

# A strategic planning model for natural gas transmission networks

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## Abstract

The design and development of natural gas transmission pipeline networks are multidisciplinary problems that require various engineering knowledge. In this problem, the type, location, and installation schedule of major physical components of a network including pipelines and compressor stations are decided upon over a planning horizon with least cost goal and subject to network constraints. Practically, this problem has been viewed as a conceptual design case and not as an optimization problem that tries to select the best design option among a set of possible solutions. Consequently, conceptual design approaches are usually suboptimal and work only for short-run development planning. We propose an integrated nonlinear optimization model for this problem. This model provides the best development plans for an existing network over a long-run planning horizon with least discounted operating and capital costs. A heuristic random search optimization method is also developed to solve the model. We show the application of the model through a simple case study and discuss how non-economic objectives may also be incorporated into model.

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## 1. Introduction

Development and planning of natural gas transmission networks as multidisciplinary projects have crucial impact on the economy of gas-rich countries like Iran. The investment costs and operation expenses of pipeline networks are so large that even small improvements in system utilization and planning can involve substantial amounts of money.

The natural gas industry services include producing, moving, and selling gas. Our main interest in this study is focused on the transportation of gas through a pipeline network. Moving gas is divided into two classes: transmission and distribution. Transmission of gas means moving a large volume of gas at high pressures over long distances from a gas source to distribution centers. In contrast, gas distribution is the process of routing gas to individual customers. For both transmission and distribution networks, the gas flows through various devices including

pipes, regulators, valves, and compressors. We further narrow down our focus in this study to transmission networks. The problem addressed in this paper involves decisions upon the development plans of an existing natural gas transmission network for a study area in a long-run horizon in order to satisfy the demand of some consumers.

Over the years, various aspects of the problem have been addressed (Larson and Wong, 1968; Graham et al., 1971; Martch and McCall, 1972; Flanigan, 1972; Mah and Schacham, 1978; Cheesman, 1971a, b; Edgar et al., 1978). Larson and Wong (1968) determined the steady-state optimal operating conditions of a straight natural pipeline with compressors in series using dynamic programming to find the optimal suction and discharge pressures. The length and diameter of the pipeline segment were assumed to be constant because of limitations of dynamic programming. Martch and McCall (1972) modified the problem by adding branches to the pipeline segments. However, the transmission network was predetermined because of the limitations of the optimization technique used. Cheesman (1971) introduced a computer optimizing code in addition

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to Martch and McCall (1972) problem. They considered the length and diameters of the pipeline segments to be variables. But their problem formulation did not allow unbranched network, so complicated network systems could not be handled. Olorunniwo (1981) and Olorunniwo and Jensen (1982) provided further breakthrough by optimizing a gas transmission network including the type and location of pipelines and compressor stations. Edgar and Himmelblau (1988) simplified the problem addressed by Olorunniwo (1981) and Olorunniwo and Jensen (1982) to make sure that the various factors involved in the design are clear. They assumed the gas quantity to be transferred along with the suction and discharge pressures to be given in the problem statement. They optimized the variables such as the number of compressor stations, the length of pipeline segments between the compressors stations, the diameters of the pipeline segments and the suction, and discharge pressures at each station. They considered the minimization of the total cost of operation per year including the capital cost in their objective function against which the above parameters are to be optimized. Edgar and Himmelblau (1988) also considered two possible scenarios: (1) the capital cost of the compressor stations is linear function of the horse power and (2) the capital cost of the compressor stations is linear function of the horsepower with a fixed capital outlay for zero horsepower.

The development planning of natural gas network requires various engineering, economics, and management knowledge. But what practically is done in developing countries like Iran, which holds the second largest natural gas resources in the world, is based on conceptual designs of engineers to satisfy short-run demand of consumers. Pursuing this policy over years has led to a complex and inefficient pipeline network that is growing annually. On the other hand in theory, the literature also lacks an integrated strategic planning method to consider different aspects of the problem over a long-run horizon. Specifically, these issues include:

1. The short-run development planning in the heart of a long-run strategic plan,
2. The best type and location of the new compressor stations,
3. The best type and routing of the pipelines,
4. The scheduling of implementing new pipelines and compressor stations in the network,
5. The best combination of natural gas procurement from available sources, and
6. The best operating conditions of compressor stations in different periods of each year of the long-run horizon.

