AN ANNOTATED BIBLIOGRAPHY OF GRASP PART II: APPLICATIONS

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Abstract. A greedy randomized adaptive search procedure (GRASP) is a metaheuristic for combinatorial optimization. It is a multi-start or iterative process, in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. Since 1989, numerous papers on the basic aspects of GRASP, as well as enhancements to the basic metaheuristic have appeared in the literature. GRASP has been applied to a wide range of combinatorial optimization problems, ranging from scheduling and routing to drawing and turbine balancing. This is the second of two papers with an annotated bibliography of the GRASP literature from 1989 to 2008. In the companion paper, algorithmic aspects of GRASP are surveyed. In this paper, we cover the literature where GRASP is applied to scheduling, routing, logic, partitioning, location, graph theory, assignment, manufacturing, transportation, telecommunications, biology and related fields, automatic drawing, power systems, and VLSI design.

1. Introduction

Optimization problems that involve a large finite number of alternatives often arise in the private and public sectors of the economy. In these problems, given a finite solution set X and a real-valued function $f : X \to \mathbb{R}$, one seeks a solution $x^* \in X$ with $f(x^*) \leq f(x)$, $\forall\, x \in X$. Common examples include designing efficient telecommunication networks, constructing cost effective airline crew schedules, and producing efficient routes for waste management pickup. To find the optimal solution in a combinatorial optimization problem it is theoretically possible to enumerate the solutions and evaluate each with respect to the stated objective. However, in practice, it is often infeasible to follow such a strategy of complete enumeration because the number of combinations often grows exponentially with the size of problem.

Much work has been done over the last six decades to develop optimal seeking methods that do not explicitly require an examination of each alternative. This research has given rise to the field of *combinatorial optimization* (see Papadimitriou and Steiglitz (1982)), and an increasing capability to solve ever larger real-world problems. Nevertheless, most problems found in industry and government are either computationally intractable by their nature, or sufficiently large so as to preclude the use of exact algorithms. In such cases, heuristic methods are usually employed to find good, but not necessarily guaranteed optimal, solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization,

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avoid entrapment at local optima, and exploit the basic structure of the problem. Building on these notions, various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. The most promising of such techniques include simulated annealing (Kirkpatrick, 1984), tabu search (Glover, 1989, 1990; Glover and Laguna, 1997), genetic algorithms (Goldberg, 1989), variable neighborhood search (Hansen and Mladenović, 1998), and GRASP, or Greedy Randomized Adaptive Search Procedures, (Feo and Resende, 1989, 1995).

A GRASP is a multi-start or iterative process (Lin and Kernighan, 1973), in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. The best overall solution is kept as the result.

An especially appealing characteristic of GRASP is the ease with which it can be implemented. Few parameters need to be set and tuned, and therefore development can focus on implementing efficient data structures to assure quick GRASP iterations.

This paper is the second of two. In both papers, we provide annotated bibliographies of the GRASP literature up to mid 2008. The papers contain references related to GRASP that have either appeared in the literature or as theses. In this paper, we review the literature of operations research and computer science applications of GRASP as well as industrial applications. We cover applications in scheduling, routing, logic, partitioning, location, graph theory, assignment, manufacturing, transportation, telecomunications, biology and related fields, automatic drawing, power systems, VLSI design, The paper conlcudes with a section on miscellaneous topics.

In the companion paper (Festa and Resende, 2008), we first look at tutorials and surveys. Papers that propose enhancements to the basic heuristic are considered next. Following that, we examine GRASP as a component of a hybrid metaheuristic. Parallelization of GRASP and GRASP source code follow.

C.H. Papadimitriou and K. Steiglitz. *Combinatorial optimization: Algorithms and complexity*. Prentice-Hall, 1982.
A classic book on combinatorial optimization.

S. Kirkpatrick. Optimization by simulated annealing: Quantitative studies. *J. of Statistical Physics*, 34:975–986, 1984.
Description of the simulated annealing metaheuristic.

F. Glover. Tabu search – Part I. *ORSA J. on Computing*, 1:190–206, 1989.
Description of the tabu search metaheuristic.

F. Glover. Tabu search – Part II. *ORSA J. on Computing*, 2:4–32, 1990.
Description of the tabu search metaheuristic.

F. Glover and M. Laguna. *Tabu Search*. Kluwer Academic Publishers, 1997.
Book on metaheuristics and in particular tabu search.

D.E Goldberg. *Genetic algorithms in search, optimization and machine learning*. Addison-Wesley, 1989.
Book on genetic algorithms.

P. Hansen and N. Mladenović. An introduction to variable neighborhood search. In S. Voss, S. Martello, I. H. Osman, and C. Roucairol, editors, *Meta-heuristics,*

Advances and trends in local search paradigms for optimization, pages 433–458. Kluwer Academic Publishers, 1998.

Description of the variable neighborhood search metaheuristic.

T.A. Feo and M.G.C. Resende. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67–71, 1989.

This is the first paper to explicitly describe a GRASP.

T.A. Feo and M.G.C. Resende. Greedy randomized adaptive search procedures. *J. of Global Optimization*, 6:109–133, 1995.

An early survey of GRASP.

S. Lin and B.W. Kernighan. An effective heuristic algorithm for the traveling-salesman problem. *Operations Research*, 21:498–516, 1973.

An early random multistart local search technique.

P. Festa and M.G.C. Resende. An annotated bibliography of GRASP, Part I: Algorithms. *International Transactions in Operational Research*, 2008.

Companion paper with an annotated bibliography of algorithmic aspects of GRASP. To appear.

2. Scheduling

GRASP has been applied to a wide variety of scheduling problems. The following papers illustrate GRASPs for scheduling.

J.F. Bard and T.A. Feo. Operations sequencing in discrete parts manufacturing. *Management Science*, 35:249–255, 1989.

This paper presents a method for efficiently sequencing cutting operations associated with the manufacture of discrete parts. The problem is modeled as an integer program. This is relaxed via Lagrangian relaxation into a min-cut problem on a bipartite network. To obtain lower bounds, a max-flow algorithm is applied and the corresponding solution is input to a GRASP.

T.A. Feo and J.F. Bard. Flight scheduling and maintenance base planning. *Management Science*, 35:1415–1432, 1989.

A GRASP for flight scheduling and maintenance base planning is described. See page 32.

T.A. Feo, K. Venkatraman, and J.F. Bard. A GRASP for a difficult single machine scheduling problem. *Computers and Operations Research*, 18:635–643, 1991.

GRASP is applied to an unusually difficult scheduling problem with flow time and earliness penalties. Two greedy functions are developed and tested. The first is the difference between the flow time and earliness penalties, normalized by the processing time. The second function evaluates the cost of scheduling a job next by estimating the cost of the remaining schedule. The local search uses 2-exchange and insertion exchange.

M. Laguna and J.L. González-Velarde. A search heuristic for just-in-time scheduling in parallel machines. *J. of Intelligent Manufacturing*, 2:253–260, 1991.

This paper presents a hybrid GRASP/tabu search metaheuristic for the weighted earliness penalty problem with deadlines in identical parallel machines.

P. De, J.B. Ghosj, and C.E. Wells. Solving a generalized model for CON due date assignment and sequencing. *International J. of Production Economics*, 34: 179–185, 1994.

This paper deals with a generalized model for assigning a constant flow allowance (CON) due date to a set of jobs and sequencing them on a single machine. The problem is viewed as a 0-1 quadratic problem and a GRASP is proposed to solve the quadratic problem. The randomization strategy used was inspired by a gradient-based variable forcing methodology proposed by Pardalos and Rodgers (1990) for a branch & bound algorithm. The local search procedure is based on a definition of neighborhood in which two solutions are neighbors if they differ in the value of exactly one variable.

T.A. Feo, J.F. Bard, and S. Holland. Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research*, 43:219–230, 1995.

A GRASP is proposed to solve a multiple machine scheduling problem. See page 30.

J.F. Bard, T.A. Feo, and S. Holland. A GRASP for scheduling printed wiring board assembly. *I.I.E. Transactions*, 28:155–165, 1996.

GRASP is used for solving the daily scheduling problem that arises in printed wiring board assembly. See page 30.

T.A. Feo, K. Sarathy, and J. McGahan. A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties. *Computers and Operations Research*, 23:881–895, 1996.

A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties is presented. The greedy function of the GRASP construction phase proposed is made up of two components: the switch over cost and the opportunity cost associated with not inserting a specific job in the next position and instead, inserting it after half of the unscheduled jobs have been scheduled. This greedy function tends to lead to a balance between the natural order and nearest neighbor approaches. The local search uses 2-exchange, insertion exchange, and a combination of the two.

J. Xu and S. Chiu. Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pages 240–248, July 1996.

The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. The greedy choice of the proposed GRASP is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule.

R.Z. Ríos-Mercado and J.F. Bard. Heuristics for the flow line problem with setup costs. *European J. of Operational Research*, pages 76–98, 1998.

This paper presents two new heuristics for the flowshop scheduling problem with sequence-dependent setup times and makespan minimization objective, one of which is a GRASP. Both heuristics are compared

with a previously proposed algorithm, based on the traveling salesman problem (TSP), on randomly generated instances. When setup times are an order of magnitude smaller than processing times, the new approaches prove superior to the TSP-based heuristic. When both processing and setup times are identically distributed, the TSP-based heuristic outperforms the proposed procedures.

L.I.D. Rivera. Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnologico y de Estudios Superiores de Monterrey, Monterrey, Mexico, 1998.

This thesis presents a parallel GRASP for a course scheduling problem. In Spanish.

R.Z. Ríos-Mercado and J.F. Bard. An enhanced TSP-based heuristic for makespan minimization in a flow shop with setup costs. *J. of Heuristics*, 5:57–74, 1999.

An enhanced heuristic for minimizing the makespan of the flow shop scheduling problem with sequence-dependent setup times is presented. To tune the parameters, each component of the heuristic is tested over a wide range of problem instances. An experimental comparison with a GRASP reveals the conditions and data attributes where the proposed procedure works best.

M.J.F. Souza, N. Maculan, and L.S. Ochi. A GRASP-tabu search algorithm to solve a school timetabling problem. In *Proceedings of the 4th Metaheuristics International Conference – MIC2001*, pages 53–58, 2001.

A hybrid GRASP and tabu search heuristic is proposed for solving a school timetabling problem. The GRASP construction greedy criterion gives priority to the lessons whose teachers have a larger number of lessons and unavailable periods. A local search technique, called Intraclasses-Interclasses, is proposed. A move involves lessons and periods.

H.R. Lourenço, J.P. Paixão, and R. Portugal. Multiobjective metaheuristics for the bus-driver scheduling problem. *Transportation Science*, 35:331–343, 2001.

GRASP is used in a genetic algorithm to implement a type of crossover called *perfect offspring*.

J.Y. Xu and S.Y. Chiu. Effective heuristic procedure for a field technician scheduling problem. *J. of Heuristics*, 7:495–509, 2001.

The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. Several heuristics, including a GRASP, are designed and tested for solving the problem. The greedy choice of the GRASP proposed is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule.

R.M. Aiex. *An experimental investigation of the probability distribution of solution time in GRASP heuristics and its application to the analysis of parallel implementations*. PhD thesis, Computer Science Department, Catholic University of Rio de Janeiro, August 2002.

This Ph.D. thesis establishes that the running time of GRASP fits a shifted exponential distribution. Parallel GRASP with path-relinking

implementations for job shop scheduling are described and tested. In Portuguese.

S. Binato, W.J. Hery, D. Loewenstern, and M.G.C. Resende. A greedy randomized adaptive search procedure for job shop scheduling. In C.C. Ribeiro and P. Hansen, editors, *Essays and surveys on metaheuristics*, pages 58–79. Kluwer Academic Publishers, 2002.

This paper proposes a GRASP for the job shop scheduling problem. The standard GRASP is enhanced with a memory based intensification scheme, that improves the local search phase around good solutions. The adaptive greedy function is the makespan resulting from the addition of an unscheduled operation to those already scheduled. The greedy choice favors the operation with the minimum value of the greedy function. The local search uses the 2-exchange based on the disjunctive graph model proposed by Roy and Sussmann (1964).

R.M. Aiex, S. Binato, and M.G.C. Resende. Parallel GRASP with path-relinking for job shop scheduling. *Parallel Computing*, 29:393–430, 2003.

This paper describes a parallel GRASP with path relinking for job shop scheduling. Independent and cooperative parallelization strategies are described and implemented.

P.L. Rocha, M.G. Ravetti, and G.R. Mateus. The metaheuristic GRASP as an upper bound for a branch and bound algorithm in a scheduling problem with non-related parallel machines and sequence-dependent setup times. In *Proceedings of the 4th EU/ME Workshop: Design and Evaluation of Advanced Hybrid Meta-Heuristics*, volume 1, pages 62–67, 2004.

