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GAS DISTRIBUTION NETWORK OPTIMIZATION BY GENETIC ALGORITHM

Berge Djebedjian

Associate Professor
Mechanical Power Engineering Dept.
Faculty of Engineering
Mansoura University
Mansoura, Egypt

E-mail: bergedje@mans.edu.eg

Islam Shahin

Process Engineer
VWSE
Cairo, Egypt

E-mail: islam1005@hotmail.com

Mohamed El-Naggar

Assistant Professor
Mechanical Power Engineering Dept.
Faculty of Engineering
Mansoura University
Mansoura, Egypt

E-mail: naggarm@mans.edu.eg

ABSTRACT

Natural gas is increasingly being used as a pure energy source. So it's valuable to reduce its total cost to be affordable for individual customer. In the past, interest was focused on efficient pipe network analysis method regardless of network cost; so many researches could be found for methods developed for network analysis. In contrast, very little researches could be developed to optimize the design of distribution networks. This, of course, reflects on software developed for the two purposes: the analysis and simulation of gas networks and the optimization purpose.

The aim of this work is to develop a computer code that simulate and optimize gas distribution networks at all pressure ranges, i.e. low, medium and high pressure networks. The aim is to reduce the network diameter sizes to a minimum value while fulfilling the constraints of maximum link velocity and minimum node pressure.

In this code, the analysis of gas distribution networks was based on the gradient algorithm which had never presented in gas networks before. In this study, the algorithm was presented and gave efficient analysis for any gas network, i.e. at all pressure ranges, at the least time any algorithm can record. Optimization of gas distribution networks was presented by the genetic algorithm. The code was applied on low pressure gas distribution networks and proved its efficiency and robustness. Therefore, this code was found to be useful at the stage of gas distribution networks design.

KEYWORDS:

Gas Network, Optimization, Genetic Algorithm, Cost, Steady-state.

INTRODUCTION

A gas pipeline network is classified as: transmission network (to transmit gas at high pressure from coastal supplies to regional demand points) and distribution network (to distribute gas to consumers at low pressure from the regional demand points). The distribution network differs from the transmission one in its small-diameter pipes, its simplicity as there are no valves, compressors or nozzles, and its operation at low and medium pressures. The main interest in this study is focused on distribution networks.

The simulation and analysis of gas networks has focused on the development of efficient algorithms for the analysis of flow and it has been widely studied in the literature, Osiadacz [1] and Osiadacz and Górecki [2] and Herrán-González et al. [3].

The gas network optimization can be divided into two main categories: the optimization of gas transmission pipelines and the optimization of gas distribution networks. The researches mainly focused on gas transmission pipelines optimization due to the high cost of equipment (compressor stations, reduction-valves stations and pipelines), and the low capability of computers to optimize gas distribution networks.

The optimization of gas networks means searching, according to a certain objective function, for optimal design parameters, optimal structures for development or optimal parameters for operation of networks, (Osiadacz [4]).

The cost of low and medium pressure networks depends mainly on network capital cost, whereas the cost of high pressure network is determined mainly by mode of operation of compressors. This operating cost of running compressor stations represents 25% to 50% of the total company's operating budget (Osiadacz [4]) which consumes over 3% to 5% of total gas transported, (Wu et al. [5]).

The optimization of gas transmission pipelines have been in many ways: in steady-state or transient, Steinbach [6]. Mainly, we emphasize on steady-state optimization. Larson and Wong [7] determined the steady-state optimal operating conditions of a straight natural pipeline with compressors in series using dynamic programming to find the optimal suction and discharge pressures. The length and diameter of the pipeline segment were assumed to be constant because of limitations of dynamic programming. Martch and McCall [8] modified the problem by adding branches to the pipeline segments. However, the transmission network was predetermined because of the limitations of the optimization technique used.

O'Neill et al. [9] introduced the problem of a transmission pipeline through compressor stations, including the optimization of the operation scheme. They used Successive Linear Programming (SLP) to optimize the problem.

Olorunniwo and Jensen [10] provided further breakthrough by optimizing a gas transmission network including the type and location of pipelines and compressor stations. Edgar and Himmelblau [11] simplified the problem addressed in [10] to make sure that the various factors involved in the design are clear. They assumed the gas quantity to be transferred along with the suction and discharge pressures to be given in the problem statement. They optimized the number of compressor stations, the length of pipeline segments between the compressors stations, the diameters of the pipeline segments and the suction and discharge pressures at each station. They considered the minimization of the total cost of operation per year including the capital cost in their objective function against which the above parameters were optimized.

Edgar and Himmelblau [11] also considered two possible scenarios: (1) the capital cost of the compressor stations is linear function of the horsepower, and (2) the capital cost of the compressor stations is linear function of the horsepower with a fixed capital outlay for zero horsepower.

Osiadacz [4] published a paper for the operation optimization for high pressure transmission lines.

De Wolf and Smeers [12] used the problem of O'Neill et al. [9] using piecewise linear programming (PLP) approach. De Wolf and Smeers [13] continued the problem and represented the piecewise linear approximations as "special ordered sets of type 2" so that the piecewise linear problem could be globally solved by a mixed-integer programming code. A nonlinear programming code was also applied directly to the nonlinear exact formulation to determine a local optimum. These two alternatives were also compared to PLP. They appeared, when converging, much slower than the PLP method.

Ríos-Mercado et al. [14] proposed a reduction technique for minimizing the fuel consumption incurred by compressor stations in steady-state natural gas transmission networks. The justification of the technique was based on a novel combination of graph theory and nonlinear functional analysis. The reduction technique can decrease the problem size by more than an order of magnitude in practice, without disrupting its mathematical structure.

Babu et al. [15] applied the differential evolution for the optimal design of gas transmission network. The differential evolution was successfully applied for this complex and highly non-linear problem. The results obtained were compared with those of nonlinear programming technique and branch and bound algorithm. The differential evolution was able to find an optimal solution satisfying all the constraints and in less computational time to converge when compared to the existing techniques.

Pietrasz et al. [16] studied the problem of reinforcing regional gas transmission networks to cope with the forecasted demand for natural gas. The objective function to minimize was the sum of reinforcement costs. They provided three optimization methods based on topological decomposition techniques (into tree-like sub networks), search space reduction (continuous relaxation, truncated branch and bound) or evolutionary algorithms. The method based on truncated branch and bound leaded to a solution which was locally optimal in the neighborhood of the relaxed solution given by the continuous relaxation. The approach based on genetic algorithms allowed specifying a computation time limit while providing a solution whose quality was equivalent to the one given by the branch and bound. Dynamic programming yielded the best results on single-source tree-like networks with acceptable computational times.

André et al. [17] presented techniques for solving the problem of minimizing investment costs on an existing gas transportation network by finding first the optimal location of pipeline segments to be reinforced and, second, the optimal sizes (among a discrete commercial list of diameters) under the constraint of satisfaction of demands with high enough pressure for all users. The new heuristics was based on a two phases approach: solving a continuous relaxation of the problem and choosing discrete values of diameters only among the set of pipes that was reinforced in the continuous relaxation. A Branch and Bound scheme was then applied to a limited number of values in order to generate good solutions with reasonable computational effort on real-world applications.

Chebouba et al. [18] proposed an ant colony optimization algorithm for operations of steady flow gas pipeline. The decisions variables were chosen to be the operating turbocompressor number and the discharge pressure for each compressing station. The results were compared with those obtained by employing dynamic programming method showing that the ant colony optimization is an interesting way for the gas pipeline operation optimization.

For the optimization of gas distribution networks, the improved capabilities of personal computers in the beginning of 1980s and the developed optimization algorithms encouraged the researches concerned with optimization of gas distribution networks.

Osiadacz et al. [2] represented the optimization of gas networks for medium and low pressure networks using dynamic programming for sizing the pipe network diameters.

