

# Simulation of Natural Gas Transmission Pipeline Network System Performance

Abraham Debebe Woldeyohannes<sup>1</sup>, Mohd Amin Abd Majid

(Department of Mechanical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Malaysia)

**Abstract:** Simulation has proven to be an effective tool for analyzing pipeline network systems (PNS) in order to determine the design and operational variables which are essential for evaluating the performance of the system. This paper discusses the use of simulation for performance analysis of transmission PNS. A simulation model was developed for determining flow and pressure variables for different configuration of PNS. The mathematical formulation for the simulation model was derived based on the principles of energy conservation, mass balance, and compressor characteristics. For the determination of the pressure and flow variables, solution procedure was developed based on iterative Newton Raphson scheme and implemented using visual C++6. Evaluations of the simulation model with the existing pipeline network system showed that the model enabled to determine the operational variables with less than ten iterations. The performances of the compressor working in the pipeline network system which includes energy consumption, compression ratio and discharge pressure were evaluated to meet pressure requirements ranging from 4000-5000kPa at various speed. Results of the analyses from the simulation indicated that the model could be used for performance analysis to assist decisions regarding the design and optimal operations of transmission PNS.

**Key words:** Energy; Transmission pipeline network; Compressor station; Natural gas; Simulation;

## 1. Introduction

Pipeline network system is the most effective way for transmitting natural gas from source to customers. When the gas moves through PNS, the pressure of the gas will decrease mainly due to friction and heat transfer. As a result, the pressure of the gas should have to be boosted by installing

compressor stations within the network to keep the gas moving. It is estimated that 3 to 5 % of the gas transmitted would be consumed by compressors in order to compensate for the lost pressure of the gas due to various reasons<sup>[1-3]</sup>. This is actually huge amount of cost especially for the network transmitting large volume of gas. The cost of natural gas burned to power the transportation of the remaining gas is equivalent to roughly 2 billion dollars per year for U.S. transmission system<sup>[3]</sup>. It has been reported that a 1% improvement on the performance of the transmission pipeline network system could result a saving of 48.6 million dollars for the U.S transmission network system<sup>[4]</sup>.

Simulation is becoming one of important tools for analyzing the performance of systems and making operating or resource policy decisions in various areas. Simulation analysis plays a significant contribution in the area of natural gas transmission network systems<sup>[5-9]</sup>.

Simulation is used to predict the behavior of the transmission PNS under different conditions which can be used as guide for decisions regarding the design and operation of the real system. During the design process, simulation could assist for selecting the structure of the network and the geometric parameters of the pipes which satisfy the requirements. Furthermore, it also facilitates the selection of sites where compressors, valves, regulators and other elements should be installed. Simulation analysis is also conducted for evaluating the performance of the components of transmission network systems<sup>[10-14]</sup>.

---

**Corresponding author:** ABRAHAM Debebe Woldeyohannes, PhD Candidate, Department of Mechanical Engineering, Universiti Teknologi PETRONAS; Research field: gas pipeline network simulation and optimization. Email: debebeabraham@yahoo.com

PNS consists of pipes and non-pipe elements like compressors, regulators, valves, scrapers, etc. The simulation of PNS without non-pipe elements is relatively easier to handle and developed in previous studies. But, the addition of non-pipe elements makes the simulation of transmission PNS more complex and requires further investigations. Pipeline network simulation without non-pipe elements is less challenging as it involves only pipes and is developed well in<sup>[9]</sup> based on graph theory.

More equations should have to be added into the governing simulation equations when non-pipe elements are considered during the analysis. Compressor station is one of the main non-pipe components of any gas transmission system and considered as a key elements. The cost of running the compressor stations represents 25% to 50% of the total transmission company's budget<sup>[15, 16]</sup>.

One of the basic differences among transmission pipeline network simulation analysis models with non-pipe elements is the way how compressor station is modeled during simulation. There have been attempts made by various researchers on modeling compressor stations within the PNS during simulation. One of the options is to consider the compressor station as a black box by setting either the suction or discharge pressures<sup>[17]</sup>. Only little information can be obtained to be incorporated into the simulation model to represent the compressor station. The effect of compressor station during simulation of PNS has been incorporated by pre-setting the discharge pressures<sup>[9, 18]</sup>. However, the speed of the compressor, suction pressure, suction temperature, and flow through the compressor were neglected during the analysis.

This study focuses on developing a simulation model for evaluating the performance of compressor within the transmission PNS for various operations by incorporating the detail characteristics of compressor stations namely; speed, suction pressure, discharge pressure, flow rates, suction temperatures.

## 2. Problem description

The pipeline configuration under study was taken from part of the existing PNS. The PNS consists of one compressor station (CS) with two centrifugal compressors working in parallel. The PNS serves eight major power plant customers and one Gas District Cooling (GDC) system. The detail specification of the transmission PNS is shown in Table 1. The gas to be transmitted is a mixture of methane (92%), ethane (5%), nitrogen (1%) and others (2%). Other related information to the gas includes gas gravity  $G=0.5$ , the average gas flowing temperature  $T=308K$  and gas compressibility factor  $Z=0.92$ . The gas flow rates in the pipe sections are designated as  $Q_1, Q_2, \dots$  etc and the flow rates passing out the gas pipeline to various customers are designated as  $Q_{C1}, Q_{C2}, \dots$  etc. as shown in Fig.1.