In this paper, we provide an optimization model to address these issues. This strategic planning model is a complement to the available studies in the literature and could bridge the gap between the theoretical researches and practical needs. In addition to the model, a specific heuristic random search algorithm is also developed which

could solve the proposed strategic planning model and provide (near) optimal development plans over the long-run planning horizon.

The remainder of this paper is organized as follows. Section 2 discusses about the model. After introducing the notation and main parameters of the model in Sections 2.2 and 2.3, we talk about the integrated model of this paper by introducing decision variables, constraints, and objective function. There is a submodel within the integrated model that finds best operating condition of the network over time. This submodel is discussed in Section 2.5. The optimization algorithm of the model is discussed in Section 3 and we use a case study in Section 4 to demonstrate the applicability of the model and the proposed solver method. Section 5 discusses about the flexibility of the model in incorporating various decision criteria and constraints in the planning model. Finally, Section 6 concludes the paper.

## 2. Proposed model

We discuss about the model of this paper in this section. Before going through the details, we briefly introduce all notations in Table 1.

### 2.1. Problem definition

The problem addressed in this paper focuses on transmission of high-pressure processed natural gas from production facilities to distribution centers or major consumption sites. The development and extension projects of a natural gas transmission network have different phases, which differ from country to country based on the rules and economic policies imposed by the governments. The major phases of a typical development project in the US are provided in Table 2 (Transportation

Table 1  
Parameters of the model

| Notation        | Parameter, sets, and variables   |
|-----------------|--|
| $\omega$        | Length of long-run planning horizon  |
| $v$             | Length of short-run planning horizon   |
| $\lambda$       | Number of short horizons in a long horizon   |
| $\Psi_{jk}$     | Set of supply nodes in $k$ th year of $j$ th short horizon   |
| $\Delta_{jk}$   | Set of demand nodes in $k$ th year of $j$ th short horizon   |
| $e_{ijk}$       | Lower bound of gas supply of $i$ th node in $k$ th year of $j$ th short horizon                                      |
| $\delta_{ijk}$  | Upper bound of gas supply of $i$ th node in $k$ th year of $j$ th short horizon                                      |
| $\tau$          | The number of periods of a year  |
| $\theta_{ijkl}$ | The demand of $i$ th node in $l$ th period of $k$ th year of $j$ th short horizon                                    |
| $\eta_{ijkl}$   | Upper bound of the pressure demanded/supplied in $i$ th node at $l$ th period of $k$ th year of $j$ th short horizon |
| $\gamma_{ijkl}$ | Lower bound of the pressure demanded/supplied in $i$ th node at $l$ th period of $k$ th year of $j$ th short horizon |
| $I_j$           | Available compressor station entrance nodes at short horizon $j$   |
| $O_j$           | Available compressor station exit nodes at short horizon $j$   |

Table 1 (continued)