De Mélo Duarte et al. [19] proposed and applied a tabu search algorithm for the optimization of constrained gas distribution networks to find the least cost combination of diameters for the pipes, satisfying the constraints related to minimum pressure requirements and upstream pipe conditions. The results of the proposed algorithm were compared with the results of a genetic algorithm and two other versions of tabu search algorithms. The results were very promising, regarding both quality of solutions and computational time.

Wu et al. [20] established a mathematical optimization model of the problem of minimizing the cost of pipelines incurred by driving the gas in a non-linear distribute network under steady-state assumptions. They presented a global approach, which was based on the GOP primal-relaxed dual decomposition method to the optimization model. The introduction of variables and adding of constraints converted the primal problem to a quadratic model.

The previous literature review reveals that the majority of researches focused on the optimization of gas transportation networks.

The main original contribution proposed in this paper is the application of the gradient algorithm of Todini and Pilati [21] for analyzing gas networks in linkage with the genetic algorithm (Holland [22] and Goldberg [23]) for optimization. The approach is applied to a case study of gas distribution network proposed by Osiadacz and Górecki [2] to demonstrate its efficiency and effectiveness.

PROPOSED STUDY

The main study was concerned with developing a computer program to simulate, analyze, solve and optimize low and medium pressure gas distribution networks.

Flow Equation

The pressure drop equations used in the design of gas pipelines have several versions, Osiadacz [1] and Coelho and Pinho [24]. The gas flow equations are divided into three categories corresponding to its pressure region: low-pressure, medium-pressure and high-pressure. In this study, the Pole's equation for low pressure region (0-75 mbar gauge) was used, [1]:

$$p_1 - p_2 = \left(11.7 \times 10^3 \frac{L}{D^5}\right) Q^2 \tag{1}$$

where the nodal pressure p is in mbar, D the diameter in mm, L the link length in m, and the flow rate Q in m^3/h.

Modeling

The method proposed by Todini and Pilati [21] is for water distribution networks. In this study, the application of this method was applied for gas networks and it constitutes of, [25]:

$$\mathbf{H}_{t+1} = -\left[\mathbf{A}_{21}(\mathbf{N}\mathbf{A}_{11})^{-1}\mathbf{A}_{12}\right]^{-1} \cdot \left[\mathbf{A}_{21}(\mathbf{N}\mathbf{A}_{11})^{-1}(\mathbf{A}_{11}\mathbf{Q}_t + \mathbf{A}_{10}\mathbf{H}_o) - (\mathbf{A}_{21}\mathbf{Q}_t - \mathbf{q}_o)\right] \tag{2}$$

$$\mathbf{Q}_{t+1} = (\mathbf{I} - \mathbf{N}^{-1})^{-1}\mathbf{Q}_t - \left[\mathbf{N}^{-1}\mathbf{A}_{11}^{-1}(\mathbf{A}_{12}\mathbf{H}_{t+1} + \mathbf{A}_{10}\mathbf{H}_o)\right] \tag{3}$$

where:

- l : Number of links
- n : Number of demand junctions
- s : Number of source nodes
- $\mathbf{A}_{10}$: Source nodes links matrix; Dimension: (l x s)
- $\mathbf{A}_{11}$: Matrix of pressure losses; Dimension: (l x l)
- $\mathbf{A}_{12}$: Demand junctions links matrix; Dimension: (l x n)
- $\mathbf{A}_{21}$: Transpose of ($\mathbf{A}_{12}$)
- $\mathbf{H}_{t+1}$: Unknown node pressure head matrix in the present iteration t + 1; its dimension is (n x l)
- $\mathbf{H}_o$: Source nodes pressure head matrix, Dimension: (s x 1)
- $\mathbf{I}$: Identity matrix
- $\mathbf{N}$: Matrix of flow rate exponent related to its flow-equation in its pressure range; Dimension: (l x l)
- $\mathbf{Q}_t$: Previous t iteration or initial flow rate matrix; Dimension (l x 1)
- $\mathbf{Q}_{t+1}$: Unknown pipe flow matrix in the present iteration t + 1; Dimension: (l x 1)
- $\mathbf{q}_o$: Flow rates demands at demand junctions; Dimension: (n x 1)

The flow rates in pipes are to be assumed initially by:

$$Q_i = \frac{\pi}{4} D_i^2 v_i \qquad i = 1,\ldots,l \tag{4}$$

where $v_i = 1$ m/s for initiation.

Data Structure

The relation between links and nodes is the most important issue to be established firmly. It's represented through the two matrices $\mathbf{A}_{12}$ and $\mathbf{A}_{10}$, so that $\mathbf{A}_{12}$ matrix represents the relationship between demand junctions (matrix rows), and all links (matrix columns), through (1,–1 and 0) numbers. It is "0" if there's no attachment between a link and a junction. It is "1" (or "−1") if the link is attached to a definite junction and is subjected to the junction direction (or the opposite direction).

After formulation, iteration starts to the model which doesn't stop until corrections in flow rates don't exceed a specified accuracy, or stops with error when exceeds the maximum trials.

Optimization

The genetic algorithm (GA), (Holland [22] and Goldberg [23]), was used as an optimization tool. In contrast of some other algorithms, the GA approach does not require certain restrictive conditions as continuity and differentiability to the second order. The major advantages of genetic algorithms are their flexibility and robustness as a global search method.

Gas distribution network optimization aims at finding the optimal pipe diameters in the network for a given layout and demand requirements to minimize the cost. The optimal pipe sizes are selected to satisfy the conservations of mass and energy, and the constraints (e.g. operating and design constraints). The optimization of a network works through objective function under some constraints. In this study, the gas piping design was concerned. The objective function was the cost and the constraints were the minimum nodal pressures and the maximum velocity in pipes to obtain as, [26].

Cost:

$$C_T = \sum_{i=1}^{l} c_i(D_i) \cdot L_i \tag{5}$$

Constraints:

$$D_{\min} \le D_i \le D_{\max} \qquad i = 1,\ldots,l \tag{6}$$

$$v_i \le v_{\max} \qquad i = 1,\ldots,l \tag{7}$$

$$p_j \ge p_{\min} \qquad j = 1,\ldots,n \tag{8}$$

Objective function:

$$F = C_T + C_{Pv} + C_{Pp} \tag{9}$$

Penalty functions:

$$\begin{aligned} C_{Pv} &= 0 && \text{if } v_i \le v_{\max} \\ C_{Pv} &= \frac{C_T}{l}\sum_{i=1}^{l}(v_i^2 - v_{\max}^2) && \text{if } v_i > v_{\max} \end{aligned} \tag{10}$$

$$\begin{aligned} C_{Pp} &= 0 && \text{if } p_j \ge p_{\min} \\ C_{Pp} &= \frac{C_T}{n}\sum_{j=1}^{n}(p_{\min} - p_j) && \text{if } p_j < p_{\min} \end{aligned} \tag{11}$$

where:

$c_i(D_i)$: Cost of link i with diameter D_i

C_T : Total cost

C_{Pp} : Penalty cost for nodal pressure

C_{Pv} : Penalty cost for velocity

D_i : Diameter of Link i

$D_{\max}$: Maximum diameter

$D_{\min}$: Minimum diameter

F : Objective function

L_i : Length of link i

l : Number of links

n : Number of nodes

p_j : Actual gas pressure at junction j

$p_{\min}$: Minimum gas pressure at junctions (18 mbar)

v_i : Actual gas velocity in link i

$v_{\max}$: Maximum gas velocity in links (10 m/s)

Code Arrangement

The ***GAGAGas.net*** software (Gradient Algorithm Genetic Algorithm Gas network) was written in C/C++ language. The developed code passes through three main stages: (a) Network graph analysis, (b) Network analysis and simulation, and (c) Network optimization. The network graph includes the nodes and links identification codes and correctness verification of given input data of network. Network analysis and simulation, in which the simulation of pressure and flow rates through the network, were formulated through gradient algorithm (Todini and Pilati [21]) and using Equations (2) and (3). In the two previous stages many programmed functions were borrowed from EPANET 2.0 (Rossman [27]) and "*Numerical Recipes in C*" (Press et al. [28]). Network optimization applied the Real-Coded Genetic Algorithm (Deb [29]) to search randomly in optimized space and then converge to better solutions.