**Table 1 Specifications of the existing pipeline network system**

Characteristic of the transmission pipeline	Value
Number of compressors	2
Demand from customer	22.65 to 48.14 MMSCMD
Number of pipes	19
Diameter of the pipes	200mm to 900mm
Length of the pipes	6km to 200km

## 3. The simulation model

The simulation model consists of two parts; mathematical formulation and solution scheme. The mathematical formulation discussed the basic components of governing simulation equations. The solution how to obtain the required flow and pressure variables which are essential for performance analysis of the compressor within the gas transmissions system is discussed under solution scheme.

### 3.1 Mathematical formulation

<sup>1</sup>The compressors were replaced with a compressor from<sup>[19]</sup> with maximum capacity of 0.98 million metric standard cubic meters per day (MMSCMD) during the analysis due to insufficient information about the field compressors. The maximum speed of the compressor is limited to 10500 rpm and the maximum head of the compressor is 108kJ/kg.

The mathematical model for the PNS simulation was developed based on the performance characteristics of the compressors, equations which govern the flow of the gas through pipes, and the principles of conservation of mass. Single phase dry gas, constant gas temperature and negligible internal corrosion of the pipes were assumed during the development of the mathematical formulation.

### 3.1.1 Flow modeling

The flow of the gas through pipes can be affected by various factors such as the gas properties, friction factor and the geometry of the pipes. The relationship between the upstream pressure, downstream pressure and flow of the gas in pipes can be described by various equations<sup>[9, 20]</sup>. The general flow equation is adopted to be used for the analysis. The general flow equation for any pipeline element relating upstream pressure  $P_i$ , downstream pressure  $P_j$  and the flow through pipe  $Q_{ij}$  can be expressed as:

$$P_i^2 - P_j^2 = K_{ij} Q_{ij}^2 \quad (1)$$

where  $K_{ij}$  is pipe flow resistance. For  $P[kPa]$ ,  $T[K]$ ,  $L[km]$ ,  $Q[m^3/hr]$  and  $D[mm]$ , the expression for  $K_{ij}$  takes the form:

$$K_{ij} = 4.3599 \times 10^8 \frac{fGZT}{D^5} \left( \frac{P_n}{T_n} \right)^2 L \quad (2)$$

For the pipeline network shown in Fig.1, all the 19 pipe flow equations are derived following the same procedures as in equation Eq. (1). For a particular case, the pipe flow equation for relating node 2 and 3 of the network is given as:

$$P_2^2 - P_3^2 = K_{23} Q_4^2 \quad (3)$$

### 3.1.2 Compressor stations modeling

Usually the data related to compressors are available in the form of compressor performance maps. In order to integrate the characteristics of the compressor into the simulation model, it is necessary to approximate the characteristics map with mathematical equation. The basic quantities related to a centrifugal compressor unit are inlet volume flow rate  $Q$ , speed  $n$ , adiabatic head  $H$ , and adiabatic

efficiency  $\eta$ . The mathematical approximation of the performance map of the compressor can be done based on the normalized characteristics. The three normalized parameters which are necessary to describe the performance map of the compressor includes,  $H/n^2$ ,  $Q/n$  and  $\eta$ . Based on the normalized parameters, the characteristics of the compressor can be approximated either by two degree<sup>[4]</sup> or three degree polynomials<sup>[6]</sup>. Three degree polynomial gives much better approximation for the characteristics map and used in this study.

Using the normalized parameter, the characteristics of the compressor can be expressed based on three degree polynomials as:

$$H/n^2 = A_1 + A_2 (Q/n) + A_3 (Q/n)^2 + A_4 (Q/n)^3 \quad (4)$$

$$\eta = B_1 + B_2 (Q/n) + B_3 (Q/n)^2 + B_4 (Q/n)^3 \quad (5)$$

For the pipeline network simulation model developed, the relationships as in Eq. (4) and Eq. (5) might not be used directly. The information from the compressor map should have to relate the discharge pressure, suction pressure and flow rate. The relationships between suction pressure  $P_s$ , and discharge pressure  $P_d$  with the head  $H$  is given as<sup>[21]</sup>:

$$H = \frac{ZRT_s}{m} \left\{ \left[ \frac{P_d}{P_s} \right]^m - 1 \right\} \quad (6)$$

where  $m = \frac{\gamma}{\gamma - 1}$

Substituting the value of  $H$  from Eq. (6) in to Eq.(4) and rearranging yields the required compressor performance equation which can be incorporated into the simulation model.

$$\left( \frac{P_d}{P_s} \right)^m = \frac{mn^2}{ZRT_s} [A_1 + A_2 (Q/n) + A_3 (Q/n)^2 + A_4 (Q/n)^3] + 1 \quad (7)$$

Based on the performance map of the compressor used, simulation analysis was performed to determine the coefficients of the mathematical approximation which represents the characteristics of the compressor. As a result, the coefficients for the mathematical approximation of the compressor were determined to be:

$$A_1 = 1.40E-006$$

$$A_2 = -1.33E-008$$

$$A_3 = 1.86E-10$$

$$A_4 = -1.23E-12$$

### 3.1.3 Formulation of mass balance equations

The mass balance equations were obtained based on the principle of conservation of mass at each junction of the pipeline network. For the given network shown in Fig.1, the mass balance equation at junction node 2 is formulated as:

$$Q_1 - Q_2 - Q_3 - Q_4 = 0 \quad (8)$$

All the other mass balance equations for each junction were formulated following the same procedures as in Eq. (8).