| Notation             | Parameter, sets, and variables  |
|----------------------|---|
| $H_j$                | The union of sets $I_j$ and $O_j$   |
| $T_j$                | The set of potential transshipment nodes for short horizon $j$  |
| $A_j$                | The set of active transshipment nodes for short horizon $j$   |
| $\rho$               | Number of different types of pipe   |
| $\sigma$             | Number of different types of compressor stations  |
| $\Gamma_{ii'j}$      | Set of pipe types between nodes $i$ and $i'$ in the available network of short horizon $j$  |
| $\mu_{ii'j}$         | Compressor station type between nodes $i$ and $i'$ in available network of short horizon $j$  |
| $P_j$                | Set of pipe edges in the available network of short horizon $j$   |
| $K_j$                | Set of compressor station edges in the available network of short horizon $j$   |
| $X_{ii'jm}$          | Type code of $m$ th new pipe between nodes $i$ and $i'$ in development plan for $j$ th short horizon  |
| $\xi$                | Maximum number of new pipes between two nodes in a development plan   |
| $\chi$               | Discount rate   |
| NPW                  | Objective function of the model (net present worth)   |
| $\alpha_{jk}$        | Capital cost paid at $k$ th year of $j$ th short horizon  |
| $\beta_{jk}^*$       | Optimum operating cost paid at $k$ th year of $j$ th short horizon  |
| $\alpha_{ad'j}^c$    | Capital cost of replacing compressor station type $a$ with compressor station type $a'$ at $j$ th short horizon   |
| $\alpha_{bi'l'j}^p$  | Capital cost of installing pipeline type $b$ between nodes $i$ and $i'$ at $j$ th short horizon   |
| $v_k^c$              | Percent of the capital cost of compressor stations in a development plan for each short horizon which occurs in $k$ th year   |
| $v_k^p$              | Percent of the capital cost of pipelines in a development plan for each short horizon which occurs in $k$ th year   |
| $B_{jk}$             | Financial budget available for $k$ th year of $j$ th short horizon  |
| $W_{ijkl}$           | The pressure in $i$ th node at $l$ th period of $k$ th year of $j$ th planning horizon  |
| $V_{hi'i'jkl}$       | The mass flow rate in $l$ th pipe between nodes $i$ and $i'$ at $l$ th period of $k$ th year of $j$ th short horizon  |
| $\mathcal{D}_{ii'j}$ | Number of pipelines between nodes $i, i'$ in the available network of short horizon $j$   |
| $t_l$                | The time point in each year when $l$ th period ends   |
| $\beta_{jk}$         | Operating cost paid at $k$ th year of $j$ th short horizon  |
| $\varpi_{jkl}$       | Operating cost paid at $l$ th period of $k$ th year of $j$ th short horizon   |
| $\varpi_{jkl}^*$     | Optimum operating cost paid at $l$ th period of $k$ th year of $j$ th short horizon   |
| $\varpi_{jkl}^c$     | Operating cost of compressor stations at $l$ th period of $k$ th year of $j$ th short horizon   |
| $\varpi_{jkl}^s$     | Operating cost of gas supply at $l$ th period of $k$ th year of $j$ th short horizon  |
| $\pi_{ijkl}^s(M, P)$ | The price function of one mass unit natural gas in $i$ th supply node at $l$ th period of $k$ th year of $j$ th short horizon if requested mass flow rate out of the node and pressure are $M$ and $P$ , respectively |
| $\pi_{jkl}^c$        | The price of one mass unit natural gas consumed in a compressor station at $l$ th period of $k$ th year of $j$ th short horizon   |
| $\pi_{jk}^f(T)$      | Annual fixed operating cost function of a compressor station of type $T$ at $k$ th year of $j$ th short horizon   |
| $\kappa(M, I, O, T)$ | Mass flow rate function consumed by a compressor station of type $T$ where the incoming mass flow rate of $M$ with pressure $I$ is delivered with an outgoing pressure of $O$ .                                       |
| $\zeta$              | Pipe resistance which is a function of type and length of pipe  |
| $\Omega(T)$          | The set of feasible vectors of (incoming mass flow rate ( $M$ ), inlet pressure ( $I$ ), outlet pressure( $O$ )) for a compressor station of type $T$   |

Table 2

Phases of a typical natural gas development project in the United States

| Phase                           | Description  |
|---------------------------------|--|
| 1. Feasibility study            | Working on economic studies and construction plans<br>Analyzing environmental impacts of the pipeline 's construction and operation              |
| 2. Certificates from FERC       | Submitting feasibility study reports to Federal Energy Regularity Commission (FERC)<br>Obtaining certificate of public convenience and necessity |
| 3. Right of way                 | Purchasing the right of way and leasing the surface property along the path of the proposed construction   |
| 4. Ordering equipments          | Ordering the pipes, and fittings and delivering them   |
| 5. Installation and utilization | Installing physical components of the network<br>Inspection<br>Utilization   |

Research Board of the National Academies, 2004). The procedures for other countries, though officially different, have similarities to the major steps described in the table. We will focus our attention in this paper on the first phase in Table 2.