The flow diagram of the proposed software is consisted of two parts; the first is the simulation and analysis engine and the other is the optimization engine. Figure (1) shows the simulation and analysis engine flow diagram; it contains the simulation, arranging matrices and network analysis. Figure (2) illustrates the optimization engine and includes the initialization, selection procedure, new generation, crossover, mutation, fitness and reporting.

A brief description of the steps of using GA for pipe network optimization is as follows:

1. ***Initiation of the first generation.*** The initial generation is adapted from the initial design provided to be optimized; now the network data is all available.
2. ***Simulation.*** The second step is to simulate this available network data so that getting the output data of velocities in links and pressures at nodes.
3. ***Costing.*** Evaluating the cost depends on two main points; the first is the actual cost of the available piping arrangement given as size-cost table. The second is to evaluate the penalties that happened due to the deviation away from constraints by velocities in links or pressures at nodes.
4. ***Fitness.*** The fitness of the coded string is taken as some function of the total network cost. For each proposed pipe network in the current population, it can be computed as the inverse or the negative value of the total network cost.
5. ***Selection.*** GA uses some techniques for selection from the population the best individuals to pass through; in this study *Tournament* technique is used.
6. ***Crossover and mutation.*** Crossover happens to the most of the selected generation in probability of 0.6 as some of good individuals may pass without making any operations on them. Very low mutation probability of about 0.04 is used.
7. ***Successive generations.*** Now there is a new generation to start again from the second step. This procedure is continued till the stopping criterion, which is the total number of generations, is met.

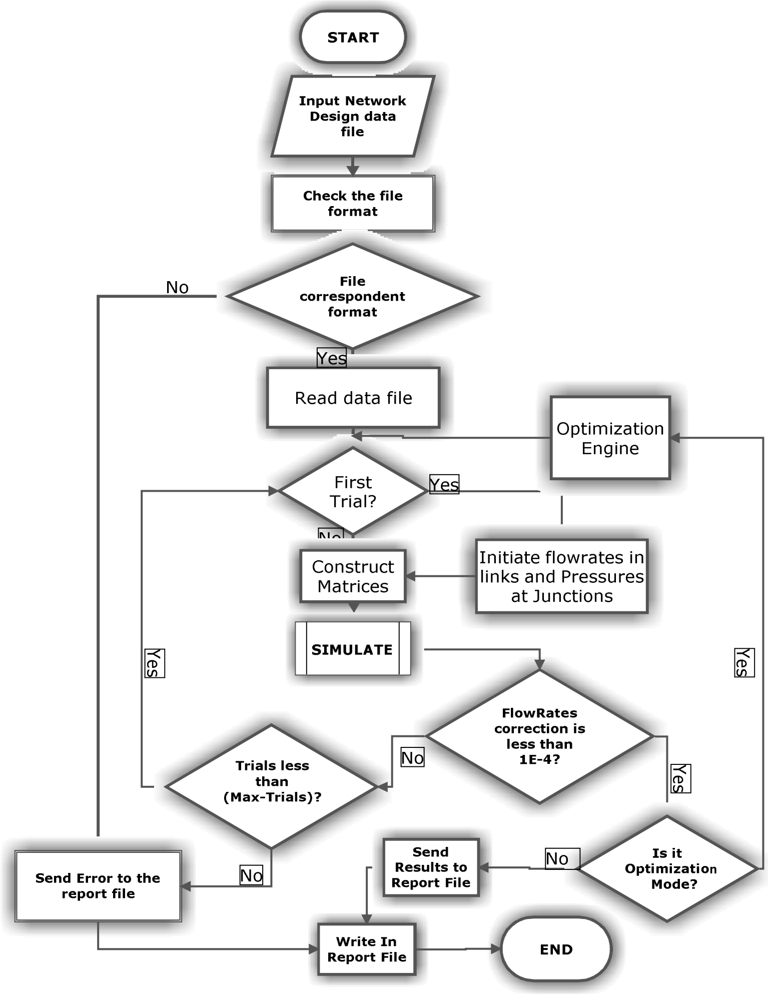

Fig. 1. Flow diagram for the simulation and analysis engine

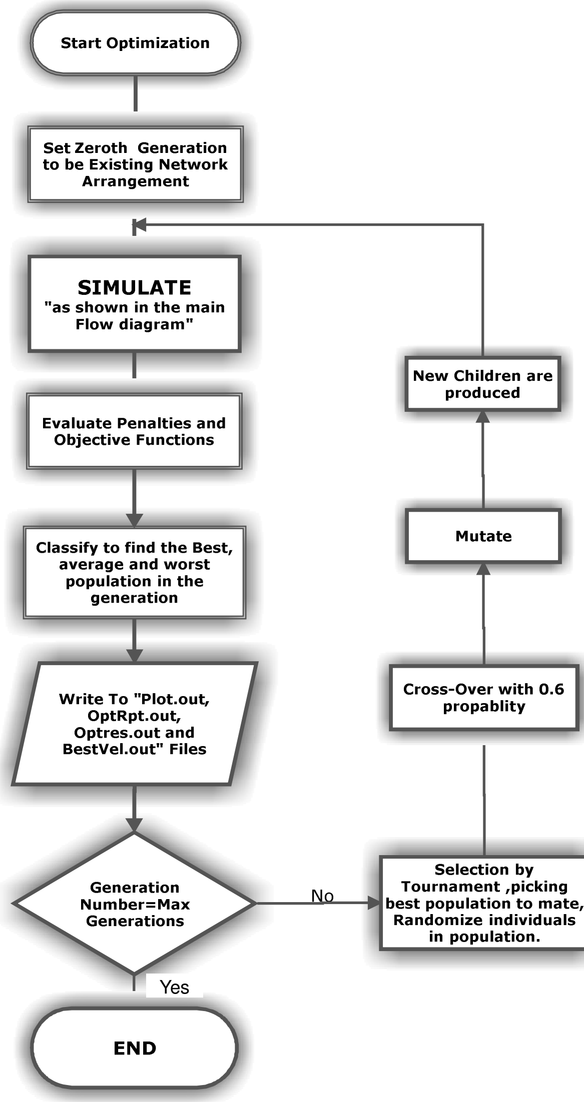

Fig. 2. Flow diagram for the optimization engine

CASE STUDY: LOW-PRESSURE DISTRIBUTION GAS NETWORK

The distribution gas network comprises 81 junctions, 108 branches and two sources of 50 mbar in pressure, Figure 3. The main case study and the parameters of the genetic algorithm are given in Table 1. This case study is given by Osiadacz and Górecki [2]. The data of nodes (demand or branch nodes) and source nodes are given in Tables A1 and A2, respectively, in the Appendix. For source node, the pressure of the network is provided, while for demand nodes the demands flow rates to the nodes are provided and the branching point have zero demand flow rate. The total demand is 1482.56 m^3/hr. The data of branches are given in Table A3.

Osiadacz and Górecki [2] simulated this case by SimNet software and optimized it by nonlinear programming.

The gas network constraints were: minimum gas pressure at junction was 18 mbar and maximum gas velocity in link was 10 m/s. The Pole's equation as pressure losses formula was used in the low-pressure region.

The cost of pipe was estimated by the relationship, [2]: $C = 2.05\,L\,D^{1.3}$ in which C is the cost in Zlotys (US$ = 2.36 Zloty, [2]), L the length in meters and D the diameter in inches. They used the continuous optimization method that supposes that any size of diameter is possible; therefore the resulted set of diameters is corrected to closest available diameter sizes. In this study, the available pipe diameters mentioned in [2] were used and the corresponding costs per meter length are mentioned in Table A4.