### 3.1.4 The looping condition

Based on the looping condition<sup>[20]</sup>, for the existing pipeline network system shown in Fig.1, the pressure drop in looped branch 2 - 8 must equal the pressure drop in pipe branch 2-4-6-7-8. This is due to the fact that both pipe branches have a common starting point (node 2) and common ending point (node 8). Based on the general flow equation, the looping condition for the pipeline network shown in Fig.1 can be expressed as:

$$Q_2^2 \frac{L_{28}}{D_{28}^5} = Q_3^2 \frac{L_{24}}{D_{24}^5} + Q_6^2 \frac{L_{46}}{D_{46}^5} + Q_7^2 \frac{L_{67}}{D_{67}^5} + Q_8^2 \frac{L_{78}}{D_{78}^5} \quad (9)$$

## 3.2 Solution procedures for the simulation model

There are 10 pressure variables and 20 flow variables to be determined for the pipeline network system shown in Fig.1. The pipeline network system consists of 19 pipe elements, 1 compressor station, 1 loop and 9 junctions. Therefore, there are 19 pipe flow equations, 1 compressor equation, 1 looping equation and 9 mass balance equations. So, there are 30 equations available for the pipeline network system. Hence, 30 equations with 30 unknowns make the pipeline network problem solvable. The result for unknown parameters was determined on the basis of the iterative Newton Raphson scheme<sup>[22, 23]</sup>.

The governing simulation equations for the pipeline network simulation in matrix form is expressed as<sup>[22]</sup>

$$\tilde{F}(\tilde{x}) = \tilde{0} \quad (10)$$

where the vector  $x$  represents the total number of unknown pressure and flow variables and  $F$  is the corresponding equations generated from pipe flow, compressor characteristics, mass balance and looping conditions.

The multivariable Newton Raphson iterative scheme for Eq. (10) takes the form

$$\tilde{A} \Big|_{x_{old}} \left[ \tilde{x}_{new} - \tilde{x}_{old} \right] = - \tilde{F}(\tilde{x}_{old}) \quad (11)$$

where  $\tilde{A}$  the Jacobian matrix is whose elements are partial derivatives of the functions with respect to the unknown pressure and flow variables.

The values of the unknown variables were calculated from Eq. (11) iteratively until the relative errors were less than specified tolerance or the number of iterations equal to the desired value. Fig. 2 shows the snapshots of the simulation model based on Visual C++6. The visual C++ code was developed for the simulation based on Newton Raphson solution scheme.

## 4. Energy consumption of the system

Based on the pressure and flow variables obtained, the simulation model was used to evaluate the energy consumption of the PNS for various configurations in order to guide for the selection of optimal system. The different alternatives were then compared based on Eq. (12) to select the alternative with minimized energy consumption.

The amount of energy input to the gas by the compressors is dependent upon the pressure of the gas and flow rate. The power required by the compressor that takes into account the compressibility of gas is given as<sup>[20]</sup>

$$HP = 4.0639 \left( \frac{k}{k-1} \right) Q T_s \left( \frac{Z_1 + Z_2}{2} \right) \left( \frac{1}{\eta_a} \right) \left[ \left( \frac{P_d}{P_s} \right)^{\frac{k}{k-1}} - 1 \right] \quad (12)$$

where  $HP$  is the compression power in kW.

### 5. Results and Discussion

Based on the data in Table 1 and the PNS shown in Fig.1, the developed simulation model was evaluated. The main modules of the developed simulation model include input parameter analysis, function evaluation module, and network evaluation module.

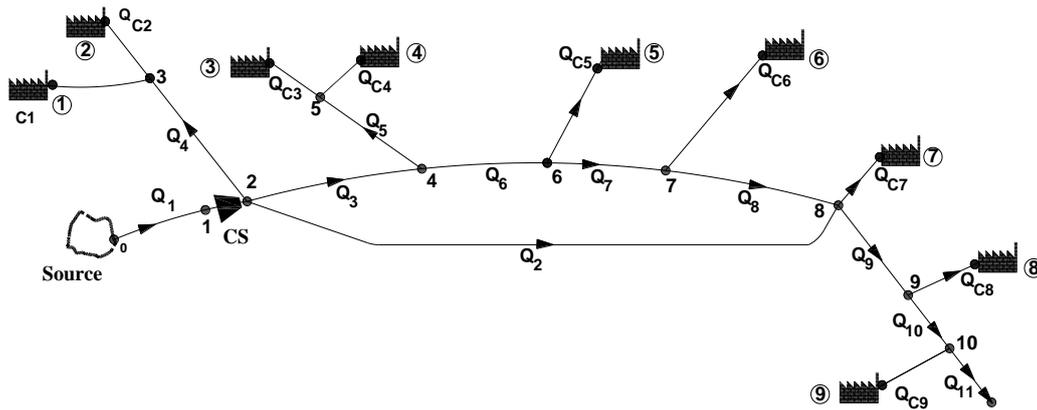


Fig. 1 Existing pipeline network for transmitting gas to various customers

#### 5.1 Input parameter analysis

This phase of the pipeline network simulation consists of taking the input from the user and making analysis in order to make the data appropriate for the next phase of the simulation. The input to the simulation includes pipe data, compressors data, customer requirements, initial estimations for the unknown variables, and the number of iterations to go.

#### 5.2 Output of the simulation

The output from the simulation model includes the value of each of the unknown pressure and flow variables, the system compression ratio and power consumption. For instance, for the source pressure of 3500kPa and end pressure requirement of 4000kPa, the results of the unknown pressure variables after ten iterations are shown in Table 2.

The corresponding result of flow variables after ten iterations is shown in Table 3. The analysis was performed with compressor speed of 8500 rpm. Note that, the analysis can be made based on any speeds of the compressor as long as the speed is within the working limits of the compressor. The initial

estimation of 4000kpa for pressure and 4000m<sup>3</sup>/hr for flow variables were taken.