The authors propose a hybridization of GRASP as an upper bound for a branch and bound procedure. At each GRASP iteration, the greedy criterion adopted to build a feasible solution consists in sorting the jobs in a non-decreasing order using the due date and then assigning each job to the machine that can finish it first. The local search switches all existing pairs of jobs assigned to different machines. The authors have also implemented path relinking at the end of each GRASP iteration to intensify the local search.

C.W. Commander, S.I. Butenko, P.M. Pardalos, and C.A.S. Oliveira. Reactive GRASP with path relinking for the broadcast scheduling problem. In *Proceedings of the 40th Annual International Telemetry Conference*, pages 792–800, 2004.

In the Broadcast Scheduling Problem, a finite set of stations must be scheduled in a time division multiple access (TDMA) frame with the objective of finding a collision free transmission schedule with the minimum number of TDMA slots and maximal slot utilization. Such a schedule must also minimize the total system delay. The authors propose several variations of GRASP with path relinking as a post-optimization strategy. They also describe a reactivity method to balance GRASP parameters.

J.D. Beltrán, J.E. Calderón, R.J. Cabrera, J.A.M. Pérez, and J.M. Moreno-Vega. GRASP/VNS hybrid for the strip packing problem. In *Proceedings of Hybrid Metaheuristics (HM2004)*, pages 79–90, 2004.

The authors propose a hybrid metaheuristic that combines GRASP and VNS procedure as a local search.

V. de Aragão Trindade and L.S. Ochi. Hybrid adaptive memory programming using GRASP and path relinking for the scheduling workover rigs for onshore oil production. In *Proceedings of the Fifth International Conference on Hybrid Intelligent Systems*, pages 500–502. IEEE Computer Society, 2005.

This paper describes a GRASP for a scheduling problem that arises in onshore oil production. At each GRASP construction iteration the RCL is created with all wells that have not been added to a route yet and arranged in decreasing priority order. An intra-route and a route-exchange neighborhoods are used by the local search procedure.

D.V. Andrade and M.G.C. Resende. A GRASP for PBX telephone migration scheduling. In *The Eighth INFORMS Telecommunication Conference*, April 2006.

The authors propose a GRASP to approximately solve a PBX telephone migration scheduling problem, where a solution is an assignment of phone numbers to time periods such that each time period has no more than a fixed number of telephones assigned to it. The construction procedure sequences the phone numbers and assigns them evenly to each time period. The greedy criterion is to minimize a penalty function associated with migrating phone numbers in different time periods. Once a feasible solution is constructed, local search is applied using three neighborhoods: swap phones, move phone, and swap periods.

A. Lim, B. Rodrigues, and C. Wang. Two-machine flow shop problems with a single server. *J. of Scheduling*, 9:515–543, 2006.

Several heuristics are proposed in this paper to solve a class of two-machine flow shop problems. They include simulated annealing, tabu search, genetic algorithms, GRASP, and other hybrids.

C.C. Ribeiro and S. Urrutia. Heuristics for the mirrored traveling tournament problem. *European J. of Operational Research*, 179:775–787, 2007.

The authors study the mirrored version of the traveling tournament problem. They propose a fast constructive algorithm and a new heuristic based on a combination of GRASP and iterated local search.

D.V. Andrade and M.G.C. Resende. GRASP with path-relinking for network migration scheduling. In *Proceedings of the International Network Optimization Conference (INOC 2007)*, 2007.

Network migration scheduling is the problem where inter-nodal traffic from an outdated telecommunications network is to be migrated to a new network. To approximately solve both versions of the problem, the authors propose a hybrid heuristic which combines GRASP with path-relinking.

S. Fernandes and H.R. Lourenço. A GRASP and branch-and-bound metaheuristic for the job-shop scheduling. In *Evolutionary Computation in Combinatorial Optimization*, volume 4446 of *Lecture Notes in Computer Science*, pages 60–71. Springer, 2007.

This paper presents a simple algorithm for the job shop scheduling problem that combines the local search heuristic GRASP with a branch-and-bound method.

M.D. Goodman, K.A. Dowsland, and J.M. Thompson. A GRASP-knapsack hybrid for a nurse-scheduling problem. *J. of Heuristics*, 2007. Published online 27 November 2007. doi:10.1007/s10732-007-9066-7

This paper describes a hybrid GRASP-knapsack heuristic for a nurse-scheduling problem. Instead of selecting the next element to be inserted in the partial solution purely on a myopic basis, the authors employ a look-ahead strategy based on a knapsack model. In the local search, a move consists in changing the shift pattern of a single nurse.

R. Alvarez-Valdes, F. Parreño, and J.M. Tamarit. Reactive GRASP for the strip-packing problem. *Computers and Operations Research*, 35(4):1065–1083, 2008a.

This paper describes a reactive GRASP for a strip-packing problem. The n pieces to pack are grouped into m types of pieces of dimensions (h_i, w_i), $i = 1, \ldots, m$, and Q_i copies of each type i are required to be packed, with $\sum_{i=1}^{m} Q_i = n$. The greedy choice is to sort the set of piece types yet to be packed in non-increasing order with respect to w_i. Ties are broken by non-increasing h_i. Four local search procedures are described.

C. Andres, C. Miralles, and R. Pastor. Balancing and scheduling tasks in assembly lines with sequence-dependent setup times. *European J. of Operational Research*, 187(3):1212–1223, 2008.

The authors consider a variant of the classical Simple Assembly Line Balancing Problem, where sequence-dependent setup times are added. In particular, whenever a task is assigned next to another at the same workstation, a setup time is added to compute the global workstation time. A GRASP is proposed whose greedy criterion is to minimize a measure of the impact of sequencing a candidate task, given the set of tasks already sequenced. The GRASP local search applies a move where it tries to exchange the positions of every two tasks in the current sequence, provided the exchange is feasible.

R. Alvarez-Valdes, E. Crespo, J.M. Tamarit, and F. Villa. GRASP and path relinking for project scheduling under partially renewable resources. *European J. of Operational Research*, 189(3):1153–1160, 2008b.

Heuristic algorithms based on GRASP and path relinking for project scheduling under partially renewable resources are developed and tested on existing test instances. The greedy selection is based on a priority rule. The improvement phase consists of two steps. First, it identifies the activities whose completion times must be reduced in order to have a new solution with the shortest makespan and labelling these activities as critical. Second, it moves critical activities in such a way that the resulting sequence is feasible according to precedence and resource constraints. In particular, the authors have designed two types of moves: 1) in a simple move, only a critical activity is moved, leaving the remaining activities unchanged; 2) in a double move, non-critical activities are moved to make the move of a critical activity possible.

R.A. Pereira, A.V. Moura, and C.C. de Souza. Comparative experiments with GRASP and constraint programming for the oil well drilling problem. In S.E. Nikoletseas, editor, *Experimental and Efficient Algorithms, 4th International Workshop, WEA 2005*, volume 3503 of *Lecture Notes in Computer Science*, pages 328–340. Springer, 2005.

A GRASP to solve a scheduling problem that arises in oil well drilling sites is presented. The GRASP heuristic is compared with a constraint

programming implementation and results show that the GRASP is better.

3. Routing

GRASP has been applied to vehicle, aircraft, telecommunications, and inventory routing problems. The following papers illustrate the application of GRASP to routing problems.

C.A. Hjorring. *The vehicle routing problem and local search metaheuristics.* PhD thesis, University of Auckland, Auckland, New Zealand, 1995.

Three metaheuristics for effectively searching through the space of cyclic orders for the vehicle routing problem are developed. They are based on GRASP, tabu search, and genetic algorithms. For tabu search, different schemes are investigated to control the tabu list length, including a reactive tabu search method. To obtain good solutions when using the genetic algorithm, specialized crossovers are developed, and a local search component is added. GRASP is used to construct an initial good solution.

G. Kontoravdis and J.F. Bard. A GRASP for the vehicle routing problem with time windows. *ORSA J. on Computing*, 7:10–23, 1995.

A GRASP is proposed for minimizing the fleet size of temporarily constrained vehicle routing problems with two types of service. The greedy function of the construction phase takes into account both the overall minimum insertion cost and the penalty cost. Local search is applied to the best solution found every five iterations of the first phase, rather than to each feasible solution.

M.F. Argüello, J.F. Bard, and G. Yu. A GRASP for aircraft routing in response to groundings and delays. *J. of Combinatorial Optimization*, 1:211–228, 1997.

This paper presents a GRASP to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. See page 32.

J.B. Atkinson. A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *J. of the Operational Research Society*, 49:700–708, 1998.

A GRASP is proposed for solving a complex vehicle-scheduling problem with tight time windows and additional constraints.

J.F. Bard, L. Huang, P. Jaillet, and M. Dror. A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32: 189–203, 1998.

A methodology is presented that decomposes the inventory routing problem with satellite facilities over the planning horizon, and then solves daily rather than multi-day vehicle routing problems. Three heuristics are proposed for solving the vehicle routing problem with satellite facilities: randomized Clarke-Wright, GRASP, and modified sweep. The GRASP proposed is a modified version of the GRASP of Kontoravdis and Bard (1995).

L.I.P. Resende and M.G.C. Resende. A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pages 397–402, July 1999.

A GRASP for a routing problem in telecommunications is described. See page 34.

C. Carreto and B. Baker. A GRASP interactive approach to the vehicle routing problem with backhauls. In C.C. Ribeiro and P. Hansen, editors, *Essays and surveys on metaheuristics*, pages 185–200. Kluwer Academic Publishers, 2002.

The incorporation of interactive tools into heuristic algorithms is investigated. A GRASP is used in the routes construction and improvement phase. The construction phase is implemented in a clustering heuristic that constructs the routes by clustering the remaining customers according to the vehicles defined by seeds while applying the 3-opt heuristic to reduce the total distance traveled by each vehicle. The greedy function takes into account routes with smallest insertion cost and costumers with biggest difference between the smallest and the second smallest insertion costs and smallest number of routes they can traverse. As the local search phase, 3-opt is used.

A. Corberán, R. Martí, and J.M. Sanchis. A GRASP heuristic for the mixed Chinese postman problem. *European J. of Operational Research*, 142(1):70–80, 2002.

The mixed postman problem, a generalization of the Chinese postman problem, is that of finding the shortest tour that traverses each edge of a given mixed graph (a graph containing both undirected and directed edges) at least once. This paper proposes a GRASP for the mixed postman problem.

M.G.C. Resende and C.C. Ribeiro. A GRASP with path-relinking for private virtual circuit routing. *Networks*, 41:104–114, 2003.

The offline Private Virtual Circuit Routing problem is formulated as an integer multicommodity flow problem with additional constraints and with an objective function that minimizes propagation delays and/or network congestion. The authors propose variants of a GRASP with path relinking.

A. Lim and F. Wang. A smoothed dynamic tabu search embedded GRASP for m-VRPTW. In *Proceedings of ICTAI 2004*, pages 704–708, 2004.

A GRASP is proposed that uses multiple initialization and solution reuse.

M.G.B. de la Peña. Heuristics and metaheuristics approaches used to solve the rural postman problem: A comparative case study. In *Proceedings of the Fourth International ICSC Symposium on Engineering of Intelligent Systems (EIS 2004)*, 2004.

A new hybrid approach based on simulated annealing, GRASP, and genetic algorithm is proposed for solving the rural postman problem.

Z. Li, S. Guo, F. Wang, and A. Lim. Improved GRASP with tabu search for vehicle routing with both time window and limited number of vehicles. In B. Orchard, C. Yang, and M. Ali, editors, *Innovations in Applied Artificial Intelligence – Proceedings of the 17th International Conference on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems (IEA/AIE 2004)*, volume 3029 of *Lecture Notes in Computer Science*, pages 552–561. Springer-Verlag,

2004.

The authors propose a hybrid GRASP for vehicle routing with both time window and limited number of vehicles. It is combined with multiple initializations, solution reuse, mutation improvement, and with four heuristics: short left time first, near customer first, short waiting time first, and long route first.

M. Boudia, M.A.O. Louly, and C. Prins. A :eactive GRASP and path relinking for a combined production-distribution problem. *Computers and Operations Research*, 34:3402–3419, 2007.

An NP-hard production-distribution problem for one product over a multi-period horizon is studied. The aim is to minimize total cost taking into account production setups, inventory levels, and distribution. A GRASP and two improved versions using either a reactive mechanism or a path relinking are proposed. See page 31.

4. Logic

GRASP has been applied to problems in logic, including SAT, MAX-SAT, and logical clause inference, as shown by the following papers.

M.G.C. Resende and T.A. Feo. A GRASP for satisfiability. In D.S. Johnson and M.A. Trick, editors, *Cliques, Coloring, and Satisfiability: The Second DIMACS Implementation Challenge*, volume 26 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 499–520. American Mathematical Society, 1996.