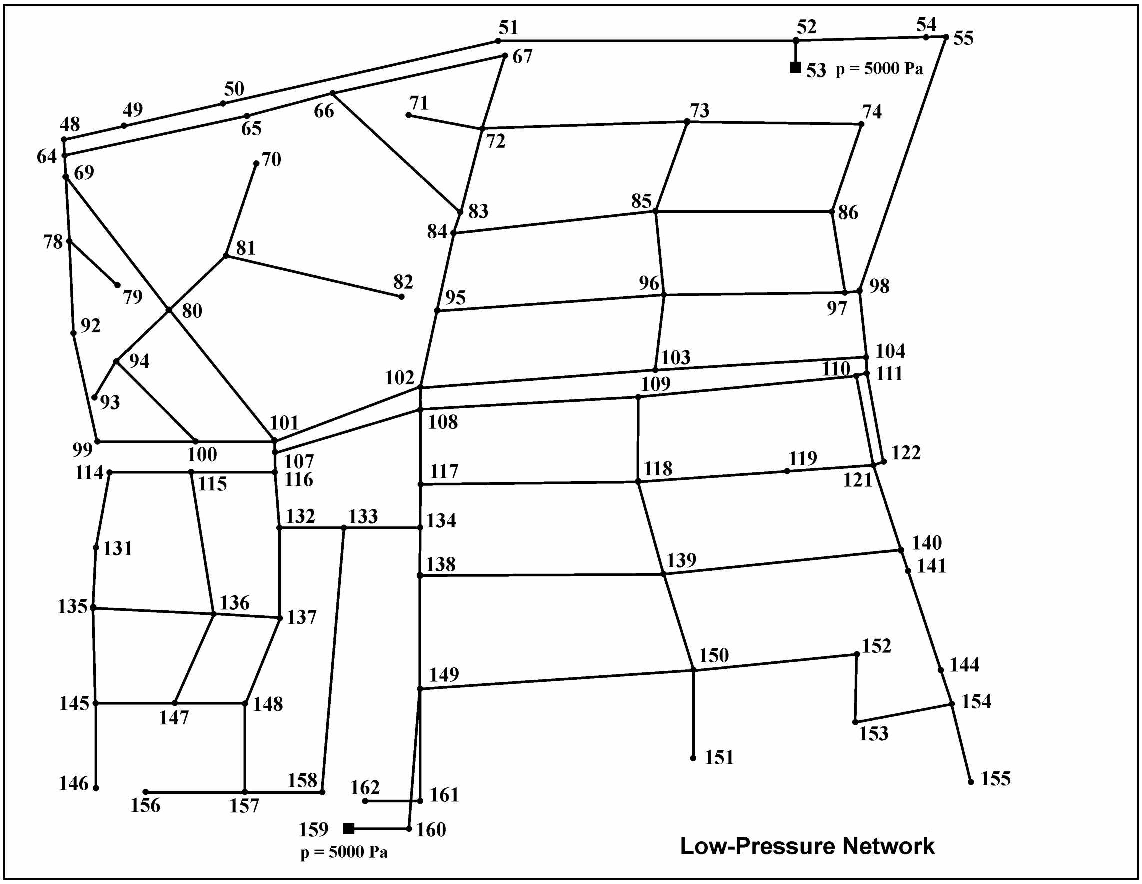

Fig. 3. Low-pressure gas distribution network layout, Osiadacz and Górecki [2]

Table 1. Case study and genetic algorithm data

Case Study		Accuracy	0.0001 m^3/hr
Pressure Range	Low Pressure	Maximum No. of Trials	40
Number of Sources	2	**Constraints**	
Number of Branches	108	Maximum Flow Velocity	10 m/s
Number of Junctions	81	Minimum Node Pressure	18 mbar
Source Pressure	50 mbar	**Genetic Algorithm**	
Fluid	Natural Gas*	Population Size	50
Temperature	Ambient (25°C)	Total No. of Generations	500
Simulation		Crossover Probability	0.8
Pressure Losses Formula	Pole’s Equation	Mutation Probability (Real)	0.06
Gas Network Analysis	Gradient Algorithm	Number of Real-Coded Variables	108

** As it is rich Methane content, the natural gas assumed to be Methane.*

RESULTS AND DISCUSSION

The constancy of the proposed code was proved by the low-pressure gas distribution network case study of Osiadacz and Górecki [2]. The simulation results of the proposed approach were compared to the results obtained by SimNet, Osiadacz and Górecki [2] and they were almost the same as there was units' conversion between the results of Osiadacz and Górecki [2] and the results of proposed study.

The ***GAGAGas.net*** software released the network analysis for the already designed network, and the optimization data for the network which fulfilled the required constraints. The simulation and optimization analyses are discussed in the following sections.

Simulation Analysis

The simulation analysis results illustrated in Tables A5 and A6 for the nodes and branches respectively, show that many flow velocities in links are very low that indicates using larger pipe sizes than suitable. This made the total capital cost of the network to be expensive, so optimizing this network sizing was required.

Optimization Analysis

A comparison between the existing design and optimized network was held, finding out how the optimization action was marking an outstanding role in reducing the network cost and assigning the most suitable diameter. The comparison between the nodal pressures and flow velocities before and after optimization are given in Tables A7 and A8, respectively. The optimal diameters obtained from the present study and that of Osiadacz and Górecki [2] study are given in Table A9. In the following paragraphs, explanatory graphs will display the results in previous sections relative to optimization effect.

Figure 4 represents the comparison between the branch sizes before and after optimization. The objective function of the optimization was the cost which depends on the link diameter; i.e. the aim of optimization was decreasing the diameter to minimize the cost. The optimal set of diameters has small diameters compared to the original ones.

Figure 5 shows the nodal pressures before and after optimization and the minimum required nodal pressure. For the designed network, all the pressures have high pressures above the nodal pressure constraint as the designed branch diameters are big and accordingly, the pressure losses are small. The small link diameters obtained from optimization decreased the nodal pressure because of the increase in pressure losses as observed from the figure.

Figure 6 illustrates the flow velocities in each link in the designed and optimum networks. Also, the maximum velocity constraint is shown. Because of the change in links-size set resulted from optimization, the flow velocities in all links were increased but they were below the maximum velocity constraint. As can be seen from Figures 5 and 6, all the constraints were fulfilled with this optimal gas distribution network.

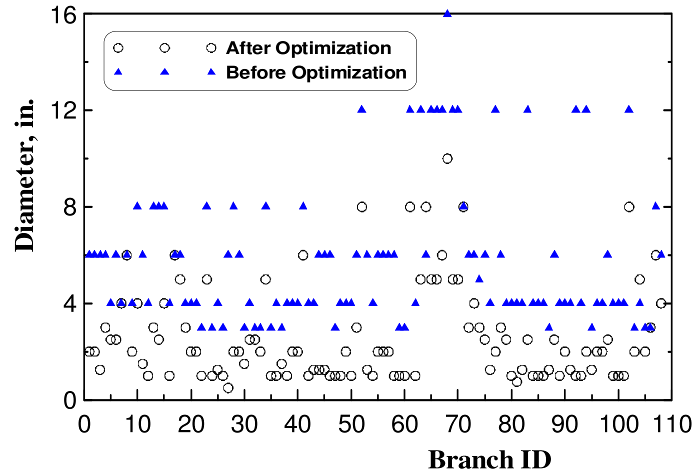

Fig. 4. Comparison between pipes diameters before (designed) and after optimization for the case study