Table 2 Result of nodal pressures after ten iterations

Node	Pressure [kPa]
0	3500.00
1	2799.26
2	4151.47
3	4101.92
4	4064.66
5	4006.72
6	4030.42
7	4022.6
8	4021.88
9	4002.87
10	4002.68

Table 3 Result of main and branch flow parameters after ten iterations

Main flow variable	Flow (m3/hr)	Branch flow variable	Flow (m3/hr)
Q1	760909	QC1	133722
Q2	140273	QC2	133722
Q3	353192	QC3	59121.7
Q4	267444	QC4	64126.5
Q5	123248	QC5	132794
Q6	229944	QC6	68645.8
Q7	97149.2	QC7	75509.5
Q8	28503.4	QC8	40739.9
Q9	93267.4	QC9	1705.01
Q10	52527.5		
Q11	50822.5		

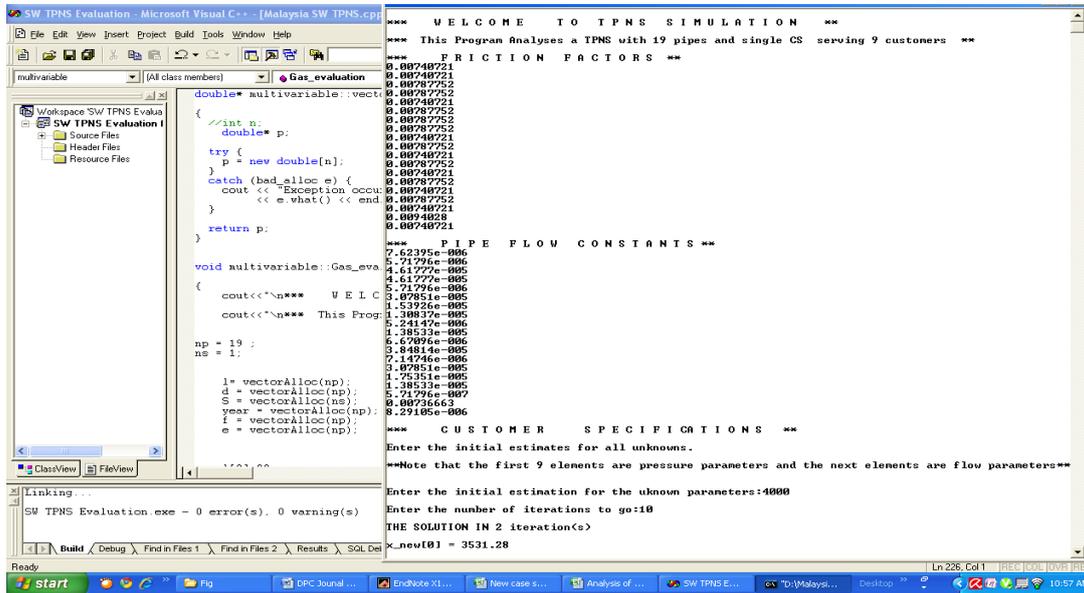


Fig. 2 Snapshot of the developed simulation tool

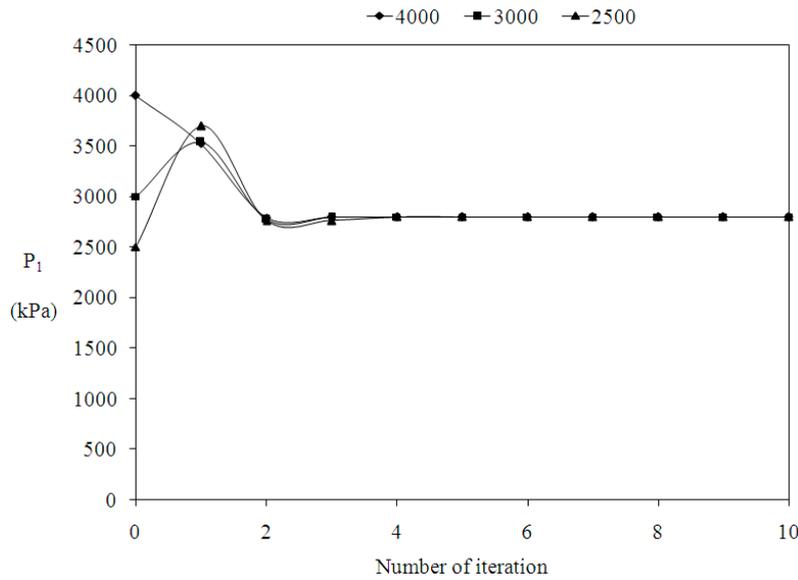


Fig. 3 Convergence of P1 for the first ten iterations

At the end of the ten<sup>th</sup> iteration, the maximum percentage relative error for both pressure and flow variables was obtained to be 8.1271E-17. The error varies depending on the initial estimations for the unknown pressure and flow variables. The simulation model was tested by giving wide ranges of initial estimations and worked well in most of the trials. From the tests conducted on the simulation model, an initial estimation near to the line pressure requirement

resulted solutions of pressure and flow variables in all the tests made.

The relative error between the successive iterations was usually high at the beginning of the iteration and decreased as the number of iteration increased. The convergences of the pressure variables at node 1,2 and the main flow variable  $Q_1$  were studied under various initial estimations. Fig.3 and Fig.4 show the convergence of nodal pressure to the

final pressure solutions for the first ten iterations at node 1 and 2 of the PNS shown in Fig.1 for initial estimations of 2500, 3000 and 4000kPa. In both figures, it was observed that the solutions to the nodal pressures were obtained starting from the third iterations. Note that the convergences of the remaining

nodal pressures follow the same trend as that of the pressure at node 2.