This paper describes a GRASP for the satisfiability problem that can be also directly applied to both the weighted and unweighted versions of the maximum satisfiability problem. The adaptive greedy function is a hybrid combination of two functions. One function seeks to maximize the number of yet-unsatisfied clauses that become satisfied after the assignment of each construction iteration, while the other maximizes the number of yet-unassigned literals in yet-unsatisfied clauses that become satisfied if opposite assignments were to be made. The local search flips the assignment of each variable, one at a time, checking if the new truth assignment increases the number of satisfied clauses.

P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science*, 1184:575–585, 1996.

A parallel GRASP is proposed for finding approximate solutions to the weighted maximum satisfiability problem.

M.G.C. Resende, L.S. Pitsoulis, and P.M. Pardalos. Approximate solution of weighted MAX-SAT problems using GRASP. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 393–405. American Mathematical Society, 1997.

This article proposes a GRASP for finding approximate solutions of weighted MAX-SAT problems. The greedy adaptive function is to maximize the total weight of yet-unsatisfied clauses that become satisfied after the assignment of each construction phase iteration. The local

search uses the 1-flip neighborhood of a vector x, defined as the set of all binary vectors that differ from x in exactly one literal.

A.S. Deshpande and E. Triantaphyllou. A greedy randomized adaptive search procedure (GRASP) for inferring logical clauses from examples in polynomial time and some extensions. *Mathematical and Computer Modelling*, 27:75–99, 1998.

Two heuristics are presented in this article for inferring a small size Boolean function from complete and incomplete examples in polynomial time. Each example can be positive or negative depending on whether it must be accepted or rejected, respectively, by the target function. Both of the proposed heuristics are randomized in the sense that instead of choosing the best candidate element, a candidate list is built whose elements are assigned with evaluative function values close to the highest one.

M.G.C. Resende, L.S. Pitsoulis, and P.M. Pardalos. Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics*, 100:95–113, 2000.

A set of Fortran subroutines for computing approximate solutions of MAX-SAT problems is described. The algorithm implemented was proposed by Resende, Pitsoulis, and Pardalos (1997). Two versions of the subroutines are distributed. One version uses a neighborhood data structure in order to speed up the local search phase, while the second version, since it does not make use of this data structure, is more memory efficient but less time efficient. Computational results improve upon those in Resende, Pitsoulis, and Pardalos (1997) using an RCL parameter α randomly chosen each GRASP iteration from the interval $[0, 1]$.

P. Festa, P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. GRASP with path-relinking for the weighted maximum satisfiability problem. In *Proceedings of the IV Workshop on Efficient and Experimental Algorithms (WEA2005)*, volume 3503 of *Lecture Notes in Computer Science*, pages 367–379, 2005.

A GRASP with path relinking for finding good quality solutions of the weighted maximum satisfiability problem (MAX-SAT) is described in this paper. Here, path relinking is used to intensify the search around good quality isolated solutions that have been produced by the GRASP heuristic. Experimental comparison of the pure GRASP (without path relinking) and the GRASP with path relinking illustrates the effectiveness of path relinking in decreasing the average time needed to find a good quality solution.

P. Festa, P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. GRASP with path-relinking for the weighted maxsat problem. *ACM J. of Experimental Algorithmics*, 11:1–16, 2006.

A GRASP with path relinking for finding good quality solutions of the weighted maximum satisfiability problem (MAX-SAT) is described. The authors designed accurate experiments to determine the effect of path relinking on the convergence of the GRASP for MAX-SAT.

5. Partitioning

Partitioning problems have been treated with GRASP. These include number partitioning, VLSI circuit partitioning, and software/hardware partitioning. The following papers describe these applications.

M.F. Argüello, T.A. Feo, and O. Goldschmidt. Randomized methods for the number partitioning problem. *Computers and Operations Research*, 23(2):103–111, 1996.

> This paper presents randomized methodologies for solving the number partitioning problem (the partitioning of a finite set of integers into two disjoint subsets such that the difference of the sums of the elements in the subsets is minimized). The greedy criterion consists in considering only large elements for differencing. Specific selection of the elements to be differenced is made at random. Differences are placed back into the list of remaining elements, and the process of selecting the next element is repeated. The proposed methods are greedy, randomized, and adaptive construction heuristics, but local search is omitted.

S. Areibi and A. Vannelli. A GRASP clustering technique for circuit partitioning. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 711–724. American Mathematical Society, 1997.

> This paper adapts a basic node interchange method for solving the circuit partitioning problem and develop a clustering technique that uses GRASP to generate clusters of moderate sizes. See page 42.

S.M. Areibi. GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, May 1999.

> This article proposes a GRASP for obtaining good initial solutions for an iterative improvement technique. At each iteration of the randomized approach, the gains associated with moving modules to the current block being filled are examined, and a restricted candidate list is built using the modules with the highest gains.

G.G. Pu, Z. Chong, Z.Y. Qiu, Z.Q. Lin, and J.F. He. A hybrid heuristic algorithm for HW-SW partitioning within timed automata. In *Proceedings of Knowledge-based Intelligent Information and Engineering Systems*, volume 4251 of *Lecture Notes in Artificial Intelligence*, pages 459–466. Springer-Verlag, 2006.

> The authors integrate the timed automata model into a hybridization of GRASP and tabu search for HW-SW partitioning. Tabu search is used as local search procedure.

6. Location and layout

Location and layout are another class of problems successfully handled by GRASP. The following papers show how GRASP is used in this context.

J.G. Klincewicz. Avoiding local optima in the p-hub location problem using tabu search and GRASP. *Annals of Operations Research*, 40:283–302, 1992.

This paper proposes two heuristics based on tabu search and GRASP for the p-hub location problem. The objective is to overcome the difficulty that local search algorithms encounter. The local search procedure of the GRASP algorithm is based on the 2-exchange.

K. Holmqvist, A. Migdalas, and P.M. Pardalos. Greedy randomized adaptive search for a location problem with economies of scale. In I.M. Bomze et al., editor, *Developments in Global Optimization*, pages 301–313. Kluwer Academic Publishers, 1997.

This paper proposes a GRASP for finding approximate solutions to a facility location problem with concave costs. The greedy function of the construction phase favors the facilities that contribute lower costs for costumers. The neighborhood function is defined as changing facility connection for one costumer. Instead of a time consuming computation of the objective function value for each neighborhood solution, the difference in cost for changing supplier is examined.

T.L. Urban. Solution procedures for the dynamic facility layout problem. *Annals of Operations Research*, 76:323–342, 1998.

The concept of incomplete dynamic programming is applied to the dynamic facility layout problem and a lower bound for the general problem is developed. A GRASP and an initialized multi-greedy algorithm are described to provide a solution methodology for large problems. The GRASP is the algorithm proposed by Li, Pardalos, and Resende (1994) for dense quadratic assignment problems.

H. Delmaire, J.A. Díaz, E. Fernández, and M. Ortega. Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem. *INFOR*, 37:194–225, 1999.

The single source capacitated plant location problem is a discrete location problem that takes into account capacities of the plants to be opened and requires that clients be served from a single open plant. The authors propose a hybrid heuristic that embeds reactive GRASP in a tabu search algorithm.

S. Abdinnour-Helm and S.W. Hadley. Tabu search based heuristics for multi-floor facility layout. *International J. of Production Research*, 38:365–383, 2000.

Two 2-stage heuristics are proposed for solving the multi-floor facility layout problem.

J.R. Cano, O. Cordón, F. Herrera, and L. Sánchez. A greedy randomized adaptive search procedure for the clustering problem. *International J. of Intelligent and Fuzzy Systems*, 12:235–242, 2002.

A GRASP for cluster analysis is described. Construction is done using a randomized greedy Kaufman procedure and local search uses the K-means algorithm. High quality solutions are found for benchmark problems. The best solutions are found with a hybrid GRASP/K-means with Kaufman initialization.

B.T. Han and V.T. Raja. A GRASP heuristic for solving an extended capacitated concentrator location problem. *International J. of Information Technology and Decision Making*, 2(4):597–617, 2003.

A GRASP heuristic for solving an extended capacitated concentrator location problem is presented. In the construction phase, the greedy

criterion looks for the minimum number of concentrators required to obtain a feasible solution. Two local search procedures are proposed and tested. The first one tries to eliminate a concentrator with low capacity consumption by trying to reassign its end-user nodes to other concentrators. The second one tries for each concentrator to replace it by a cheaper one while not violating capacity constraints.

C. Cotta and A.J. Fernández. A hybrid GRASP with evolutionary algorithm approach to Golomb ruler search. In *Parallel Problem Solving from Nature - PPSN VIII*, volume 3242 of *Lecture Notes in Computer Science*, pages 481–490. Springer-Verlag, 2004.

This paper proposes a GRASP that takes as input a conflict graph associated with the problem. The construction phase greedy criterion is based on vertex degrees and the RCL is built by applying a fixed-size criterion. The local search procedure tests for each point another valid candidate position and performs the best change.

J.A. Pacheco and S. Casado. Solving two location models with few facilities by using a hybrid heuristic: A real health resources case. *Computers and Operations Research*, 32:3075–3091, 2005.

A hybrid scatter search algorithm is proposed that incorporates procedures based on different strategies, such as GRASP and path relinking.

M. Pérez, F. Almeida, and J.M. Moreno-Vega. A hybrid GRASP-path relinking algorithm for the capacitated p-hub median problem. In *Hybrid Metaheuristics*, volume 3636 of *Lecture Notes in Computer Science*, pages 142–153. Springer, 2005.

A hybrid GRASP with path-relinking algorithm for the capacitated p-hub median problem is presented. The greedy evaluation function has two phases: a location phase and an allocation phase. In the location phase, one element is added at each time to the set of hubs, while the allocation phase consists of an allocation to the nearest hub. As local search, the authors use a greedy procedure for location and an allocation to the nearest hub.

J.A. Díaz and E. Fernández. Hybrid scatter search and path relinking for the capacitated p-median problem. *European J. of Operational Research*, 169:570–585, 2006.

GRASP is used to generate the initial reference set both for scatter search and path relinking.

G.L. Cravo, G.M. Ribeiro, and L.A. Nogueira Lorena. A greedy randomized adaptive search procedure for the point-feature cartographic label placement. *Computers and Geosciences*, 34(4):373–386, 2008.

A GRASP based on an underlying conflict graph is described for the point-feature cartographic label placement problem. See page 41.

I.A. Contreras and J.A. Díaz. Scatter search for the single source capacitated facility location problem. *Annals of Operations Research*, 157:73–89, 2008.

This paper proposes a scatter search approach to provide upper bounds for the optimal solution of a single source capacitated facility location problem. The proposed approach uses GRASP to initialize the reference set. Solutions of the reference set are combined using a procedure that consists of an initialization and an improvement phase. During

the initialization phase each client is assigned to an open facility. The local search procedure explores two types of neighborhood structures. The first type is related to the assignment of clients within the set of selected facilities while neighborhood of the second type is related to solutions where the set of open facilities changes.

7. Graph theoretic applications

Perhaps the largest class of problems for which GRASP has been applied is graph theory. The following papers illustrate the wide applicability of GRASP to these problems.

T.A. Feo, M.G.C. Resende, and S.H. Smith. A greedy randomized adaptive search procedure for maximum independent set. *Operations Research*, 42:860–878, 1994.

A GRASP for approximately solving the maximum independent set problem is described. The greedy function chosen orders admissible vertices with respect to the minimum admissible vertex degree. The adjective admissible is referred to a vertex that is not adjacent to any vertex in the current independent set. The neighborhood definition used in the local search is $(2, 1)$-exchange, where two nonadjacent vertices can be added to the current solution if a single vertex from the solution is removed.

M. Laguna, T.A. Feo, and H.C. Elrod. A greedy randomized adaptive search procedure for the two-partition problem. *Operations Research*, 42:677–687, 1994.

A GRASP for the network 2-partition problem is proposed. The greedy function of the construction phase minimizes the augmented weight of the partition. For the local improvement phase, four alternative procedures are considered: best swap, first swap, slight swap, and slightest swap. The best strategies are slight and slightest swaps. Slight swap selects a near-minimum gain exchange at each iteration, while slightest swap chooses the absolute minimum gain.

E.M. Macambira and C.C. de Souza. A GRASP for the maximum clique problem with weighted edges. In *Proceedings of the XXIX Brazilian Symposium on Operations Research*, page 70, October 1997.

The authors propose a branch-and-cut algorithm for the maximum clique problem with weighted edges. The initialization phase of the algorithm uses a GRASP to generate good starting solutions. The greedy function minimizes the sum of weights of the edges outgoing from vertices in the partition. The local search uses the exchange of one element in the current partition with one not in it.