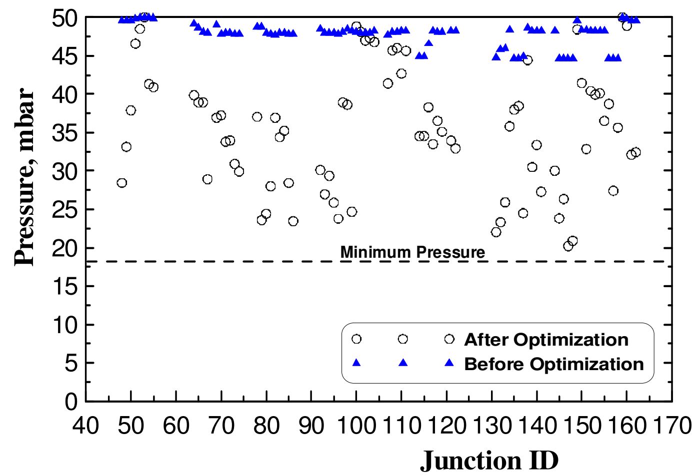

Fig. 5. Comparison between nodal pressures before (designed) and after optimization for the case study

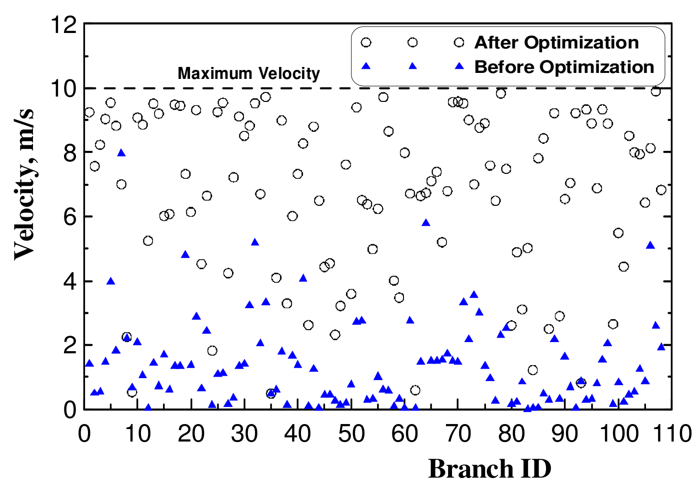

Fig. 6. Comparison between flow velocities before (designed) and after optimization for the case study

Figure 7 represents the comparison between the optimal branch sizes obtained by the present study and that of Osiadacz and Górecki [2]. There are very close results in many links; however big diameters are eliminated. The maximum diameter for the optimal network obtained by the software is 10 inches.

Osiadacz and Górecki [2] obtained an optimal cost of \$ 54,350.580 whereas the present study found an optimal cost of \$ 36,200.39457. The new found optimal cost is 66.6% of their optimal cost.

Figure 8 shows the best, average, and worst fitnesses for each fifth increasing generation for clarity. It should be noted that the best fitness is always feasible solution but for the average and worst solutions, they may be feasible and may be not. The code doesn't deviates between feasible and infeasible solution automatically.

Figure 9 illustrates the best solution for each generation which resulted by taking the minimum costs from Fig. 8 and disregarding the higher costs. This network containing 108 pipes and with 20 available commercial pipe sizes has a total solution space of 20^{108} different network designs. The GA optimization technique found the best solution \$ 36,200.39457 at generation no. 321 which is very small fraction of the total search space. The original (design) cost is \$ 98,963.614; therefore the optimal cost is approximately 36.6% of the original cost.

The optimization run time was 16 minutes and obtained by a computer with Intel Pentium 4 (2 GHz) processor and 256 MB of Ram. Generally, there is strong computing time saving compared to those obtained with other optimization techniques.

CONCLUSIONS

This study presents the optimization of gas distribution networks to determine the optimal diameter for each pipe in order to minimize the investment cost. The optimization of gas distribution network is computationally complex as the constraints of maximum flow velocity and minimum required nodal pressure should be fulfilled. The optimal diameters of pipes were chosen from available size diameters. The gradient algorithm was used for the network analysis whereas the real-coded genetic algorithm for the optimization. The following conclusions can be deduced:

1. The genetic algorithm is a very efficient, robust, and flexible algorithm to reach solutions very fast.
2. The gradient algorithm used for the gas network simulation extremely reduces the computational time when it is compared to other numerical schemes.
3. The app1ication of the developed software for the optimization of a case study minimizes the cost to \$ 36,200.39457 which is approximately 36.6% of the original cost (\$ 98,963.614).
4. The previous optimal cost of Osiadacz and Górecki [2] is \$ 54,350.580 compared to that of the present study \$ 36,200.39457, which is 66.6% of their optimal cost.

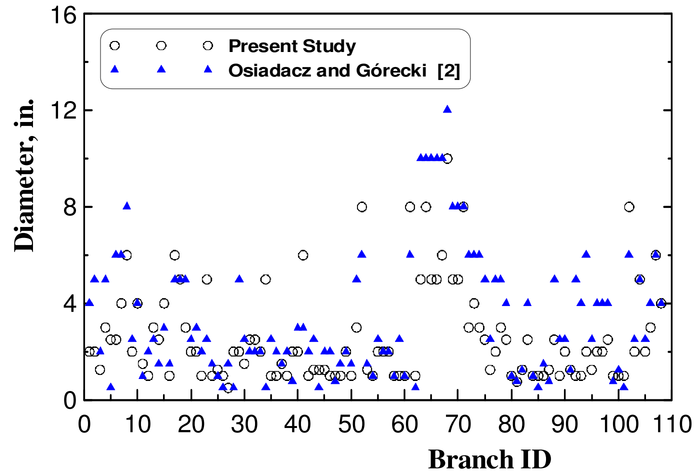

Fig. 7. Comparison between optimal diameters obtained by Osiadacz and Górecki [2] and present study

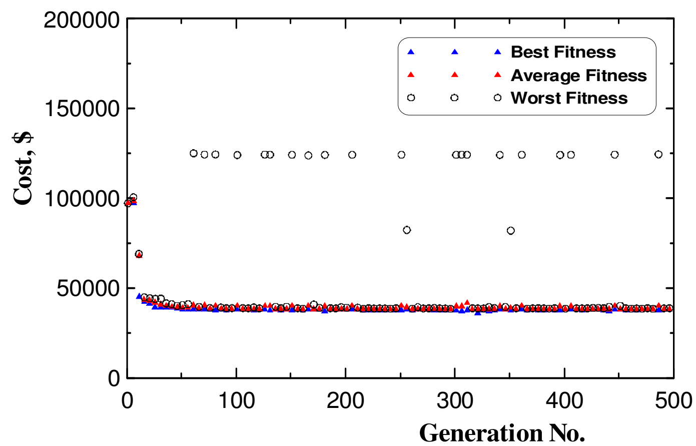

Fig. 8. Best, average and worst fitnesses for the case study

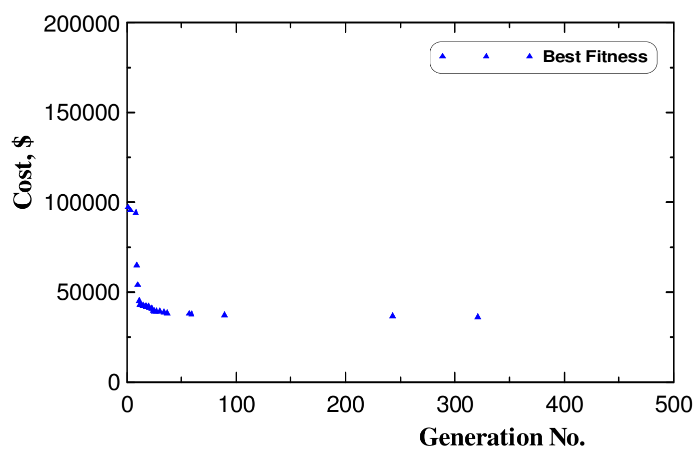

Fig. 9. Best fitness for the case study

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APPENDIX A: CASE STUDY DATA AND RESULTS