The convergence of the main flow parameter  $Q_1$  for an initial estimation of 2500, 3000 and 4000m<sup>3</sup>/hr for the first ten iterations is shown in Fig.5.

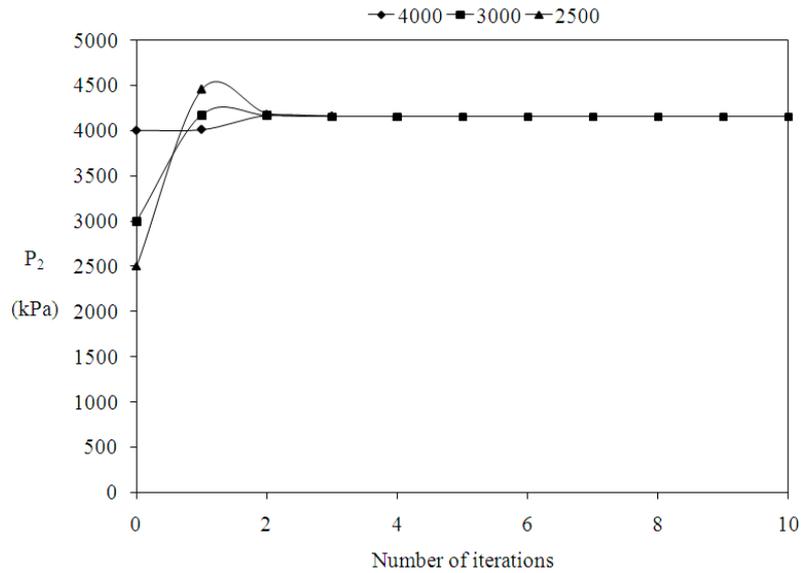


Fig. 4 Convergence of P2 for the first ten iterations

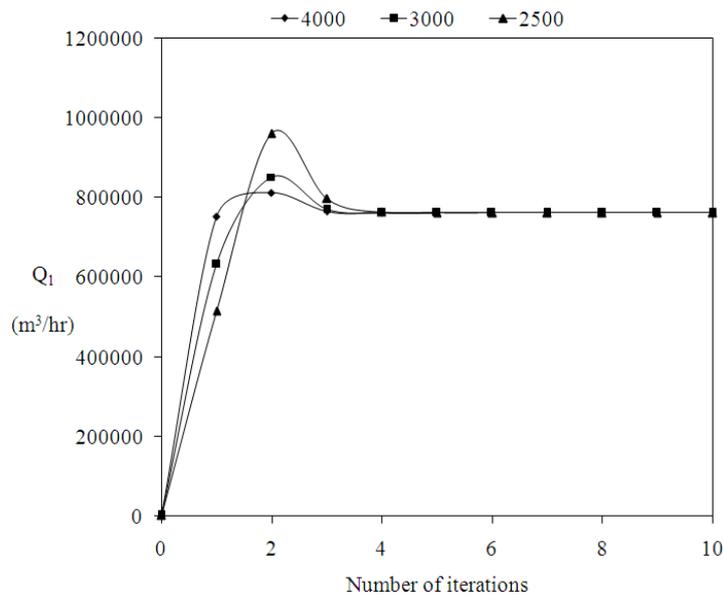


Fig. 5 Convergence of Q1 for the first ten iterations

### 5.3 Validation of the simulation model

The results of the PNS simulation model were compared with two other models based on two

different pipeline network configurations. In the first case, the simulation model was compared to an exhaustive optimization technique based on gunbarrel

pipeline networks system<sup>[1]</sup>. The pipeline network consists of six nodes, three pipes, two compressor stations and with no-loop. The detail of the various

inputs and description of the problem is shown in Table 4.

**Table 4 Gunbarrel network configuration problem instance**

Item	Wu [1]	Developed simulation model
Problem of the network	Finding optimal nodal pressures	Finding the nodal pressures and flow parameters
Input parameters	Flow rate	Demand and source pressure
Number of compressors	5	8
Speed of the compressors	-	5775 rpm
Flow equation	General	General
Characteristics of compressor	not included	Integrated with governing simulation equations
Method of solution	Exhaustive search	Newton Raphson scheme

It was observed that as the speed of the compressors increased, the results of nodal pressures from the PNS simulation model were getting closer to the results of nodal pressures obtained in<sup>[1]</sup>. After conducting various simulation experiments, compressor speed at 5775rpm gave better results of nodal pressures with maximum percentage error of 2.37 % which was obtained at node 5 of the network. The corresponding flow deviation was 10.7%. Table 5 shows the detail comparison of the results from<sup>[1]</sup> and the developed PNS simulation model.

**Table 5 Comparison of results of simulation and result of [1] based on gunbarrel configuration**

Nodal pressures and flow variables	Result of from [1]	Results of the simulation model	absolute % error
P1	4112.88	4112.88	0.00
P2	3433.59	3497.23	1.85
P3	3640.43	3616.10	0.67
P4	2850.70	2896.84	1.62
P5	3088.85	3015.71	2.37
P6	2101.13	2100.00	0.05
Q	707921	783715	10.71

The second comparison was made based on the problem instance taken from<sup>[9]</sup>. The network consists of 10 pipes, two CSs and two loops. For this problem, Newton loop-node method was applied in order to get the flow and pressure variables by assuming fixed pressure ratios for each compressor. However, the final nodal pressures obtained after meeting the predefined error limits failed to satisfy the previously

assumed pressure ratios. The pressure ratio at CS1 was assumed to be 1.8 and that of CS2 was assumed to be 1.4. However, the pressure ratios after the solutions were obtained were actually 1.34 and 1.1832 for CS1 and CS2, respectively.