M.G.C. Resende and C.C. Ribeiro. A GRASP for graph planarization. *Networks*, 29:173–189, 1997.

A GRASP for the graph planarization problem is described, extending the two-phase heuristic of Goldschmidt and Takvorian (*Networks*, v. 24, pp. 69–73, 1994). Computational experience on a large set of standard test problems is presented. On almost all test problems considered, the heuristic either matches or finds a better solution than previously described graph planarization heuristics. In several cases, previously unknown optimal solutions are found.

E.M. Macambira and C.N. Meneses. A GRASP algorithm for the maximum weighted edge subgraph problem. In *Proceedings of the IX Latin-Iberoamerican Conference of Operations Research (IX CLAIO)*, 1998.

A GRASP for the maximum weighted edge subgraph problem is proposed to overcome the difficulties encountered by local search methods. The greedy function of the construction phase favors vertices corresponding to the maximum sum of the weights associated with its outgoing edges. The local search tries to improve the actual solution by simply swapping one element in the solution set with one not belonging to the solution. In Portuguese.

S.L. Martins, C.C. Ribeiro, and M.C. Souza. A parallel GRASP for the Steiner problem in graphs. In A. Ferreira and J. Rolim, editors, *Proceedings of IRREGULAR'98 – 5th International Symposium on Solving Irregularly Structured Problems in Parallel*, volume 1457 of *Lecture Notes in Computer Science*, pages 285–297. Springer-Verlag, 1998.

A parallel GRASP for the Steiner problem in graphs is described.

P.M. Pardalos, M.G.C. Resende, and J. Rappe. An exact parallel algorithm for the maximum clique problem. In R. De Leone et al., editor, *High performance algorithms and software in nonlinear optimization*, pages 279–300. Kluwer Academic Publishers, 1998.

GRASP is used to produce good-quality upper bounds in an exact method for maximum clique.

M.G.C. Resende. Computing approximate solutions of the maximum covering problem using GRASP. *J. of Heuristics*, 4:161–171, 1998.

A GRASP for the maximum covering problem is described. The greedy function is the total weight of the yet-uncovered demand points that become covered after the selection. The local search procedures uses a 2-exchange neighborhood structure. The GRASP is shown to find near optimal solutions. See page 33.

M.G.C. Resende, T.A. Feo, and S.H. Smith. Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software*, 24:386–394, 1998.

Fortran subroutines for finding approximate solutions of the maximum independent set problem with GRASP are described in detail.

J. Abello, P.M. Pardalos, and M.G.C. Resende. On maximum clique problems in very large graphs. In J. Abello and J. Vitter, editors, *External memory algorithms and visualization*, volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 199–130. American Mathematical Society, 1999.

An approach for clique and quasi-clique computations in very large multi-digraphs is presented. The authors discuss graph decomposition schemes that break up the original problem into several pieces of manageable dimensions. A semi-external memory GRASP is presented to approximately solve the maximum clique problem and maximum quasi-clique problem. The construction phase uses vertex degrees as a guide for construction. The local search uses a simple $(2, 1)$-exchange.

S.L. Martins, P.M. Pardalos, M.G.C. Resende, and C.C. Ribeiro. Greedy randomized adaptive search procedures for the Steiner problem in graphs. In P.M.

Pardalos, S. Rajasekaran, and J. Rolim, editors, *Randomization methods in algorithmic design*, volume 43 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 133–145. American Mathematical Society, 1999.

Four versions of a GRASP for approximately solving general instances of the Steiner problem in graphs are proposed. One is implemented and tested. The construction phase is based on the distance network heuristic. A distance network corresponding to the original graph is built. Associated with each edge of the distance network is a weight that takes into account the shortest distances in the original graph. With respect to the new weight distribution, Kruskal's algorithm is used to solve the minimum spanning tree (MST) problem and the edges in the MST so computed are replaced by the edges in the corresponding shortest paths in the original graph. The local search is based on insertions and eliminations of nodes to and from the current solution.

P.M. Pardalos, T. Qian, and M.G.C. Resende. A greedy randomized adaptive search procedure for the feedback vertex set problem. *J. of Combinatorial Optimization*, 2:399–412, 1999.

This paper describes a GRASP for finding approximate solutions to the feedback vertex set problem on a digraph. Several greedy functions are tested, all of them favoring vertices with high degree. The local search procedure tries at each iteration to eliminate redundant vertices. Some efficient problem reduction techniques are also described. They are useful both to simplify the problem instance and to determine if a digraph is acyclic.

C.C. Ribeiro and M.G.C. Resende. Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software*, 25:341–352, 1999.

This paper describes Fortran subroutines for finding approximate solutions to the problem of minimizing the number of edges that need to be removed from a graph to make it planar.

R.A. Patterson, H. Pirkul, and E. Rolland. A memory adaptive reasoning technique for solving the capacitated minimum spanning tree problem. *J. of Heuristics*, 5: 159–180, 1999.

A GRASP for the capacitated minimum spanning tree problem is proposed. Testing, however, is limited to only a semi-greedy version of the GRASP.

S.L. Martins, M.G.C. Resende, C.C. Ribeiro, and P.M. Pardalos. A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *J. of Global Optimization*, 17:267–283, 2000.

This paper presents a GRASP for the Steiner problem in graphs. The construction phase is based on the Mehlhorn distance network heuristic, which consists of computing the modified distance network graph and using Kruskal's algorithm to solve the minimum spanning tree problem for the resulting graph. The local search is done by using a combination of key-path based local search and node based local search.

L.S. Ochi, M.B. Silva, and L. Drummond. Distributed parallel metaheuristics based on GRASP and VNS for solving the traveling purchaser problem. In *Proceedings*

of the 4th Metaheuristics International Conference – MIC2001, pages 489–494, 2001.

In this paper hybrid metaheuristics based on GRASP and VNS are described to solve the traveling purchaser problem, a generalization of the traveling salesman problem.

R.K. Ahuja, J.B. Orlin, and D. Sharma. Multi-exchange neighborhood structures for the capacitated minimum spanning tree problem. *Mathematical Programming*, 91:71–97, 2001.

This paper presents generalizations of new neighborhood structures for the capacitated minimum spanning tree problem. The new structures allow simultaneous cyclic exchanges of nodes among multiple subtrees. As the size of these structures grows exponentially with the problem size, the authors propose a heuristic search technique based on the concept of an *improvement graph* which converts each possible cyclic exchange into a subset-disjoint cycle in the improvement graph and a profitable cyclic exchange into a negative cost cycle, heuristically identified using a modification of the shortest-path label-correcting algorithm. To judge the efficacy of the neighborhoods local improvement and tabu search algorithms have been developed. Local improvement uses a GRASP construction mechanism to generate repeated starting solutions for local improvement.

P. Festa, P.M. Pardalos, and M.G.C. Resende. Algorithm 815: FORTRAN subroutines for computing approximate solution to feedback set problems using GRASP. *ACM Transactions on Mathematical Software*, 27:456–464, 2001a.

This article describes a set of ANSI standard Fortran 77 subroutines to find approximate solutions of both the feedback vertex set problem and the feedback arc set problem. The GRASP of Pardalos, Qian, and Resende (1999) is used to produce the approximate solutions of the feedback set problem. Feedback arc set problems are converted into feedback vertex set problems and solved.

P. Festa, P.M. Pardalos, M.G.C. Resende, and Ribeiro. GRASP and VNS for MAX-CUT. In J.P. Sousa, editor, *Proceedings of the IV Metaheuristics International Conference (MIC2001)*, pages 371–376, 2001b.

In this paper, a GRASP and a variable neighborhood search (VNS) for MAX-CUT are proposed and tested.

M. Laguna and R. Martí. A GRASP for coloring sparse graphs. *Computational Optimization and Applications*, 19:165–178, 2001a.

A GRASP for graph coloring is presented. The construction phase constructs the next coloring, one color at time. The greedy function chooses the vertex having the maximum degree among the uncolored vertices adjacent to at least one colored vertex. At each step, the local search combines the two smallest cardinality color classes into one and tries to find a valid color for each violating vertex.

R. Martí and V. Estruch. Incremental bipartite drawing problem. *Computers and Operations Research*, 28:1287–1298, 2001.

A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. See page 41.

S.A. Canuto, M.G.C. Resende, and C.C. Ribeiro. Local search with perturbations for the prize-collecting Steiner tree problem in graphs. *Networks*, 38:50–58, 2001.

The authors describe a multi-start local search algorithm for the prize-collecting Steiner tree problem, based on the generation of initial solutions by a primal-dual algorithm using perturbed node prizes. Path relinking is used to improve the solutions found by local search and variable neighborhood search is used as a post-optimization procedure. The neighborhood of a solution is formed by all minimum spanning trees whose sets of nodes differ by exactly one node. The proposed perturbation algorithm is similar to a GRASP procedure, in which the greedy randomized construction is replaced by the construction of initial solutions using perturbed cost functions.

M. Laguna and R. Martí. A GRASP for coloring sparse graphs. *Computational Optimization and Applications*, 19(2):165–178, 2001b.

A GRASP for coloring sparse graphs is described. During the construction phase the greedy criterion is defined on vertex degrees. The local search procedure combines into one the smallest and second smallest cardinality color classes and then, for each violating vertex, it tries to rebuild feasibility.

J. Abello, M.G.C. Resende, and S. Sudarsky. Massive quasi-clique detection. In S. Rajsbaum, editor, *LATIN 2002: Theoretical Informatics*, volume 2286 of *Lecture Notes in Computer Science*, pages 598–612. Springer-Verlag, 2002.

The authors propose techniques that are useful for the detection of dense subgraphs (quasi-cliques) in massive sparse graphs whose vertex set, but not the edge set, fits in memory. The algorithms rely also on greedy randomized adaptive search procedures (GRASP) to extract the dense subgraphs.

C.C. Ribeiro, E. Uchoa, and R.F. Werneck. A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS J. on Computing*, 14:228–246, 2002.

This paper proposes a hybrid GRASP with path-relinking for the Steiner problem in graphs.

R. Martí. Arc crossing minimization in graphs with GRASP. *IEEE Transactions*, 33:913–919, 2002.

This paper presents a GRASP for minimizing straight-line crossings in hierarchical graphs. See page 40.

P. Festa, P.M. Pardalos, M.G.C. Resende, and C.C. Ribeiro. Randomized heuristics for the MAX-CUT problem. *Optimization Methods and Software*, 7:1033–1058, 2002.

In this paper, a GRASP, a variable neighborhood search (VNS), and a path relinking heuristic for MAX-CUT are proposed and tested. New hybrid heuristics that combine GRASP, VNS as GRASP local search, and path relinking are also proposed and tested.

L. Drummond, L.S. Vianna, M.B. Silva, and L.S. Ochi. Distributed parallel metaheuristics based on GRASP and VNS for solving the traveling purchaser problem. In *Proceedings of the Ninth International Conference on Parallel and Distributed Systems (ICPADS02)*, pages 257–266, 2002.

This paper describes several strategies for parallel implementations of GRASP and VNS applied to the traveling purchaser problem. Parallel algorithms based on master-worker, completely distributed and independent models, using static and dynamic load balance are proposed.

R.K. Ahuja, J.B. Orlin, and D. Sharma. A composite very large-scale neighborhood structure for the capacitated minimum spanning tree problem. *Operations Research Letters*, 31:185–194, 2003.

The authors designed a GRASP for the capacitated minimum spanning tree problem, whose local search procedure uses a composite very large-scale neighborhood structure.

R. Martí and M. Laguna. Heuristics and meta-heuristics for 2-layer straight line crossing minimization. *Discrete Applied Mathematics*, 127(3):665–678, 2003.

This paper presents extensive computational experiments to compare several different randomized heuristics for the problem of minimizing straight-line crossings in a 2-layer graph. A tabu search yields the best results for relatively dense graphs, while a GRASP outperforms all other approaches when tackling low-density graphs. See page 41.

I.H. Osman, B. Al-Ayoubi, and M. Barake. A greedy random adaptive search procedure for the weighted maximal planar graph problem. *Computers and Industrial Engineering*, 45(4):635–651, 2003.

The weighted maximal planar graph (WMPG) problem seeks to find a subgraph from a given weighted complete graph such that the subgraph is planar – it can be embedded on the plane without any arcs intersecting. The subgraph is maximal if no additional arc can be added to the subgraph without destroying its planarity and it also has the maximal sum of arc weights. In this paper, the authors describes a GRASP, where the RCL is dynamically updated.

M.C. Souza, C. Duhamel, and C.C. Ribeiro. A GRASP heuristic for the capacitated minimum spanning tree problem using a memory-based local search strategy. In M.G.C. Resende and J. Sousa, editors, *Metaheuristics: Computer Decision-Making*, pages 627–658. Kluwer Academic Publisher, 2004.