Table A1. Nodes data for the case study

Junction ID	Demand (m^3/h)	Junction ID	Demand (m^3/h)	Junction ID	Demand (m^3/h)
100	8.20	150	64.64	84	44.10
101	17.93	151	8.79	85	52.87
102	26.24	152	26.33	86	25.01
103	24.26	153	1.98	92	8.05
104	10.27	154	0.00	93	4.68
107	3.82	155	6.71	94	12.89
108	18.82	156	5.16	95	34.44
109	24.96	157	16.76	96	35.71
110	14.81	158	31.85	97	23.27
111	8.59	160	0.00	98	1.00
114	0.25	161	16.67	99	8.52
115	11.08	162	1.17		
116	8.59	48	7.19		
117	32.30	49	4.95		
118	39.89	50	6.51		
119	19.98	51	2.80		
121	35.41	52	6.61		
122	6.97	54	6.61		
131	0.00	55	1.00		
132	12.22	64	3.55		
133	27.60	65	4.63		
134	17.46	66	20.21		
135	60.66	67	18.86		
136	71.72	69	7.37		
137	15.18	70	4.87		
138	42.53	71	6.47		
139	45.63	72	38.15		
140	38.24	73	23.70		
141	26.39	74	12.55		
144	0.59	78	5.14		
145	8.89	79	4.42		
146	3.89	80	21.74		
147	13.37	81	21.96		
148	20.29	82	19.44		
149	55.55	83	30.65		

Table A2. Source nodes for the case study

Source ID	159	53
Pressure (mbar)	50.00	50.00

Table A3. Branches data for the case study

Branch ID	Start Node	End Node	Length (m)	Diameter (in.)
1	100	101	73.2	6
2	101	102	139.0	6
3	102	103	214.0	6
4	103	104	199.0	6
5	107	101	13.1	4
6	107	108	135.9	6
7	107	116	20.1	4
8	108	102	22.9	6
9	108	109	216.1	6
10	108	117	81.1	8
11	109	110	199.9	6
12	109	118	92.0	4
13	110	111	10.1	8
14	110	121	98.1	8
15	111	104	18.0	8
16	111	122	96.0	4
17	114	115	78.0	6
18	114	131	70.1	6
19	115	116	75.9	4
20	115	136	153.0	4
21	116	132	93.9	4
22	117	118	199.9	3
23	117	134	46.9	8
24	118	119	139.9	3
25	118	139	100.0	4
26	119	121	78.9	3
27	121	122	7.0	6
28	121	140	93.0	8
29	131	135	63.1	6
30	132	133	60.0	3
31	132	137	96.9	4
32	133	134	70.1	3
33	133	158	267.9	3
34	134	138	53.0	8
35	135	136	110.0	3
36	135	145	100.9	4
37	136	137	60.0	3
38	136	147	96.0	4
39	137	148	92.0	4
40	138	139	223.1	4

Branch ID	Start Node	End Node	Length (m)	Diameter (in.)
41	138	149	118.9	8
42	139	140	221.9	4
43	139	150	103.9	4
44	140	141	15.8	6
45	141	144	111.9	6
46	144	154	36.0	6
47	145	146	89.9	3
48	145	147	73.2	4
49	147	148	67.1	4
50	148	157	95.1	4
51	149	150	252.1	6
52	149	160	152.1	12
53	149	161	121.9	6
54	150	151	89.9	4
55	150	152	150.0	6
56	152	153	70.1	6
57	153	154	88.1	6
58	154	155	84.1	6
59	156	157	98.1	3
60	157	158	67.1	3
61	159	160	56.1	12
62	162	161	50.0	4
63	48	49	56.1	12
64	48	64	15.8	6
65	49	50	93.9	12
66	50	51	249.0	12
67	51	52	281.0	12
68	52	53	31.1	16
69	52	54	120.1	12
70	54	55	18.9	12
71	55	98	270.1	8
72	64	65	168.9	6
73	64	69	21.9	6
74	65	66	78.0	5
75	66	67	160.0	6
76	66	83	220.1	4
77	67	72	78.0	12
78	69	78	70.1	6
79	69	80	175.0	4
80	70	81	57.0	4

Table A3. (Continued)

Branch ID	Start Node	End Node	Length (m)	Diameter (in.)
81	71	72	88.1	4
82	72	73	189.9	4
83	72	83	91.1	12
84	73	74	160.0	4
85	73	85	93.9	4
86	74	86	86.9	4
87	78	79	68.9	3
88	78	92	98.1	6
89	80	101	161.8	4
90	80	81	78.0	4
91	81	82	181.1	4
92	83	84	20.1	12
93	84	85	187.1	4
94	84	95	82.0	12

Branch ID	Start Node	End Node	Length (m)	Diameter (in.)
95	85	86	163.1	3
96	85	96	89.9	4
97	86	97	86.9	4
98	92	99	92.0	6
99	93	94	61.0	4
100	94	100	110.0	4
101	94	80	78.0	4
102	95	102	84.1	12
103	95	96	210.0	3
104	96	103	82.0	4
105	96	97	167.9	3
106	97	98	14.0	3
107	98	104	70.1	8
108	99	100	89.9	6

Table A4. Piping cost per 1 meter length

D (mm)	D (inch)	Cost (Zloty) / L (m) = $2.05\ D^{1.3}$
12.5	0.50	0.8326
18.75	0.75	1.4104
25	1.00	2.0500
31.25	1.25	2.7399
37.5	1.50	3.4727
50	2.00	5.0477
62.5	2.50	6.7465
75	3.00	8.5509
100	4.00	12.4289
125	5.00	16.6117
150	6.00	21.0548
200	8.00	30.6035
250	10.00	40.9029
300	12.00	51.8429
400	16.00	75.3546

Table A5. Simulation results data for "Case study": Nodes results

Junction ID	Demand (m^3/h)	Pressure (mbar)
100	8.20	48.03
101	17.93	47.94
102	26.24	47.96
103	24.26	48.00
104	10.27	48.26
107	3.82	47.74
108	18.82	48.03
109	24.96	48.09
110	14.81	48.23
111	8.59	48.24
114	0.25	44.83
115	11.08	44.92
116	8.59	46.55
117	32.30	48.19
118	39.89	48.09
119	19.98	48.08
121	35.41	48.21
122	6.97	48.21
131	0.00	44.75
132	12.22	45.83
133	27.60	45.98
134	17.46	48.32
135	60.66	44.68
136	71.72	44.65
137	15.18	44.89
138	42.53	48.59
139	45.63	48.20
140	38.24	48.20
141	26.39	48.20
144	0.59	48.21
145	8.89	44.65
146	3.89	44.64
147	13.37	44.64
148	20.29	44.65
149	55.55	49.50

Junction ID	Demand (m^3/h)	Pressure (mbar)
150	64.64	48.35
151	8.79	48.34
152	26.33	48.25
153	1.98	48.24
154	0.00	48.22
155	6.71	48.22
156	5.16	44.58
157	16.76	44.60
158	31.85	44.60
160	0.00	49.87
161	16.67	49.50
162	1.17	49.50
48	7.19	49.49
49	4.95	49.53
50	6.51	49.59
51	2.80	49.77
52	6.61	49.98
54	6.61	49.89
55	1.00	49.88
64	3.55	49.16
65	4.63	48.67
66	20.21	48.14
67	18.86	47.95
69	7.37	48.99
70	4.87	47.76
71	6.47	47.95
72	38.15	47.95
73	23.70	47.82
74	12.55	47.82
78	5.14	48.76
79	4.42	48.75
80	21.74	47.95
81	21.96	47.76
82	19.44	47.68
83	30.65	47.95

Junction ID	Demand (m^3/h)	Pressure (mbar)
84	44.10	47.95
85	52.87	47.82
86	25.01	47.84
92	8.05	48.47
93	4.68	47.95
94	12.89	47.96
95	34.44	47.95
96	35.71	47.87
97	23.27	48.03
98	1.00	48.48
99	8.52	48.23
159*	-703.45	50
53*	-779.11	50
* Source Nodes		