After conducting various simulation experiments, compressor speed at 5025 rpm for CS1 and 4750 rpm for CS2 gave results of nodal pressures and flow parameters near to the results obtained in<sup>[9]</sup>. Table 6 shows comparison of nodal pressures and flow variables for the looped configurations between the developed simulation model and the result from<sup>[9]</sup>. Mean absolute percent error of 5.10% was observed between the two methods. The variations of the flow and nodal pressure parameters could be resulted from the type of flow equations that were used in the analysis and the oversimplification of compressor stations in the case of the method in<sup>[9]</sup>. Panhandle ‘A’ flow equation was used in<sup>[9]</sup> where as the general flow equation has been applied in the developed simulation model. Furthermore, the PNS simulation model consists of detailed characteristics of the compressor rather than only limited to compression ratio.

Based on the comparison made, the developed simulation model is superior as it contains detailed information regarding the compressor stations which are essential for evaluating the performance of the network system.

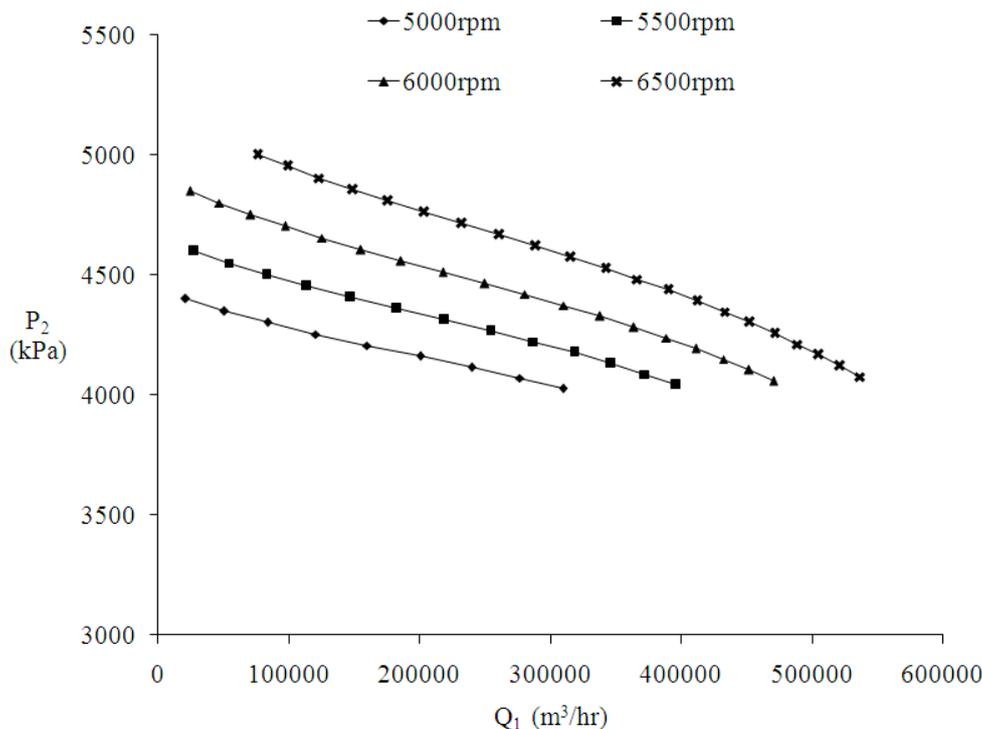
**Table 6 Comparison of results of simulation and result of [9] based on gunbarrel configuration**

Nodal P[kPa]	Result of from [16]	Result of the simulation model	abs. % error	Flow[m3/hr]	Results from[16]	Results of the simulation model	abs.% error
P1	5000.00	5000.00	0.00	Q1	64715.00	57984.301	0.40
P2	4790.60	4991.81	4.20	Q2	65283.10	67015.70	2.65
P3	4818.10	4990.62	3.58	Q3	-18713.10	-19469.10	4.04
P4	4468.70	4979.06	11.42	Q4	63429.00	57453.30	9.42
P5	5995.40	5936.70	0.98	Q5	26570.90	27546.70	3.67
P6	4762.30	4987.98	4.74	Q6	63429.00	57453.30	9.42
P7	5634.90	5921.97	5.09	Q7	26570.90	27546.70	3.67
P8	5637.70	5921.70	5.04	Q8	17924.80	17763.30	0.90
P9	5549.90	5918.02	6.63	Q9	30504.50	29690.00	2.67
P10	5486.30	5915.10	7.82	Q10	14495.50	15310.00	5.62

**5.4 Application of the simulation model for compressor performance analysis**

Further simulation study was conducted in order to apply the model for performance analysis of the compressor used for the PNS shown in Fig.1. For the performance analysis of the compressor, discharge pressure ( $P_2$ ) and the flow rate through the compressor ( $Q_1$ ) were considered. The performance of the

compressor was studied for source pressure of 3500kPa to meet pressure requirements of various demand pressures ranging from 4000 to 5000kPa. The analysis was performed with compressor speeds of 5000, 5500, 6000 and 6500rpm. Note that the analysis can also be made at any speed of the compressor as long as the speed is within the working limit of the compressor.



**Fig. 6 Discharge pressure variations with flow**

The variation of the discharge pressure ( $P_2$ ) of the compressor with flow rate ( $Q_1$ ) is shown in Fig. 6. As it is shown in Fig.6, for a constant speed operation

of the compressor, an increased in discharge pressure gave rise to a decrease in flow capacity of the system and vice versa. The results of the analysis in Fig.6

shows that the characteristic map generated using the simulation model is similar in shape to the characteristics maps of the compressors described

in[21,24]. As a result, the simulation model could be used to analyze the performance of the compressors.

