	gas pipeline diameter (m)
e	specific internal energy (J kg^{-1})
h	specific enthalpy (J kg^{-1})
M	molecular mass of gas (kg mol^{-1})
P	gas pressure (bar)
q	mass flow rate (kg s^{-1})
R_u	universal gas constant (J mol^{-1}K^{-1})
R_g	gas constant (J kg^{-1}K^{-1})
A	gas pipeline cross-sectional area (m^2)
T	temperature of gas (K)
v	velocity (m/s)
x	axial coordinate (m)
Z	compressibility factor
D	gas pipeline diameter (m)
Ω	heat flow per unit length (J m^{-1} s^{-1})
ρ	gas density (kg m^{-3})
θ	inclination angle of the pipeline to horizon (°)
τ	tangential stress (N)

Two important cases usually are considered in literature to solve the above Equations:

(a) Isothermal flow (T = constant) corresponding to slow dynamic changes, in which the value of Ω can be calculated through energy equation

(b) Adiabatic flow (Ω= 0) corresponding to fast dynamic changes, that includes the particular isentropic flow case

Rewriting the equations (1), (2) and (3) in function of Fanning friction factor, f, and mass flow, q, and assuming isothermal flow, the one-dimensional gas flow dynamics inside a gas pipeline is described by the set of PDE's shown in Equation (5).

$$f = \frac{|\tau|}{\frac{1}{2}\rho v^2} \quad (20)$$

$$q = \rho v A = \rho Q \quad (21)$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial \rho}{\partial x} = 0 \\ \frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(Sp + \frac{q^2}{A\rho}\right) + \frac{2fq|q|}{DA\rho} + \rho A g \sin\theta = 0 \end{cases} \quad (22)$$

Friction term turns the equations of the non viscous gas classic dynamics to viscous ones. Inertia term changes the creeping motion to undulatory propagation phenomenon [6].

As the first simplification the third term in equation (22) is neglected, making reference to horizontal pipeline. Then, the gas flow dynamics through a gas pipeline can be represented by the system of PDE's shown in Equation (23) in which it is considered to have isothermal process, $p = a^2\rho$. Besides the relation q = qvS = constant = qQ = $q_n Q_n$ is used to express the model in function of flow rate in normal conditions, Q_n(x, t) and pressure p(x, t), where the subscript n refers to quantities at standard conditions of pressure P_n=0.1 MPa and temperature T_n=288K. This system was used by Herran-Gonzalez, Cruz, Andres-Toto, and Risco-Martin [6] for unsteady flow simulation.

$$\begin{cases} \frac{\partial p}{\partial t} = -\frac{a^2\rho_n}{A}\frac{\partial Q_n}{\partial x} \\ \frac{\partial Q_n}{\partial t} = -\frac{A}{\rho_n}\frac{\partial p}{\partial x} - \frac{2fa^2\rho_n Q_n^2}{DAp} \end{cases} \quad (23)$$

where

L	L gas pipeline length (m)
Q	Q volumetric flow rate (m^3 s^{-1})
Q_n	volumetric flow rate in normal conditions (m^3 s^{-1})
ρ_n	gas density in normal conditions (kg m^3)

Moving the x-derivatives to the left had side the above system of equations becomes the system shown in equation (24).

$$\begin{cases} \frac{\partial Q_n}{\partial x} = -\frac{A}{a^2\rho_n}\frac{\partial p}{\partial t} \\ \frac{\partial p}{\partial x} = -\frac{\rho_n}{A}\frac{\partial Q_n}{\partial t} - \frac{2fa^2\rho_n Q_n|Q|}{DA^2} \end{cases} \quad (24)$$

9. Proposing an Analytical Scheme to Solve the PDEs

Rewriting the set of equations (23) with new constant coefficients the set of equations (25) are obtained.

$$\begin{cases} \frac{\partial p}{\partial t} = -\frac{a^2\rho_n}{A}\frac{\partial Q_n}{\partial x} = \alpha_1\frac{\partial Q_n}{\partial x} \\ \frac{\partial Q_n}{\partial t} = -\frac{A}{\rho_n}\frac{\partial p}{\partial x} - \frac{2fa^2\rho_n Q_n^2}{DAp} = \beta_1\frac{\partial p}{\partial x} + \beta_2\frac{Q_n^2}{p} \end{cases} \quad (25)$$

$p_x(x)$ and $Q_x(x)$ are assumed to be the steady state solutions of the system. From the first equation of equation (25) it can be inferred that Q_n is constant under steady state condition.

$$\frac{\partial p}{\partial t} = 0;\ \frac{\partial Q_n}{\partial x} = 0 \quad (26)$$

$$Q_n = cte \quad (27)$$

Now p(x) can be found from the second equation in equation (25).

$$\frac{\partial Q_n}{\partial t} = 0;\ -\frac{A}{\rho_n}\frac{\partial p}{\partial x} - \frac{2fa^2\rho_n Q_n^2}{DAp} = 0 \quad (28)$$

$$p\frac{\partial p}{\partial x} = -\frac{2fa^2\rho_n Q_n^2}{DAp} \quad (29)$$

By choosing new variables F and ξ we get

$$p^2 = F \quad (30)$$

$$\frac{4fa^2\rho_n Q_n^2}{DAp} = \xi \quad (31)$$

Equation (29) leads to

$$\frac{\partial F}{\partial x} = \xi \quad (32)$$

And the steady state solution of p is calculated through Equation (33).

$$p(x) = \sqrt{p(0)^2 - \xi x} \quad (33)$$

Using the set of separation relations (34) in Equation (25) the model that governs the dynamic of the system under transient condition can be written by Equation (35).

$$\begin{cases} p(x,t) = p_t(t).\,p_x(x) \\ Q_n(x,t) = Q_t(t).\,Q_x(x) \end{cases} \quad (34)$$

$$\begin{cases} \dot{p}_t p_x = \alpha_1 Q_t \dot{Q}_x;\ \frac{\dot{Q}_x}{p_x} = \lambda;\ \dot{p}_t = \alpha_1 \lambda Q_t \\ \dot{Q}_t Q_x = \beta_1 \dot{p}_x p_t + \beta_2 \frac{{Q_t}^2 {Q_x}^2}{p_t p_x};\ \dot{Q}_t = \gamma_1 p_t + \gamma_2 \frac{{Q_t}^2}{p_t} \end{cases} \quad (35)$$

Differentiating the first equation in Equation (35) with respect to t and using the second one to eliminate the term Q_t , the model can be written by Equation (36)

$$\ddot{p}_t - \gamma_2 \left(\frac{1}{\alpha_1 \lambda} \frac{\dot{p}_t}{p_t} \right) \dot{p}_t - \frac{\gamma_1}{\alpha_1 \lambda} p_t = 0 \quad (36)$$

11. Conclusions

In order to generate an optimal dispatching system for natural gas pipeline network a proficient decision making system was proposed as well as a mathematical cost evaluating model. The proficient system is responsible for specifying the level of pressure in the pipeline in addition to finding out the needed horsepower of the costumers. The fuel cost minimization model was also presented via a mathematical model. Fuzzy sets and fuzzy logic has been used in this model that leads to a fuzzy programming problem for defining the proper speed of each compressor in the network to satisfy the horsepower requirement. Furthermore, the dynamic modeling of a gas distribution pipeline network was demonstrated besides an analytical scheme to solve the set of equations governing the pressure and flow dynamics of the system.

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