Table A6. Simulation results data for "Case study": Branches results

Branch ID	Flow (m³/h)	Velocity (m/s)	Head loss (mbar)
1	89.53	1.4073	0.09
2	-31.63	0.4972	-0.02
3	-33.75	0.5305	-0.04
4	-93.49	1.4696	-0.27
5	-112.40	3.9753	-0.19
6	-116.61	1.8329	-0.28
7	225.19	7.9643	1.19
8	140.48	2.2083	0.07
9	-42.18	0.6631	-0.06
10	-233.73	2.0666	-0.16
11	-68.10	1.0704	-0.14
12	0.95	0.0338	0.00
13	-164.46	1.4541	-0.01
14	81.55	0.7211	0.02
15	-190.38	1.6833	-0.02
16	17.33	0.6129	0.03
17	-85.62	1.3458	-0.09
18	85.37	1.3419	0.08
19	-135.62	4.7964	-1.63
20	38.92	1.3764	0.27
21	80.98	2.8641	0.72
22	10.21	0.6423	0.10
23	-276.24	2.4425	-0.13
24	2.23	0.1404	0.00
25	-30.95	1.0948	-0.11
26	-17.75	1.1158	-0.12
27	-10.36	0.1628	0.00
28	38.75	0.3427	0.01
29	85.37	1.3419	0.07
30	-22.38	1.4070	-0.15
31	91.14	3.2233	0.94
32	-82.32	5.1762	-2.34
33	32.35	2.0339	1.38
34	-376.03	3.3248	-0.27
35	7.85	0.4938	0.03
36	16.86	0.5961	0.03
37	-28.62	1.7992	-0.24
38	3.67	0.1297	0.00
39	47.34	1.6744	0.24
40	38.93	1.3769	0.40
41	-457.49	4.0451	-0.91
42	-2.68	0.0947	0.00
43	-34.98	1.2370	-0.15
44	-2.16	0.0340	0.00
45	-28.55	0.4489	-0.01
46	-29.14	0.4581	0.00
47	3.89	0.2446	0.01
48	4.08	0.1441	0.00
49	-5.63	0.1991	0.00
50	21.42	0.7577	0.05
51	172.57	2.7126	1.16
52	-703.45	2.7644	-0.36
53	17.84	0.2804	0.01
54	8.79	0.3109	0.01
55	64.16	1.0086	0.10
56	37.83	0.5947	0.02
57	35.85	0.5636	0.02
58	6.71	0.1055	0.00
59	-5.16	0.3244	-0.01
60	-0.50	0.0312	0.00
61	703.45	2.7644	0.13
62	-1.17	0.0414	0.00
63	-374.72	1.4726	-0.04
64	367.53	5.7772	0.33
65	-379.67	1.4920	-0.07
66	-386.18	1.5176	-0.18
67	-388.98	1.5286	-0.20
68	-779.11	1.7222	-0.02
69	383.52	1.5072	0.09
70	376.91	1.4812	0.01
71	375.91	3.3238	1.40
72	138.00	2.1693	0.50
73	225.98	3.5521	0.17
74	133.37	3.0189	0.53
75	86.33	1.3570	0.18
76	26.83	0.9491	0.19
77	67.47	0.2651	0.00
78	147.38	2.3167	0.23
79	71.22	2.5189	1.04
80	-4.87	0.1722	0.00
81	-6.47	0.2288	0.00
82	23.99	0.8486	0.13
83	-1.15	0.0045	0.00
84	-1.12	0.0395	0.00
85	1.41	0.0500	0.00
86	-13.67	0.4834	-0.02
87	4.42	0.2779	0.01
88	137.82	2.1665	0.29
89	9.17	0.3243	0.02
90	46.27	1.6365	0.20
91	19.44	0.6875	0.08
92	-4.96	0.0195	0.00
93	24.19	0.8557	0.13
94	-73.26	0.2879	0.00
95	-4.92	0.3092	-0.02
96	-22.35	0.7903	-0.05
97	-43.60	1.5419	-0.19
98	129.77	2.0399	0.24
99	-4.68	0.1655	0.00
100	-23.53	0.8321	-0.07
101	5.96	0.2107	0.00
102	-116.36	0.4573	-0.01
103	8.67	0.5448	0.08
104	-35.48	1.2549	-0.12
105	-13.91	0.8746	-0.16
106	-80.77	5.0788	-0.45
107	294.14	2.6008	0.22
108	121.25	1.9060	0.20

Table A7. Optimized versus existing simulation pressure for "Case study"

Junction ID	Design Pressure (mbar)	Optimization Pressure (mbar)
100	48.03	48.81
101	47.94	48.09
102	47.96	46.98
103	48.00	47.30
104	48.26	46.76
107	47.74	41.38
108	48.03	45.68
109	48.09	45.98
110	48.23	42.67
111	48.24	45.63
114	44.83	34.49
115	44.92	34.52
116	46.55	38.28
117	48.19	33.49
118	48.09	36.50
119	48.08	35.09
121	48.21	33.91
122	48.21	32.91
131	44.75	22.02
132	45.83	23.30
133	45.98	25.90
134	48.32	35.78
135	44.68	37.96
136	44.65	38.44
137	44.89	24.48
138	48.59	44.42
139	48.20	30.48
140	48.20	33.37
141	48.20	27.27
144	48.21	30.01
145	44.65	23.81
146	44.64	26.33
147	44.64	20.21
148	44.65	20.89
149	49.50	48.42

Junction ID	Design Pressure (mbar)	Optimization Pressure (mbar)
150	48.35	41.42
151	48.34	32.84
152	48.25	40.40
153	48.24	39.92
154	48.22	40.10
155	48.22	36.50
156	44.58	38.71
157	44.60	27.40
158	44.60	35.62
160	49.87	48.90
161	49.50	32.09
162	49.50	32.43
48	49.49	28.43
49	49.53	33.12
50	49.59	37.87
51	49.77	46.56
52	49.98	48.49
54	49.89	41.29
55	49.88	40.88
64	49.16	39.84
65	48.67	38.91
66	48.14	38.92
67	47.95	28.91
69	48.99	36.91
70	47.76	37.22
71	47.95	33.77
72	47.95	33.94
73	47.82	30.90
74	47.82	29.91
78	48.76	37.03
79	48.75	23.59
80	47.95	24.42
81	47.76	27.99
82	47.68	36.91
83	47.95	34.36

Junction ID	Design Pressure (mbar)	Optimization Pressure (mbar)
84	47.95	35.22
85	47.82	28.42
86	47.84	23.45
92	48.47	30.10
93	47.95	26.95
94	47.96	29.34
95	47.95	25.85
96	47.87	23.78
97	48.03	38.91
98	48.48	38.60
99	48.23	24.67
159*	50	50
53*	50	50

* Source Nodes

Table A8. Optimized versus existing simulation for "Case study"