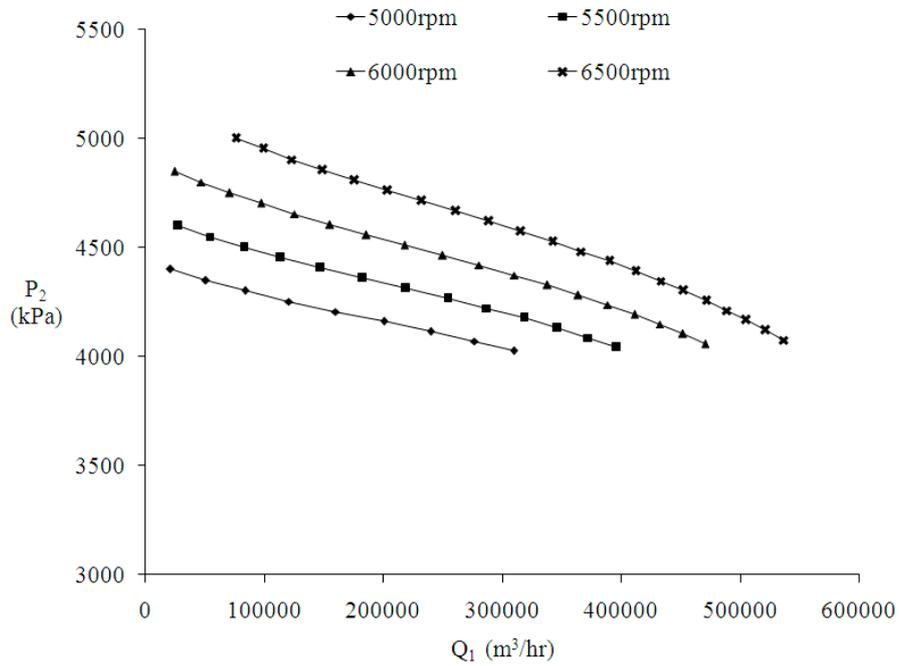


Fig. 7 Compression ratio variations with flow

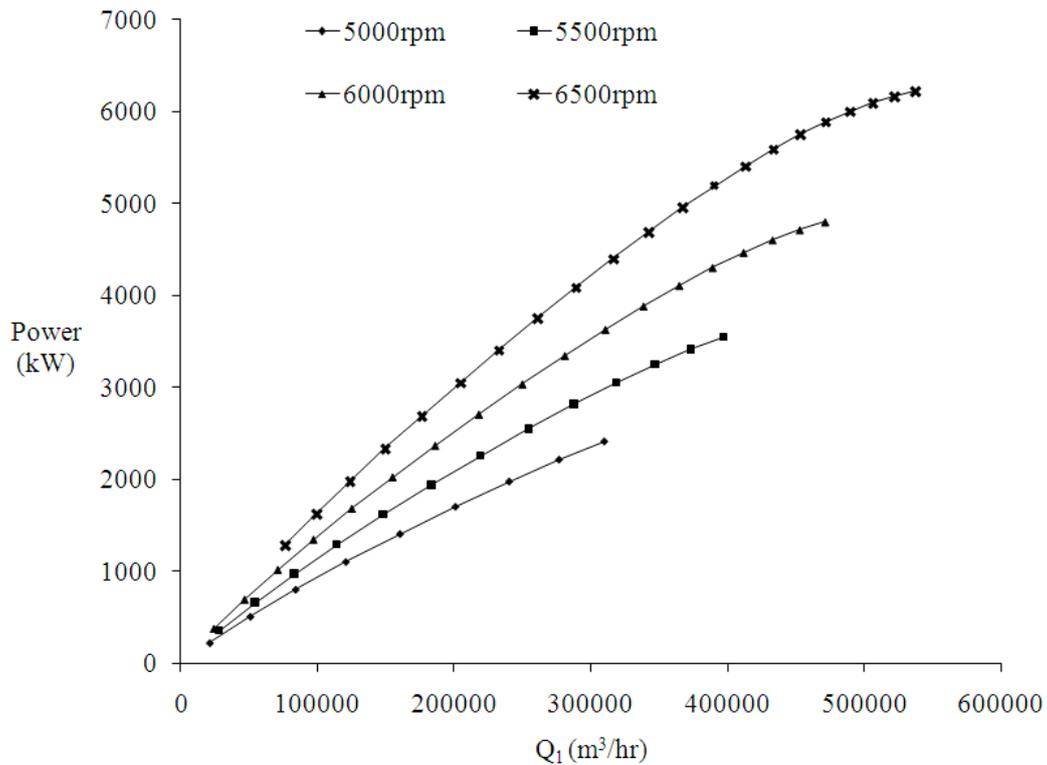


Fig. 8 Variation of energy consumption with flow

The variation of compression ratio (CR) with flow and speed is shown in Fig 7. It was observed from Fig.7 that, higher CR for the system was achieved at lower flow capacities. For a constant flow operation, an increase in speed of the compressor increased the CR of the system. The results shown in Fig.7 also shows the characteristic map of the compressor based on CR which is similar in shape with that of the characteristic map plotted in<sup>[21]</sup>.

Power consumption variation with flow through the compressors based on various speed is shown in Fig.8 It was observed that an increase in flow rate increased the power consumption. For a constant flow operation, an increase in speed of the compressor increased the power consumption of the system. As it was given in[20], power consumption by the system is function of the flow rate, CR and properties of the gas. When the speed of the compressor increased, it was observed in Fig. 7 that the CR of the compressor was also increased. Hence, the power consumption of the system increased due to an increment in CR of the system.