The authors propose a GRASP with path-relinking heuristic. It uses a randomized version of a savings heuristic in the construction phase and an extension of the local search strategy, incorporating some short term memory elements of tabu search.

T.C.S. Dias, G.F. de Sousa Filho, E.M. Macambira, L.A.F. Cabral, and M.H.C. Fampa. An efficient heuristic for the ring star problem. In *Experimental Algorithms*, volume 4007 of *Lecture Notes in Computer Science*, pages 24–35. Springer, 2006.

A hybrid heuristic is proposed for the Ring Star Problem. See page 37.

A.M. Campbell and B.W. Thomas. Probabilistic traveling salesman problem with deadlines. *Transportation Science*, 42(1):1–21, 2008.

The probabilistic traveling salesman problem with deadlines (PTSPD) is an extension of the well-known probabilistic traveling salesman problem (PTSP) in which, in addition to the presence of randomness, customers must also be visited before a given deadline. The authors carry out several computational experiments, whose aim is not to empirically

show the efficiency of the proposed algorithms but rather to demonstrate the difference between TSPD and PTSPD solutions. They propose also a GRASP with a 1-shift neighborhood and a restricted candidate list of size 3. To avoid premature convergence, a best-improving 2-Opt neighborhood is run each time the 1-shift neighborhood fails to find an improving solution. If the 2-Opt procedure finds an improving solution, a new GRASP local search is applied from this new solution.

J.E.C. Arroyo, P.S. Vieira, and D.S. Vianna. A GRASP algorithm for the multicriteria minimum spanning tree problem. *Annals of Operations Research*, 159: 125–133, 2008.

Given a vector of costs associated with each edge of the input graph, the multi-criteria minimum spanning tree problem is to find all Pareto optimal or efficient spanning trees. The authors propose a GRASP whose construction procedure uses different weighted utility functions. The GRASP local search tries to improve the current solution by defining a drop-and-add neighborhood, where the spanning trees are represented by Prufer numbers.

8. Quadratic and other assignment problems

GRASP has been applied to several hard assignment problems, including quadratic, biquadratic, multidimensional, and frequency assignment. The following articles cover these problems.

Y. Li, P.M. Pardalos, and M.G.C. Resende. A greedy randomized adaptive search procedure for the quadratic assignment problem. In P.M. Pardalos and H. Wolkowicz, editors, *Quadratic assignment and related problems*, volume 16 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 237–261. American Mathematical Society, 1994.

A GRASP for the quadratic assignment problem is described. Construction first makes two assignments, and then completes the solution by making assignments, one at a time. The greedy function is assignment interaction cost. The local search procedure is a 2-assignment exchange.

T.A. Feo and J.L. González-Velarde. The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science*, 29:330–341, 1995.

A GRASP is proposed for solving the problem of assigning highway trailers to railcar hitches in intermodal transportation. See page 32.

P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP implementation for the quadratic assignment problem. In A. Ferreira and J. Rolim, editors, *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*, pages 115–130. Kluwer Academic Publishers, 1995.

This paper discusses an efficient parallel implementation of GRASP for sparse quadratic assignment problems.

M.G.C. Resende, P.M. Pardalos, and Y. Li. Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 22:104–118, 1996.

This paper presents Fortran subroutines for solving dense quadratic assignment problems with GRASP.

P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 23:196–208, 1997a.

This paper presents Fortran subroutines for solving sparse quadratic assignment problems with GRASP.

P.M. Pardalos, K.G. Ramakrishnan, M.G.C. Resende, and Y. Li. Implementation of a variance reduction based lower bound in a branch and bound algorithm for the quadratic assignment problem. *SIAM J. on Optimization*, 7:280–294, 1997b.

This paper describes a branch and bound algorithm for the quadratic assignment problem which uses GRASP to compute upper bounds.

R.A. Murphey, P.M. Pardalos, and L.S. Pitsoulis. A parallel GRASP for the data association multidimensional assignment problem. In P.M. Pardalos, editor, *Parallel processing of discrete problems*, volume 106 of *The IMA Volumes in Mathematics and Its Applications*, pages 159–180. Springer-Verlag, 1998.

A GRASP for finding good solutions for the data association multidimensional assignment problem is described. At each discrete time interval, the data set is formulated as a multidimensional assignment problem (MAP) with a maximum likelihood cost function. A near-optimal solution to each MAP is obtained with a GRASP. The proposed method can be easily parallelized to substantially decrease the running time.

T. Mavridou, P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A GRASP for the biquadratic assignment problem. *European J. of Operational Research*, 105: 613–621, 1998.

This paper proposes a GRASP for finding approximate solutions of the biquadratic assignment problem. As in the case of GRASP for the quadratic assignment problem, the construction phase has two stages. The first stage simultaneously makes four assignments, selecting the pairs corresponding to the smallest interaction costs, while the second stage makes the remaining assignments, one at time. The greedy function in the second stage selects the assignment corresponding to the minimum interaction cost with respect to the already-made assignments. In the local search phase, 2-exchange local search is applied to the permutation constructed in the first phase.

C. Fleurent and F. Glover. Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS J. on Computing*, 11:198–204, 1999.

Adaptive memory strategies that are the heart of tabu search methods are shown to be a foundation for alternative, enhanced, multistart approaches. This paper illustrates that constructive multistart methods, such as Random Restart and GRASP, can be improved by the addition of memory and associated heuristic search principles. The improved results indicate that these principles (learning, intensification, candidate list strategies, POP) are not limited to applications with transition neighborhoods, as in local search, but can also be useful for applications characterized by constructive (and destructive) neighborhoods. The paper shows that the GRASP for QAP of Li, Pardalos, and Resende (1994) can be improved upon by using these memory strategies.

L.S. Pitsoulis. *Algorithms for nonlinear assignment problems*. PhD thesis, Department of Industrial and Systems Engineering, University of Florida, 1999.

This dissertation presents GRASPs for solving the following NP-hard nonlinear assignment problems (NAPs): quadratic assignment problem (QAP), biquadratic assignment problem (BiQAP), turbine balancing problem (TBP), and multidimensional assignment problem (MAP). Computational results indicate that all of the suggested algorithms are among the best in the literature in terms of solution quality and computational time.

R.K. Ahuja, J.B. Orlin, and A. Tiwari. A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research*, 27:917–934, 2000.

This paper presents a genetic algorithm for the QAP that incorporates the construction phase of the GRASP for QAP of Li, Pardalos, and Resende (1994) to generate the initial population.

M.C. Rangel, N.M.M. de Abreu, P.O. Boaventura Netto, and M.C.S. Boeres. A modified local search for GRASP in the quadratic assignment problem. *Pesquisa Operacional*, 20(1):55–76, 2000a.

An improvement is proposed for the local search phase of the GRASP of Li, Pardalos, and Resende (1994) to solve the quadratic assignment problem. The new strategy amplifies the local search range and improves the local search's efficiency. In Portuguese.

X. Liu, P.M. Pardalos, S. Rajasekaran, and M.G.C. Resende. A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P.M. Pardalos, and F. Hsu, editors, *Mobile Networks and Computing*, volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 195–201. American Mathematical Society, 2000.

A GRASP for frequency assignment is described in this paper. Local search is done by simulated annealing. See page 34.

M. Prais and C.C. Ribeiro. Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS J. on Computing*, 12: 164–176, 2000.

This paper describes a GRASP for matrix decomposition problem arising in the context of traffic assignment in communication satellites. The local search phase of the GRASP proposed is based on a new neighborhood, defined by constructive and destructive moves. See page 34.

M.C. Rangel, N.M.M. Abreu, and P.O. Boaventura Netto. GRASP in the QAP: An acceptance bound for initial solutions. *Pesquisa Operacional*, 20:45–58, 2000b.

This paper presents a modified version of the GRASP algorithm proposed by Li, Pardalos, and Resende (1994) for the quadratic assignment problem. The new GRASP uses a criterion to accept or reject a given initial solution, thus trying to avoid searches that eventually can be fruitless. It computes a normalized limit cost, defined with the aid of QAP upper and lower bounds easily obtained and discards all solutions with cost less than the computed limit. In Portuguese.

A. Srinivasan, K.G. Ramakrishnan, K. Kumaram, M. Aravamudam, and S. Naqvi. Optimal design of signaling networks for Internet telephony. In *IEEE INFOCOM 2000*, volume 2, pages 707–716, March 2000.

GRASP is used in an approach for efficient design of a signaling network for a network of software switches supporting Internet telephony. See page 35.

L.S. Pitsoulis, P.M. Pardalos, and D.W. Hearn. Approximate solutions to the turbine balancing problem. *European J. of Operational Research*, 130:147–155, 2001.

The turbine balancing problem is formulated as a standard quadratic assignment problem and a GRASP implementation for solving the resulting problem is presented.

A.J. Robertson. A set of greedy randomized adaptive local search procedure (GRASP) implementations for the multidimensional assignment problem. *Computational Optimization and Applications*, 19:145–164, 2001.

This paper introduces four GRASP implementations for the multidimensional assignment problem by combining two construction methods (randomized greedy and randomized max regret) and two local search methods (2-exchange and variable depth exchange). At each iteration of the randomized greedy construction phase, a set of best assignments is constructed from which a random element is selected and added to the solution set. The greedy function of the randomized max regret construction method is a measure of the competition between the two leading cost candidates. The maximum regret value corresponds to the candidate assignment that has the largest winning margin between itself and its next highest competitor. The variable depth exchange is an extension of the 2-exchange method that allows a more extensive search of the surrounding neighborhood space.

R.M. Aiex. *An experimental investigation of the probability distribution of solution time in GRASP heuristics and its application to the analysis of parallel implementations*. PhD thesis, Computer Science Department, Catholic University of Rio de Janeiro, August 2002.

This Ph.D. thesis establishes that the running time of GRASP fits a shifted exponential distribution. Parallel GRASP with path-relinking implementations for three-index assignment are described and tested. In Portuguese.

H.R. Lourenço and D. Serra. Adaptive approach heuristics for the generalized assignment problem. *Mathware and Soft Computing*, 9(2-3):209–234, 2002.

This paper presents several hybrid algorithms consisting of adaptive construction heuristics and subsequent application of local search to solve the GAP. The basic elements are extracted from a specific ant colony optimization algorithm (MAX-MIN Ant System) and GRASP, which are are embedded in a general framework.

C.A. Oliveira, P.M. Pardalos, and M.G.C. Resende. GRASP with path-relinking for the quadratic assignment problem. In *Efficient and Experimental Algorithms*, volume 3059 of *Lecture Notes in Computer Science*, pages 356–368, 2004.

A GRASP for the quadratic assignment problem is described. Path relinking is invoked at each GRASP iteration as intensification procedure.

R. Aiex, M.G.C. Resende, P.M. Pardalos, and G. Toraldo. GRASP with path relinking for three-index assignment. *INFORMS J. on Computing*, 17(2):224–247, 2005.

This paper describes variants of GRASP with path relinking for the three index assignment problem (AP3).

S.A. de Carvalho Jr. and S. Rahmann. Microarray layout as a quadratic assignment problem. In Hudson et al., editor, *Proceedings of the German Conference on Bioinformatics (GCB)*, volume P-83 of *Lecture Notes in Informatics*, pages 11–20. Gesellschaft für Informatik, 2006.

The production of commercial DNA microarrays is based on a light-directed chemical synthesis driven by a set of masks or micromirror arrays. The arrangement of the probes on the chip and the order in which their nucleotides are synthesized play an important role on the quality of the final product. This paper proposes a new model called conflict index for evaluating microarray layouts, and shows that the probe placement problem is an instance of the quadratic assignment problem. GRASP is used to design the layout of small artificial chips. The authors compare GRASP with the best known algorithm and describe how it can be combined with other algorithms to design million-probe microarrays.

9. Manufacturing

GRASP has been used to address several optimization problems in manufacturing. The following papers are examples of this.

J.F. Bard and T.A. Feo. Operations sequencing in discrete parts manufacturing. *Management Science*, 35:249–255, 1989.

This paper presents a method for efficiently sequencing the cutting operations associated with the manufacture of discrete parts.

T.A. Feo and J.F. Bard. The cutting path and tool selection problem in computer-aided process planning. *J. of Manufacturing Systems*, 8:17–26, 1989.

The purpose of this paper is to provide a method for minimizing the sum of tool setup and volume removal times associated with metal cutting operations on a flexible machine. The problem is modeled as an integer program, then relaxed into a min-cut problem on a simple network. After obtaining a tentative solution, the authors use a GRASP to identify good feasible points corresponding to alternative process plans. These are seen to speed convergence during branch & bound.

J.F. Bard and T.A. Feo. An algorithm for the manufacturing equipment selection problem. *IIE Transactions*, 23:83–92, 1991.