Branch ID	Design		Optimization		ΔD (in.)	Branch ID	Design		Optimization		ΔD (in.)
	Diameter (in.)	Velocity (m/s)	Diameter (in.)	Velocity (m/s)			Diameter (in.)	Velocity (m/s)	Diameter (in.)	Velocity (m/s)	
1	6	1.4073	2	9.2456	-4	41	8	4.0451	6	8.2720	-2
2	6	0.4972	2	7.5662	-4	42	4	0.0947	1	2.6230	-3
3	6	0.5305	1.25	8.2315	-4.75	43	4	1.2370	1.25	8.8010	-2.75
4	6	1.4696	3	9.0345	-3	44	6	0.0340	1.25	6.4931	-4.75
5	4	3.9753	2.5	9.5430	-1.5	45	6	0.4489	1.25	4.4337	-4.75
6	6	1.8329	2.5	8.8189	-3.5	46	6	0.4581	1	4.5437	-5
7	4	7.9643	4	7.0015	0	47	3	0.2446	1	2.3201	-2
8	6	2.2083	6	2.2501	0	48	4	0.1441	1	3.2195	-3
9	4	0.6631	2	0.5325	-2	49	4	0.1991	2	7.6163	-2
10	8	2.0666	4	9.0810	-4	50	4	0.7577	1	3.5900	-3
11	6	1.0704	1.5	8.8521	-4.5	51	6	2.7126	3	9.4000	-3
12	4	0.0338	1	5.2413	-3	52	12	2.7644	8	6.5100	-4
13	8	1.4541	3	9.5111	-5	53	6	0.2804	1.25	6.3841	-4.75
14	8	0.7211	2.5	9.2000	-5.5	54	4	0.3109	1	4.9836	-3
15	8	1.6833	4	6.0156	-4	55	6	1.0086	2	6.2411	-4
16	4	0.6129	1	6.0772	-3	56	6	0.5947	2	9.7133	-4
17	6	1.3458	6	9.4821	0	57	6	0.5636	2	8.6529	-4
18	6	1.3419	5	9.4467	-1	58	6	0.1055	1	4.0100	-5
19	4	4.7964	3	7.3202	-1	59	3	0.3244	1	3.4726	-2
20	4	1.3764	2	6.1345	-2	60	3	0.0312	1	7.9837	-2
21	4	2.8641	2	9.3133	-2	61	12	2.7644	8	6.7101	-4
22	3	0.6423	1	4.5235	-2	62	4	0.0414	1	0.5936	-3
23	8	2.4425	5	6.6439	-3	63	12	1.4726	5	6.6401	-7
24	3	0.1404	1	1.8231	-2	64	6	5.7772	8	6.7326	2
25	4	1.0948	1.25	9.2531	-2.75	65	12	1.4920	5	7.1021	-7
26	3	1.1158	1	9.5421	-2	66	12	1.5176	5	7.3921	-7
27	6	0.1628	0.5	4.2382	-5.5	67	12	1.5286	6	5.2000	-6
28	8	0.3427	2	7.2190	-6	68	16	1.7222	10	6.7919	-6
29	6	1.3419	2	9.1091	-4	69	12	1.5072	5	9.5643	-7
30	3	1.4070	1.5	8.5126	-1.5	70	12	1.4812	5	9.5763	-7
31	4	3.2233	2.5	8.8218	-1.5	71	8	3.3238	8	9.5203	0
32	3	5.1762	2.5	9.5221	-0.5	72	6	2.1693	3	9.0025	-3
33	3	2.0339	2	6.7001	-1	73	6	3.5521	4	7.0019	-2
34	8	3.3248	5	9.7213	-3	74	5	3.0189	3	8.7651	-2
35	3	0.4938	1	0.4896	-2	75	6	1.3570	2.5	8.9020	-3.5
36	4	0.5961	1	4.0918	-3	76	4	0.9491	1.25	7.5901	-2.75
37	3	1.7992	1.5	8.9871	-1.5	77	12	0.2651	2	6.4910	-10
38	4	0.1297	1	3.2891	-3	78	6	2.3167	3	9.8290	-3
39	4	1.6744	2	6.0123	-2	79	4	2.5189	2.5	7.4802	-1.5
40	4	1.3769	2	7.3219	-2	80	4	0.1722	1	2.6092	-3

Table A8. (Continued)

Branch ID	Design		Optimization		ΔD (in.)
	Diameter (in.)	Velocity (m/s)	Diameter (in.)	Velocity (m/s)	
81	4	0.2288	0.75	4.8921	-3.25
82	4	0.8486	1.25	3.1090	-2.75
83	12	0.0045	2.5	5.0192	-9.5
84	4	0.0395	1	1.2230	-3
85	4	0.0500	1	7.8109	-3
86	4	0.4834	1	8.4313	-3
87	3	0.2779	1.25	2.5000	-1.75
88	6	2.1665	2.5	9.2190	-3.5
89	4	0.3243	1	2.9010	-3
90	4	1.6365	2	6.5459	-2
91	4	0.6875	1.25	7.0425	-2.75
92	12	0.0195	1	9.2198	-11
93	4	0.8557	1	0.8221	-3
94	12	0.2879	2	9.3290	-10
95	3	0.3092	1.25	8.8980	-1.75
96	4	0.7903	2	6.8798	-2
97	4	1.5419	2	9.3330	-2
98	6	2.0399	2.5	8.8910	-3.5
99	4	0.1655	1	2.6500	-3
100	4	0.8321	1	5.4920	-3
101	4	0.2107	1	4.4401	-3
102	12	0.4573	8	8.5145	-4
103	3	0.5448	2	7.9982	-1
104	4	1.2549	5	7.9371	1
105	3	0.8746	2	6.4306	-1
106	3	5.0788	3	8.1290	0
107	8	2.6008	6	9.8925	-2
108	6	1.9060	4	6.8290	-2

Cost (Design) = 233,554.129 Zlotys ~ \$ 98,963.614
Cost (Optimization) = 85,433.05 Zlotys ~ \$ 36,200.4
Profit = \$ 62,763.2

Table A9. Comparison between optimized diameters obtained by Osiadacz and Górecki [2] and present study

Branch ID	Optimized Diameters (in.) Osiadacz and Górecki [2]	Optimized Diameters (in.) Present Study	ΔD (in.)
1	4	2	2
2	5	2	3
3	2	1.25	0.75
4	5	3	2
5	0.5	2.5	-2
6	6	2.5	3.5
7	6	4	2
8	8	6	2
9	2.5	2	0.5
10	4	4	0
11	1	1.5	-0.5
12	2	1	1
13	2.5	3	-0.5
14	1.5	2.5	-1
15	3	4	-1
16	1.5	1	0.5
17	5	6	-1
18	5	5	0
19	5	3	2
20	2.5	2	-0.5
21	3	2	1
22	2	1	1
23	2.5	5	-2.5
24	1.5	1	0.5
25	1	1.25	-0.75
26	0.5	1	-0.5
27	1.5	0.5	1
28	0.5	2	-1.5
29	5	2	3
30	2.5	1.5	1
31	2	2.5	-0.5
32	2	2.5	-0.5
33	2	2	0
34	0.5	5	-4.5
35	2.5	1	1.5
36	2	1	1
37	1.5	1.5	0
38	2	1	1
39	0.75	2	-1.25
40	3	2	1
41	3	6	-3
42	2	1	1
43	2.5	1.25	1.25
44	0.5	1.25	-0.75
45	2	1.25	0.75
46	2	1	1
47	0.75	1	-0.25
48	1.5	1	0.5
49	2	2	0
50	1.5	1	0.5
51	5	3	2
52	6	8	-2
53	1.5	1.25	0.25
54	1	1	0
55	2.5	2	0.5
56	2	2	0
57	2	2	0
58	1	1	0
59	2.5	1	1.5
60	1	1	0
61	6	8	-2
62	0.5	1	-0.5
63	10	5	5
64	10	8	2
65	10	5	5
66	10	5	5
67	10	6	4
68	12	10	2
69	8	5	3
70	8	5	3
71	8	8	0
72	6	3	3
73	6	4	2
74	6	3	3
75	5	2.5	2.5
76	2.5	1.25	1.25
77	5	2	3
78	5	3	2
79	4	2.5	1.5
80	1	1	0
81	0.75	0.75	0
82	1.25	1.25	0
83	4	2.5	1.5
84	1	1	0
85	0.5	1	-0.5
86	1.5	1	0.5
87	0.75	1.25	-0.5
88	5	2.5	2.5
89	2.5	1	1.5
90	2.5	2	0.5
91	1.25	1.25	0
92	5	1	4
93	4	1	3
94	6	2	4
95	2.5	1.25	1.25
96	4	2	2
97	4	2	2
98	4	2.5	1.5
99	0.75	1	-0.25
100	1.25	1	0.25
101	0.5	1	-0.5
102	6	8	-2
103	2.5	2	0.5
104	5	5	0
105	2.5	2	0.5
106	4	3	1
107	6	6	0
108	4	4	0

Cost (Osiadacz and Górecki [2]) = 128,267.368 Zlotys ~ $ 54,350.580
Cost (Present Study) = 85,433.1 Zlotys ~ $ 36,200.4
Profit = $ 18,150.2