## 6. Conclusion

Simulation model for performance analysis of transmission pipeline network was developed by incorporating the detailed characteristics of compressor. The model applied to obtain pressure and nodal flow variables which are essential for evaluating the performance of compressor in the pipeline network system. The results for the pressure and flow variables were obtained with less than ten iterations to reasonable relative percentage errors. Analyses of the performance of compressor for existing pipeline network configuration were conducted using the developed simulation model. The performance characteristics maps generated using the developed simulation model is similar to the one available in literatures. Hence, the simulation model could be

applied to evaluate the performance of gas transmission network systems which is essential for making operational decisions.

The developed simulation model could be easily extended to be applied for performance analysis of PNS for other petroleum products. When pipeline network operates, there are cases where the system may be subjected to various severe environmental factors like change in temperature and corrosion. The simulation model for performance analysis which takes into account these effects is an important issue. Transient model is another important problem to be addressed from the simulation perspective.

## References:

- [1] WU S., RIOS-MERCADO R. Z., BOYD E. A., SCOTT L. R. Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. *Mathematical and Computer Modeling*. 2000, 31: 197-220.
- [2] BORRAZ-S'ANCHEZ C., R'IOS-MERCADO R. Z. A. Hybrid Meta-heuristic approach for natural gas pipeline network optimization. *LNCS*, 2005, 3636: 54-65.
- [3] CARTER R. G. Pipeline optimization: Dynamic programming after 30 years. Proceedings of the 30th PSIG Annual Meeting, Denver October 1998, 28-30.
- [4] WU S. Steady-state simulation and fuel cost minimization of gas pipeline networks. PhD Thesis, University of Houston. 1998
- [5] NIMMANONDA P., URAIKUL V., CHAN C. W., TONTIWACHWUTHIKUL P. A Computer-aided model for design of a simulation system for the natural gas pipeline network system. Proceedings of the 2002 IEEE Canadian Conference on Electrical & Computer Engineering, Winnipeg, 2002, 1634 -1639.
- [6] ABBASPOUR M. Simulation and optimization of non-isothermal, one dimensional single/two phase flow in natural gas pipeline. PhD Thesis, Kansas State University. 2005.
- [7] HOEVEN T. v. d. Constrained network simulation. The 35th Annual Meeting of Pipeline Simulation Interest Group (PSIG), Switzerland, October, 2003.15-17.
- [8] ABDOLAH F., MESBAH A., BOOZARJOMEHRY R. B., SVRCEK W. Y. The effect of major parameters on simulation results of gas pipelines. *International Journal of Mechanical Sciences*, doi: 10.1016/j.ijmecsci.2006.12.001. 2007.

- [9] OSIADACZ A. J. Simulation and analysis of gas networks, Gulf Publishing Company, Houston, TX 77252, USA. 1987.
- [10] VALERIANO A., CASTRO R. C. M. Turbine and compressor performance analysis through simulation. 38th Annual Meeting of Pipeline Simulation Interest Group (PSIG), 11-13 October, Williamsburg, Virginia. 2006.
- [11] BROWN R., RAHMAN K. Turbine/Compressor performance monitoring software and flow capacity. 34th Annual Meeting of Pipeline Simulation Interest Group (PSIG), Portland, October, 2002, 24-25.
- [12] PATEL V., FENG J., DASGUPTA S., RAMDOSS P., WU J. Application of dynamic simulation in the design, operation, and troubleshooting of compressor systems. Proceedings of the thirty-sixth turbomachinery symposium, 2007, 95-105.
- [13] TIJL P. Modeling, simulation and evaluation of a centrifugal compressor with surge avoidance control. DCT 2004.039, Technische Universiteit Eindhoven, Eindhoven. 2004.
- [14] HERBOLD J. W. Centrifugal turbo-compressor performance model. M.Sc Thesis, The University of Texas at Arlington. 1998.
- [15] RÍOS-MERCADO R. Z., KIM S., BOYD E. A. Efficient operation of natural gas transmission systems: A network-based heuristic for cyclic structures. Computers & Operations Research, 2006, 33: 2323–2351.
- [16] [16]KIM S. B. Minimum cost fuel consumption on natural gas transmission network problem. PhD Thesis, Texas A & M University. 1999.
- [17] LETNIEWSKI F. W. Compressor station modeling in networks. 25th Annual Meeting of Pipeline Simulation Interest Group (PSIG), Pittsburgh, October 1993, 14-15.
- [18] NIMMANONDA P. A computer-aided simulation model for natural gas pipeline network system operations. MSc Thesis, University of Regina. 2003.
- [19] SANTOS S. P. d., LUBOMIRSKY M. Gas Composition Effect on Centrifugal Compressor Performance. 35th Annual Meeting of Pipeline Simulation Interest Group (PSIG), Virginia, October, 2006, 11-13.
- [20] MENON E. S. Gas pipeline hydraulics, CRC Press, New York. 2005.
- [21] GRESH M. T. Compressor performance Aerodynamics for the user, Butterworth Heintemann, Boston. 2000.
- [22] GHOSH P. Numerical methods with computer programs in C++, Prentice-Hall, New Delhi. 2006.
- [23] SALLEH S., ZOMAYA A. Y., BAKAR S. A. Computing for numerical methods using visual C++, John Wiley & Sons.
- [24] KURZ R. 2004, The Physics of Centrifugal Compressor Performance. The 34th Annual Meeting of Pipeline Simulation Interest Group (PSIG), California, October, 2008, 20-22.