This paper provides a unified framework in which product and process demands can be related to manufacturing system requirements. The authors develop a nonlinear cost minimization model. The objective is to determine how many of each machine type to purchase and what fraction of the time each piece of equipment will be configured for a particular type of operation. Once the original problem is converted into a MILP, a depth-first branch & bound algorithm is used, employing the greedy randomized set covering heuristic of Feo and Resende (1989), to implicitly search for optimality. Viewing the contribution that any machine makes to satisfy the demand of any process as the unit benefit associated with that machine, a benefit-to-cost ratio is computed for

each machine. To derive a feasible solution, the heuristic iteratively selects machines with largest ratio and updates benefits to take into account the remaining demand.

J.G. Klincewicz and A. Rajan. Using GRASP to solve the component grouping problem. *Naval Research Logistics*, 41:893–912, 1994.

Two new heuristics are proposed for solving a particular set partitioning problem that arises in robotics assembly, as well as in a number of other manufacturing and material logistics application areas. The heuristics are GRASPs involving two alternate procedures for determining starting points: component-based and code-based.

T.A. Feo, J.F. Bard, and S. Holland. Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research*, 43:219–230, 1995.

This paper describes a decision support system known as INSITES, designed to assist Texas Instruments in the day-to-day assembly operations of their printed wiring board (PWB) facilities. A GRASP is used to solve the underlying multiple machine scheduling problem. The schedule produced at each GRASP iteration is evaluated based on one of five different optimization criteria. The choice of the criterion to be followed is made by the user to rank order the schedules provided by multiple GRASP iterations.

J.F. Bard, T.A. Feo, and S. Holland. A GRASP for scheduling printed wiring board assembly. *IIE Transactions*, 28:155–165, 1996.

The assembly of printed wiring boards (PWBs) typically involves the coordination of thousands of components and hundreds of part numbers in a job shop environment with more than 50 different processes and workstations. The authors propose a GRASP for solving the daily scheduling problem that arises in such environment. The first phase of GRASP obtains a user-specified number of schedules. The greedy function is the product between the weighted processing time and and the slack time window.

T.L. Urban, W.-C. Chiang, and R.A. Russel. The integrated machine allocation and layout problem. *International J. of Production Research*, 38:2913–2930, 2000.

GRASP is used to solve quadratic assignment sub-problems in a model that aggregates quadratic assignment problems with several network flow problems with side constraints. This model is used to produce machine layouts where machines are not required to be placed in a functional or cellular layout.

J. Yen, M. Carlsson, M. Chang, J.M. Garcia, and H. Nguyen. Constraint solving for inkjet print mask design. *J. of Imaging Science and Technology*, 44:391–397, 2000.

Print masks are used to control the firing of the nozzles, i.e., to determine which nozzles on an inkjet printer cartridge are to spit an ink droplet at each particular instant in a multiple-pass print mode. Masks are generated by minimizing the total costs. A GRASP is proposed for for automatic generation of print masks. It has been used to design the print masks for Hewlett Packard's wide format printers (DeskJet 2500C and 2500CM). This approach can shorten the turn-around time for print mask design.

R. Alvarez-Valdes, F. Parreño, and J.M. Tamarit. A GRASP algorithm for constrained two-dimensional non-guillotine cutting problems. *J. of the Operational Research Society*, 56(4):414–425, 2005.

A GRASP for constrained two-dimensional non-guillotine cutting problems is presented. In the construction phase, the authors take the smallest rectangle breaking the ties by the nearest distance to a corner of the stock rectangle. Then, two criteria have been considered to select the piece: 1) The first piece in a list ordered, giving priority to pieces which must be cut; 2) The piece producing the largest increase in the objective function. Three alternative local search procedures are proposed.

M. Boudia, M.A.O. Louly, and C. Prins. A reactive GRASP and path relinking for a combined production-distribution problem. *Computers and Operations Research*, 34:3402–3419, 2007.

In this paper, an NP-hard production-distribution problem for one product over a multi-period horizon is studied. The aim is to minimize total cost taking into account production setups, inventory levels, and distribution. A GRASP and two improved versions using either a reactive mechanism or path-relinking are proposed. See page 12.

S.K. Monkman, D.J. Morrice, and J.F. Bard. A production scheduling heuristic for an electronics manufacturer with sequence-dependent setup costs. *European J. of Operational Research*, 187(3):1100–1114, 2008.

The authors propose a three-step heuristic to schedule multiple product families on parallel, identical production lines so as to minimize setup costs. The heuristic involves assignment, sequencing, and time scheduling steps. The problem is modeled as a traveling salesman subset-tour problem. The authors propose a GRASP for the sequencing step. The construction phase uses a cardinality based RCL and the greedy function takes into account the cost associated with the arcs of the underlying graph. The local search uses two different neighborhoods: a node elimination and a node swap neighborhood.

M.C.V. Nascimento, M.G.C. Resende, and F.M.B. Toledo. GRASP with path-relinking for the multi-plant capacitated lot sizing problem. *European J. of Operational Research*, 2008.

This paper describes a GRASP with path-relinking for the independent multi-plant, multi-period, and multi-item capacitated lot sizing problem where transfers between the plants are allowed. In addition, the paper addresses applications of the proposed heuristics for the special case of the capacitated lot sizing problem with parallel machines. The computational tests show that the proposed heuristics outperform other heuristics previously described in the literature. To appear.

10. Transportation

GRASP has been used to find approximate solutions of problems in air, rail, and intermodal transportation. The following papers illustrate these applications.

T.A. Feo and J.F. Bard. Flight scheduling and maintenance base planning. *Management Science*, 35:1415–1432, 1989.

This paper presents a model that can be used by planners to both locate maintenance stations and develop flight schedules that better meet the cyclical demand for maintenance. The problem is formulated as large-scale mixed integer program, i.e. a minimum cost, multicommodity flow network with integral constraints, where each airplane represents a separate commodity and each arc has an upper and lower capacity of flow. Since obtaining feasible solutions from the relative LP relaxation is difficult, the authors propose a GRASP.

T.A. Feo and J.L. González-Velarde. The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science*, 29:330–341, 1995.

This paper deals with the problem of optimally assigning highway trailers to railcar hitches in intermodal transportation. Using a set covering formulation, the problem is modeled as an integer linear program, whose linear programming relaxation yields a tight lower bound. This formulation also provides the basis for developing a branch-and-bound algorithm and a GRASP for solving the problem. The greedy strategy of the construction phase of GRASP consists in selecting at each step a feasible assignment of the most difficult to use available railcar together with the most difficult to assign trailer. To improve the constructed solution, a 2-exchange local search is applied, carrying out a complete enumeration of the solutions in the neighborhood.

M.F. Argüello, J.F. Bard, and G. Yu. A GRASP for aircraft routing in response to groundings and delays. *J. of Combinatorial Optimization*, 1:211–228, 1997.

This paper presents a GRASP to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. The proposed heuristic is a neighborhood search technique that takes as input an initial feasible solution, so that the construction phase is omitted. Two types of partial route exchange operations are described. The first type is the exchange of flight sequences with identical endpoints. In the second type, the sequence of flights being exchanged must have the same origination airport, but the termination airports are swapped.

J.F. Bard. An analysis of a rail car unloading area for a consumer products manufacturer. *J. of the Operational Research Society*, 48:873–883, 1997.

This paper discusses how to design and analyze the railcar unloading area of Procter & Gamble's principal laundry detergent plant. The related combinatorial problem of assigning railcars to positions on the platform and unloading equipment to railcars is modeled as a mixed-integer nonlinear program. To approximately solve the problem, four alternatives are proposed and evaluated with the help of a GRASP.

D. Sosnowska. Optimization of a simplified fleet assignment problem with metaheuristics: Simulated annealing and GRASP. In P.M. Pardalos, editor, *Approximation and complexity in numerical optimization*, pages 477–488. Kluwer Academic Publishers, 2000.

Two heuristics based on simulated annealing and GRASP are presented for finding approximate solutions for a simplified fleet assignment problem. Both methods are based on swapping parts of sequence of flight legs

assigned to an aircraft (rotation cycle) between two randomly chosen aircrafts. In simulated annealing, the exchange is such that a solution is accepted according to a probability distribution, while in GRASP only exchanges leading to a better solution are permitted and the potentially best part of the assignment is conserved and the rest is randomly reattributed. The construction phase does not use a restricted candidate list explicitly, but a solution is built by simply trying to make the time interval between two flights as small as possible.

11. Telecommunications

Telecommunications, including network design, is a field in which much work with GRASP has been done. The papers below illustrate this.

J.G. Klincewicz. Avoiding local optima in the p-hub location problem using tabu search and GRASP. *Annals of Operations Research*, 40:283–302, 1992.

This paper proposes two heuristics based on tabu search and GRASP for the p-hub location problem. See page 15.

J. Xu and S. Chiu. Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pages 240–248, July 1996.

A GRASP for the field technician scheduling problem is described. See page 5.

F. Poppe, M. Pickavet, P. Arijs, and P. Demeester. Design techniques for SDH meshrestorable networks. In *Proceedings of the European Conference on Networks and Optical Communications (NOC'97), Volume 2: Core and ATM Networks*, pages 94–101, 1997.

To design low cost reliable telecommunication networks, the authors propose three algorithms: an integer linear programming algorithm (branch-and-cut-and-price), a GRASP, and a zoom-in approach that combines a genetic algorithm with deterministic optimization routines. The greedy choice of the proposed GRASP is to favor paths having lowest additional cost. The local search iteratively tries to reroute some paths in order to further decrease the overall network cost.

M.G.C. Resende. Computing approximate solutions of the maximum covering problem using GRASP. *J. of Heuristics*, 4:161–171, 1998.

A GRASP for facility location on a network with the objective of maximizing service coverage is proposed. See page 19.

L.I.P. Resende and M.G.C. Resende. A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pages 397–402, July 1999.

This paper describes a GRASP for routing permanent virtual circuits (PVC) for frame relay in telecommunications systems. The objective is to minimize PVC delays while balancing trunk loads. The greedy choice selects from the set of not yet routed PVCs the one that minimizes the delay while balancing the trunk loads. The local search procedure reroutes each PVC, one at a time, checking each time if the new route taken together with the remaining fixed routes improves the objective function.

J. Abello, P.M. Pardalos, and M.G.C. Resende. On maximum clique problems in very large graphs. In J. Abello and J. Vitter, editors, *External memory algorithms and visualization*, volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 119–130. American Mathematical Society, 1999.

GRASP is used to identify cliques and quasi-cliques in very large multi-digraphs that arise from a telephone call detail database. See page 19.

M. Armony, J.G. Klincewicz, H. Luss, and M.B. Rosenwein. Design of stacked self-healing rings using a genetic algorithm. *J. of Heuristics*, 6:85–105, 2000.

A hybrid genetic algorithm for design of stacked self-healing rings is proposed and tested. The initial population is made up of randomly generated solutions as well as solutions generated by a GRASP.

X. Liu, P.M. Pardalos, S. Rajasekaran, and M.G.C. Resende. A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P.M. Pardalos, and F. Hsu, editors, *Mobile Networks and Computing*, volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 195–201. American Mathematical Society, 2000.

A GRASP for frequency assignment is described. Local search uses simulated annealing. The construction phase uses two greedy functions. The first chooses a vertex from the set of unselected vertices with high saturation degrees. The second function is used to assign a frequency to the selected vertex. A frequency is selected from a set of permissible frequencies that contribute little additional cost to the objective function.

M. Prais and C.C. Ribeiro. Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS J. on Computing*, 12: 164–176, 2000.

This paper describes a GRASP for matrix decomposition problem arising in the context of traffic assignment in communication satellites. A geostationary communication satellite has a number of spot beam antennas covering geographically distributed areas. According to the slot switching configuration on the on-board switch, the uplink traffic received at the satellite has to be immediately sent to ground areas through a set of transponders. The slot switching configurations are determined through the solution of a time slot assignment problem, which is equivalent to the decomposition of a nonnegative traffic matrix into the sum of a family of switching mode matrices. See page 27.

A. Srinivasan, K.G. Ramakrishnan, K. Kumaram, M. Aravamudam, and S. Naqvi. Optimal design of signaling networks for Internet telephony. In *IEEE INFOCOM 2000*, volume 2, pages 707–716, March 2000.

This paper presents an approach for efficient design of a signaling network for a network of software switches supporting Internet telephony. Optimal load balancing for given demand forecast is formulated as a quadratic assignment problem, which is solved with a GRASP.

J.Y. Xu and S.Y. Chiu. Effective heuristic procedure for a field technician scheduling problem. *J. of Heuristics*, 7:495–509, 2001.

A GRASP for a field technician scheduling problem is described. See page 6.

J.G. Klincewicz. Enumeration and search procedures for a hub location problem with economies of scale. *Annals of Operations Research*, 110:107–122, 2002.

An optimal enumeration scheme, as well as other heuristics based on tabu search and GRASP are proposed for locating hubs in a communications or transportation network.

E. Amaldi, A. Capone, F. Malucelli, and F. Signori. Optimization models and algorithms for downlink umts radio planning. In *Proceedings of Wireless Communications and Networking, (WCNC 2003)*, volume 2, pages 827–831, 2003a.

To efficiently solve the NP-hard downlink base station (BS) location problem, a GRASP and a tabu search are proposed. Each GRASP construction phase starts from an empty set of active BSs and at each iteration randomly selects an available candidate set (in which to install an additional BS) from a set of available candidate set elements which yield the best improvements in the objective function. In the local search the following moves are considered to explore the solution space: removing a BS, installing a new BS, removing an existing BS, and installing a new one (swap). The output GRASP solution is used as initial solution for a tabu search algorithm.

E. Amaldi, A. Capone, and F. Malucelli. Planning umts base station location: Optimization models with power control and algorithms. *IEEE Transactions on Wireless Communications*, 2(5):939–952, 2003b.

Two randomized greedy procedures and a tabu search algorithm for the uplink (mobile to base station) direction are described. In particular, the authors devise a greedy randomized procedure and a reverse greedy randomized procedure that construct a solution, i.e. a subset of candidate sites where to activate base stations by iteratively adding and removing base stations from the current solution, respectively. The greedy criterion takes into account the number of connections they could service. The authors define swap moves that amount to installing a new base station in one of the empty sites while deleting one of the active base stations by only considering swaps between candidate sites that are relatively close to each other.

S.K. Sasi Kumar. A multi-exchange heuristic for formation of balanced disjoint rings. Master's thesis, Graduate Studies of Texas A&M University, August 2005.

A set of construction heuristics (Break-MST, Distance Method, Hybrid Method, and GRASP-Based Distance Method) and improvement heuristics (Multi-Exchange, Single Move) are proposed. Different combinations of construction and improvement heuristics are described and the quality of solution thus obtained have been compared to standard Branch and Cut Technique.

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C. Commander, C.A.S. Oliveira, P.M. Pardalos, and M.G.C. Resende. A GRASP heuristic for the cooperative communication problem in ad hoc networks. In *Proceedings of the VI Metaheuristics International Conference (MIC2005)*, pages 225–330, 2005.

A GRASP heuristic for the cooperative communication problem in ad hoc networks is described. This paper is a preliminary version of Commander et al. (2006).

C. Commander, P. Festa, C.A.S. Oliveira, P.M. Pardalos, M.G.C. Resende, and M. Tsitselis. A greedy randomized algorithm for the cooperative communication problem on ad hoc networks. In *Eighth INFORMS Telecommunications Conference*, April 2006.

Ad hoc networks are composed of a set of wireless units that can communicate without the use of a pre-established server infrastructure. Each client has the capacity of accessing network nodes that are within its reach. The problem consists in maximizing the amount of connectivity among a set of users, subject to constraints on the maximum distance traveled, as well as restrictions on what types of movement can be performed. The greedy function value of each candidate element is a measure of additional connections created by its insertion in the partial solution under construction. The local search procedure is based on a perturbation function consisting of selecting a wireless agent and rerouting.

D.V. Andrade and M.G.C. Resende. A GRASP for PBX telephone migration scheduling. In *Eighth INFORMS Telecommunication Conference*, April 2006.

The PBX telephone migration problem arises when an enterprise acquires a new PBX to replace an existing one. The objective is to assign phones to time periods such that no more than a given number of phones is assigned to any period and the total penalty is minimized. The authors propose a GRASP for approximatively solve this problem, where a solution is an assignment of phone numbers to time periods such that each time period has no more than a fixed telephone numbers assigned to it. See page 7.

L.F. Santos, R. Milagres, C.V. Albuquerque, S. Martins, and A. Plastino. A hybrid GRASP with data mining for efficient server replication for reliable multicast. In *49th Annual IEEE GLOBECOM Technical Conference*, 2006.

The authors describe how to combine GRASP with data mining techniques for efficient server replication for reliable multicast. Data mining refers to the extraction of new and potentially useful knowledge from datasets.

T.C.S. Dias, G.F. de Sousa Filho, E.M. Macambira, L.A.F. Cabral, and M.H.C. Fampa. An efficient heuristic for the ring star problem. In *Experimental Algorithms*, volume 4007 of *Lecture Notes in Computer Science*, pages 24–35. Springer, 2006.

This paper considers the Ring Star Problem where one aims to locate a simple cycle through a subset of vertices of a graph with the objective of minimizing the sum of two costs: a routing cost proportional to the length of the cycle, and an assignment cost from the vertices not in the cycle to their closest vertex on the cycle. A hybrid metaheuristic is proposed. It uses a General Variable Neighborhood Search (GVNS) to improve the quality of the solution obtained with a GRASP. Computational experiments on instances from the classical TSP library and randomly generated that the proposed hybrid metaheuristic is efficient and outperforms other available methods.

12. Biology

A number of papers on GRASP have appeared in biology and related fields.

V.I. Iorvik, E. Triantaphyllou, T.W. Liao, and S.M. Waly. Predicting muscle fatique via electromyography: A comparative study. In *Proceedings of the 25th International Conference on Computers and Industrial Engineering*, pages 277–280, March 1999.

The authors present a comparison of some state-of-the-art AI predictive and statistical techniques, including a GRASP.

D.G. Brown. *Algorithmic methods in genetic mapping*. PhD thesis, Cornell University, Ithaca, NY, USA, 2000.

This thesis is a survey of existing methods for genetic mapping problems and proposed several new algorithms. A GRASP has been also investigated. The greedy function is defined on bin length, while the local search first removes from the sample those population members that do not affect on the objective function value. Then, it tries to greedily augment the sample until no member can been removed without increasing the minimum bin size.

D.G. Brown, T.J. Vision, and S.D. Tanksley. Selecting mapping: A discrete optimization approach to select a population subset for use in a high-density genetic mapping project. *Genetics*, 155:407–420, 2000.

The authors propose a GRASP for selecting a population subset for use in a high-density genetic mapping project. At each iteration of the construction phase, they add to the partial solution one among the r unchosen population members which most improve the objective function value. The authors investigated very small sized RCL (i.e. $r = 3$ and $r = 5$). The implemented local search removes from the current solution some members and greedily includes other members, until no further improving exchange can be done.

A.A. Andreatta and C.C. Ribeiro. Heuristics for the phylogeny problem. *J. of Heuristics*, 8:429–447, 2002.

The phylogeny problem consists in finding a phylogeny with the minimum number of evolutionary steps (the so-called parsimony criterion). The authors propose different heuristic approaches to the phylogeny problem under the parsimony criterion, including a GRASP and a variable neighborhood search.

M. Andronescu and B. Rastegari. Motif-GRASP and Motif-ILS: Two new stochastic local search algorithms for motif finding. In *Mini Workshop on Stochastic Search Algorithms*, Computer Science Department, University of British Columbia, Vancouver, Canada, 2003.

A motif is a conserved pattern thought to exist in several biosequences such as DNA, RNA, and proteins. Given N biosequences S_i, $i = 1, 2, \ldots, N$ with length n_i and a number L, the problem of motif finding consists in finding a sequence M_i of length L for each biosequence such that their similarity grade is maximized. A candidate solution is represented as a set $a_1, a_2, \ldots, a_N$, where $a_k \in [1, n_k - L + 1]$, for each $k \in [1, N]$. All candidate solutions correspond to all possible combinations of a_i assignment. Several greedy functions are proposed

based on the a weight defined on the starting point of the motif. One of such greedy function is the ratio between the probability of generating a certain subsequence x from the current motif and the probability of generating x from the background. The neighborhood structure used in the local search procedure is a 1-exchange.

A.P. Reynolds, J.L. Dicks, I.N. Roberts, J.J. Wesselink, B. de la Iglesia, V. Robert, T. Boekhout, and V.J. Rayward-Smith. Algorithms for identification key generation and optimization with application to yeast identification. In *Applications of Evolutionary Computing*, volume 2611 of *Lecture Notes in Computer Science*, pages 107–118. Springer-Verlag, 2003.

For the automated creation of low cost identification keys, several algorithms are described in this paper. One of them applies the greedy randomized strategy of the GRASP framework.

C. Fried, W. Hordijk, S.J. Prohaska, C.R. Stradler, and P.F. Stradler. The footprint sorting problem. *Journal of Chemical Information and Computer Sciences*, 44 (2):332–338, 2004.

Phylogenetic footprints are short pieces of no-coding DNA sequence in genes that are conserved between evolutionary distant species. The authors of this paper show that solving the footprint sorting problem requires the solution of a minimum weight vertex feedback set problem. For solving the latter, they use the GRASP provided in Festa et al. (2001).

C.C. Ribeiro and D.S. Vianna. A GRASP/VND heuristic for the phylogeny problem using a new neighborhood structure. *International Transactions in Operational Research*, 12:325–338, 2005.

A phylogeny is a tree that relates taxonomic units, based on their similarity over a set of characters. The phylogeny problem consists in finding a phylogeny with the minimum number of evolutionary steps. The authors propose a GRASP that uses variable neighborhood descent for local search.

P. Festa. On some optimization problems in molecular biology. *Mathematical Bioscience*, 207(2):219–234, 2007.

This paper provides a detailed description of some among the most interesting molecular biology problems that can be formulated as combinatorial optimization problems and proposes a GRASP to find improved solutions for a particular class of them, known as the far from most string problem. The greedy function takes into account the contribution to the objective function achieved by selecting a particular element. In the case of the far from most string problem, it is intuitive to relate the greedy function to the occurrence of each character in a given position. To realize the local search phase the 2-exchange algorithm is used.

M.J. Hirsch, C.N. Meneses, P.M. Pardalos, M.A. Ragle, and M.G.C. Resende. A continuous GRASP to determine the relationship between drugs and adverse reactions. In O. Seref, O.E. Kundakcioglu, and P.M. Pardalos, editors, *Data mining, systems analysis, and optimization in biomedicine*, volume 953 of *AIP Conference Proceedings*, pages 106–121. Springer, 2007.

The authors formulate the drug-reaction relationship problem as a continuous optimization problem and utilize continuous GRASP (C-GRASP), a continuous global optimization heuristic, to approximately determine the relationship between drugs and adverse reactions. C-GRASP works by discretizing the domain into a uniform grid. A RCL is formed containing unfixed coordinates whose objective function value is better or equal to a threshold value computed by applying the min-max α-percentage rule. The local improvement phase can be seen as approximating the role of the gradient of the objective function.

A. Goëffon, J.-M. Richer, and J.-K. Hao. Progressive tree neighborhood applied to the maximum parsimony problem. *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, 5(1):136–145, 2008.

The authors compare a GRASP/VND with other alternative procedures. An initial solution is built with a greedy randomized algorithm and is then improved with a VND heuristic using the Multiple Subtree Pruning and Regrafting neighborhood l-SPR.

13. Automatic drawing

GRASP has been used to find approximate solutions to problems that arise in automatic graph drawing. The following papers illustrate this.

M.G.C. Resende and C.C. Ribeiro. A GRASP for graph planarization. *Networks*, 29:173–189, 1997.

GRASP is applied to a graph planarization problem. See page 18.

E. Fernández and R. Martí. GRASP for seam drawing in mosaicking of aerial photographic maps. *J. of Heuristics*, 5:181–197, 1999.

Commercial aerial photographic maps are often so large that it is necessary to produce one map from two or even more photographs. These are combined, two at a time, in a process called *mosaicking*. The objective is to make the final map appear to be the product of a single photograph. The most difficult step in the mosaicking process is *seam-drawing*. This paper proposes a GRASP for solving the seam-drawing process.

M. Laguna and R. Martí. GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS J. on Computing*, 11:44–52, 1999.

This paper develops a GRASP for the problem of minimizing straight line crossings in a 2-layer graph. The method proposed can be coupled with a path relinking strategy to search for improved outcomes. The greedy criterion of the construction phase is based on the degree of the vertices and a value based restricted candidate list is used. Each step of the improvement phase consists in selecting each vertex to be considered for a move. A probabilistic selection rule is used such that vertices with high degree are more likely to be selected first at each step of this process.

C.C. Ribeiro and M.G.C. Resende. Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software*, 25:341–352, 1999.

This paper describes Fortran subroutines implementing the GRASP for graph planarization of Resende and Ribeiro (1997).

R. Martí. Arc crossing minimization in graphs with GRASP. *IEE Transactions*, 33:913–919, 2001.

A GRASP for minimizing straight-line crossings in hierarchical graphs is presented. GRASP is shown to be faster than more complex heuristics but produces lower-quality solutions. It is not as fast as simple heuristics, but finds better-quality solutions. Hence, it is a candidate for practical implementation in graph drawing systems.

R. Martí and V. Estruch. Incremental bipartite drawing problem. *Computers and Operations Research*, 28:1287–1298, 2001.

The goal of limiting the number of arc crossings is a well accepted criterion of how well a graph is drawn. Incremental graph drawing supports interactive updates by users. A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. Computational experiments are done on 450 instances and results are compared with a branch and bound algorithm.

R. Martí and M. Laguna. Heuristics and meta-heuristics for 2-layer straight line crossing minimization. *Discrete Applied Mathematics*, 127(3):665–678, 2003.

This paper presents extensive computational experiments to compare several randomized metaheuristics for the problem of minimizing straight-line crossings in a 2-layer graph. Extensive computational results are presented using 12 heuristics and two meta-heuristics. On dense graphs, a tabu search meta-heuristic does best, with GRASP a close second. On low-density graphs, GRASP outperforms all other approaches. See page 23.

G.L. Cravo, G.M. Ribeiro, and L.A. Nogueira Lorena. A greedy randomized adaptive search procedure for the point-feature cartographic label placement. *Computers and Geosciences*, 34(4):373–386, 2008.

This paper describes a GRASP based on an underlying conflict graph for the point-feature cartographic label placement problem. The construction phase uses a cardinality based RCL and a greedy function defined on vertex degree, since it is a measure of labels in conflict. Improving swap moves are attempted in the local search.

14. Electrical power systems

GRASP has been applied to a transmission expansion problem in electrical power systems. The three papers below describe this application.

S. Binato, G.C. Oliveira, and J.L. Araújo. A greedy randomized adaptive search procedure for transmission expansion planning. *IEEE Transactions on Power Systems*, 16:247–253, 2001.

This paper presents a GRASP for a long term transmission expansion planning problem. The greedy function is to minimize the load curtailment required to eliminate all operational violations. The local search phase is based on circuit exchanges, i.e. the procedure exchanges selected additions with unselected additions.

S. Binato and G.C. Oliveira. A Reactive GRASP for transmission network expansion planning. In C.C. Ribeiro and P. Hansen, editors, *Essays and surveys on metaheuristics*, pages 81–100. Kluwer Academic Publishers, 2002.

The GRASP previously proposed by Binato, Oliveira, and Araújo (1998) for solving a transmission network expansion problem is enhanced with the reactive scheme of Prais and Ribeiro (2000) to self-adjust the GRASP RCL parameter α. The authors also propose to apply a bias distribution function of Bresina (1996) to bias the random greedy construction phase towards the most promising variables.

H. Faria Jr., S. Binato M.G.C. Resende, and D.J. Falcão. Power transmission network design by a greedy randomized adaptive path relinking approach. *IEEE Transactions on Power Systems*, 20(1):43–49, 2005.

This paper proposes a Greedy Randomized Adaptive Path Relinking (GRAPR) to solve the static power transmission network design problem, which consists in choosing, from a pre-defined set of candidate circuits, those that should be built in order to minimize the investment and operational costs, and to supply the forecasted demand along a planning horizon.

15. VLSI design

GRASP has been applied to circuit partitioning problems in VLSI design, as indicated by the papers below.

S. Areibi and A. Vannelli. A GRASP clustering technique for circuit partitioning. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 711–724. American Mathematical Society, 1997.

This paper adapts a basic node interchange scheme for solving the circuit partitioning problem and develops a clustering technique that uses GRASP to generate clusters of moderate sizes. The number of clusters is predetermined as a function of the number of partitions required. Initially, the heuristic reads the circuit description and resizes the blocks to be used by GRASP, which utilizes only the construction phase to generate the number of required clusters. The GRASP construction phase is followed by a post-processing stage, in which a simple dynamic hill climbing algorithm is used as local search to improve the initial solution generated.

S.M. Areibi. GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, May 1999.

A GRASP for VLSI circuit partitioning is proposed.

16. Miscellaneous applications

A number of papers on GRASP have appeared in fields as diverse as consumer choice theory and multitarget multisensor tracking. The following papers could not be classified into one of the previous categories and are grouped as miscellaneous applications.

T.A. Feo and M.G.C. Resende. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67–71, 1989.

GRASP is proposed for set covering. A value based restricted candidate list is used to construct solutions of difficult set covering problems that arise in computing the 1-width of the incidence matrix of Steiner triple systems. The local search is based on the elimination of redundant elements in the cover. This is the first paper on what later would be known as GRASP.

J.B. Ghosh. Computational aspects of the maximum diversity problem. *Operations Research Letters*, 19:175–181, 1996.

This paper addresses two variants of the maximum diversity problem. This problem arises when m elements are to be selected from an n-element population based on inter-element distances. Using a reduction from the vertex cover problem, the authors prove that the problem is NP-hard and propose a GRASP for approximately solving it. The neighborhood of a solution is the set of all solutions that can be obtained by replacing an element in the incumbent solution with one element that is not in it.

K. Holmqvist, A. Migdalas, and P.M. Pardalos. A GRASP algorithm for the single source uncapacitated minimum concave-cost network flow problem. In P.M. Pardalos and D.-Z. Du, editors, *Network design: Connectivity and facilities location*, volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 131–142. American Mathematical Society, 1998.

This paper is concerned with the single source uncapacitated version of the minimum concave-cost network flow problem, that requires establishing a minimum cost flow through a given network from a single source to a set of sinks. The authors propose a GRASP that can be trivially implemented on parallel processors. The construction phase iteratively builds a tree starting from the source node. The elements of the restricted candidate list are end nodes of arcs with a cost close to the best one. The local search phase applies either of the two local search variants proposed by Guisewite and Pardalos (1990).

R.A. Murphey, P.M. Pardalos, and L.S. Pitsoulis. A greedy randomized adaptive search procedure for the multitarget multisensor tracking problem. In P.M. Pardalos and D.-Z. Du, editors, *Network design: Connectivity and facilities location*, volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 277–301. American Mathematical Society, 1998.

A GRASP is presented for approximately solving the multitarget multisensor tracking problem, which can be interpreted as a collection of multidimensional assignment problems. Since the objective is to select a target hypothesis and partition of the measurements that is most likely to occur, a likelihood cost function and partitioning constraint set are developed. The GRASP construction phase creates the restricted candidate list containing the most likely to occur (lower cost) tuples. The local search explores all 2-exchange permutations.

R. Colomé and D. Serra. Consumer choice in competitive location models: Formulations and heuristics. *Papers in Regional Science*, 80:439–464, 2001.

This paper studies the importance of customer behavior with respect to distance or transportation costs in the optimality of locations obtained by traditional state-of-art competitive location models. The authors propose four models to represent the problem and propose a hybrid metaheuristic to solve it. The proposed method consists of two phases. In the first phase, a good initial solution is found by applying a GRASP procedure, while in the second phase the previous solution found is improved by applying a tabu search heuristic.

P.L. Hammer and D.J. Rader, Jr. Maximally disjoint solutions of the set covering problem. *J. of Heuristics*, 7:131–144, 2001.

This paper describes the problem of finding two solutions of a set covering problem that have a minimum number of common variables. It is proved that this problem is NP-complete and three heuristics are proposed for solving it. Two of these algorithms find the solutions sequentially. One of them is a GRASP. The third algorithm finds solutions simultaneously. Each proposed heuristic is a variant of the standard greedy method for set covering problems, whose greedy choice favors unselected variables that maximize the number of uncovered rows that become covered. To reduce the overlap of any pair of solutions, a local search algorithm is used that swaps at each iteration parts of the solution found with a set of variables not in it.

M.C. Medeiros, M.G.C. Resende, and A. Veiga. Piecewise linear time series estimation with GRASP. *Computational Optimization and Applications*, 19:127–144, 2001.

This paper describes a GRASP to build piecewise linear statistical models with multivariate thresholds. The construction phase consists of sequentially choosing hyperplanes until the maximum number of hyperplanes is reached. The greedy function orders the possible hyperplanes with respect to the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic.

M.C. Medeiros, A. Veiga, and M.G.C. Resende. A combinatorial approach to piecewise linear time series analysis. *J. of Computational and Graphical Statistics*, 11: 236–258, 2002.

This paper presents a new approach to modeling threshold processes, based on a linear model with time-varying parameters. The authors show that this formulation is closely related to the self-exciting threshold autoregressive models (SETAR) with the advantage that it incorporates linear multivariate thresholds. A GRASP is proposed to estimate the parameters of the model. The greedy choice takes into account the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic.

E. Pinãna, I. Plana, V. Campos, and R. Martí. GRASP and path relinking for the matrix bandwidth minimization. *European J. of Operational Research*, 153(1): 200–210, 2004.

This paper proposes a GRASP for the problem of reducing the bandwidth of a matrix. The GRASP can apply five different construction methods and is combined with a path-relinking strategy as intensification to search for improved outcomes.

X. Delorme, X. Gandibleux, and J. Rodriguez. GRASP for set packing problems. *European J. of Operational Research*, 153(3):564–580, 2004.

A GRASP is described for the set packing problem. The proposed framework uses a self-tuning procedure (reactive GRASP), an intensification procedure (using path-relinking), and a procedure involving the diversification of the selection (using a learning process). A set of test problem instances includes real-world railway planning instances, never solved before by means of metaheuristics.

G.C. Silva, L.S. Ochi, and S.L. Martins. Experimental comparison of greedy randomized adaptive search procedures for the maximum diversity problem. In *Proceedings of the IV Workshop on Efficient and Experimental Algorithms (WEA2004)*, volume 3059 of *Lecture Notes in Computer Science*, pages 498–512, 2004.

In this paper, several new GRASPs are described and compared. They are based on three construction algorithms and two other techniques: reactive GRASP and filtering of constructed solutions. The authors measure the diversity between any two elements by the Euclidean distance and coherently define the GRASP greedy function. Two local search procedures are described. The first one simply defines a neighborhood as a set of all solutions obtained by replacing an element in the solution by another that does not belong to the set associated with the solution itself. The second local search procedure is a variable neighborhood search based on a new neighborhood defined as the set of all solutions obtained by replacing two elements in the solution by another two that are not in the solution.

L.F. Santos, M.H. Ribeiro, A. Plastino, and S.L. Martins. A hybrid GRASP with data mining for the maximum diversity problem. In *Hybrid Metaheuristics*, volume 3636 of *Lecture Notes in Computer Science*, pages 116–127. Springer, 2005.

The authors describe how to combine GRASP with data mining techniques to solve the maximum diversity problem. Data mining refers to the extraction of new and potentially useful knowledge from datasets in terms of patterns and rules.

A.M. Alvarez, J.L. Gonzalez-Velarde, and K. De Alba. GRASP embedded scatter search for the multicommodity capacitated network design problem. *J. of Heuristics*, 11:233–257, 205.

Two strategies are proposed for GRASP: a traditional approach and a memory based technique. The local search procedure consists in sorting the paths to try to obtain a better distribution with lower cost.

M.J. Hirsch, P.M. Pardalos, and M.G.C. Resende. Sensor registration in a sensor network by continuous GRASP. *IEEE Military Communications Conference (MILCOM 2006)*, 2006.

Sensor registration can be seen as the process of removing (accounting for) non-random errors, or biases, in the sensor data. The authors describe a continuous GRASP, when not all data is seen by all sensors, and the correspondence of data seen by the different sensors is not known a priori. The authors test their algorithm on test problems from the literature.

A. Duarte and R. Martí. Tabu search and GRASP for the maximum diversity problem. *European J. of Operational Research*, 178(1):71–84, 2007.

In this paper, the authors propose a new hybrid heuristic based on the tabu search methodology and incorporating memory structures for both construction and improvement.

M.J. Hirsch, P.M. Pardalos, and M.G.C. Resende. Solving systems of nonlinear equations with continuous GRASP. *Nonlinear Analysis: Real World Applications*, 2008. Published online 12 March 2008. DOI: 10.1016/j.nonrwa.2008.03.006.

A method for finding all roots of a system of nonlinear equations is described. It uses continuous GRASP (C-GRASP). Given a nonlinear system, the authors solve a corresponding adaptively modified global optimization problem multiple times, each time using C-GRASP, with areas of repulsion around roots that have already been found. The heuristic makes no use of derivative information.

M.G.C. Resende, R. Martí, M. Gallego, and A. Duarte. GRASP and path relinking for the Max-Min diversity problem. *Computers and Operations Research*, 2008.

The Max-Min Diversity Problem (MMDP) consists in selecting a subset of elements from a given set in such a way that the diversity among the selected elements is maximized. It has applications in the social and biological sciences. This paper proposes a GRASP with path relinking heuristic for finding approximate solutions to the Max-Min diversity problem. To appear.

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