We assume that there exists a study area with an existing natural gas pipeline network. There is also a long-run planning horizon for this study area. The study area has a number of consumption centers/sources of gas supply with known demand/supply range over the long-run horizon. The problem addressed here is finding the ‘‘best feasible’’ development plans for the gas transmission network over the long-run horizon based on the existing network. ‘‘Best’’ here means least discounted operating and capital costs paid over the long horizon and by ‘‘feasible’’ we mean satisfying all structural, technical, and planning constraints.

## 2.2. Planning horizons

The model assumes a long-run planning horizon consisting of a number short-run planning horizons with equal length. We assume that the lengths of long and short horizons are  $\omega$  and  $v$  years respectively, and the long horizon has  $\lambda$  short horizons. Simply, we have

$$\omega = \lambda v.$$

The key assumption here is that the structure of the network at the end of short horizon  $j$  must be capable of satisfying the total demand of all consumers during all years of short horizon  $j + 1$ . Hence, the available useable structure of the network for a short horizon is fixed during all years of the short horizon. But the structure of the network could be changed during short horizons; these changes are either in the combination of pipelines or in compressor stations.

The model plans some changes in the network structure for each short horizon. These changes could be considered as “development plans”. We assume that the development plan for short horizon  $j + 1$  are implemented during previous short horizons and finished no later than the end of short horizon  $j$ . So the changes in the structure are available to be utilized in short horizon  $j + 1$ . The planned-ahead changes for the first short horizon are implemented before the start of the long horizon. The model considers a short horizon called “short horizon zero” before the first short horizon, so that the required changes for the first short horizon are implemented until the end of the short horizon zero. The hidden assumption here is that the demand of all consumers before the start of the long horizon is met by the available network before long horizon.

### 2.3. Network structure

The model assumes a study area that has some natural gas consumers and suppliers during long horizon.

The major physical components of a real gas transmission network are pipelines and compressor stations. These two components are represented by some edges in the network of the model; one edge for each pipe and one edge for each compressor station.

Along with the mentioned edges, the network has five types of nodes:

- Demand nodes.
- Supply nodes.
- Compressor station entrance nodes.
- Compressor station exit nodes.
- Transshipment nodes.

The demand nodes are major locations of consuming natural gas in the study area. The demand nodes are either the central point of major consumption regions in the study area or export terminals of natural gas from the study area to outside. In contrast, supply nodes are locations of resources of processed natural gas in the study area. These nodes are either refineries or plants producing natural gas or the import terminals of natural gas from outside of the study area.

Entrance nodes of compressor stations are points where gas enters a compressor station edge to be compressed. Another set of nodes, exit nodes, are considered at the end of compressor station edges, where compressed gas flows through the network by compressor stations.

Transshipment nodes are the places where more than 2 pipe edges are joined. In each transshipment node, the gas flows into the node via some pipe(s) and flows out through some other(s).

Each node in the network is assigned a unique code in the model. The edges are recognized by the codes of their corresponding end nodes.

#### 2.3.1. Demand and supply parameters

The combination of locations of demand and supply nodes must be forecasted by the modeler. We assume that these combinations are fixed during each year of long horizon, but can be modified at the end of each year; for instance, a new gas supplier may be added at the end of year  $j$  to the combination of supply nodes for this year so that the new supplier may start to feed gas into network from the beginning of year  $j + 1$ . We denote the sets of supply and demand nodes in  $k$ th year of  $j$ th short horizon by  $\Psi_{jk}$  and  $\Delta_{jk}$ , respectively, where

$$j = 1, 2, \dots, \lambda; \quad k = 1, 2, \dots, v.$$

We also need to define the sets of supply and demand nodes for the last year of planning horizon zero; these two sets are denoted by  $\Psi_{0,v}$  and  $\Delta_{0,v}$ , respectively.

In this paper, the demand and supply are measured by their mass flow rate (kg/year). We assume that the amount of potential gas supply in each supply node is limited to a range. These ranges must be forecasted in order to construct and run the model. The parameters  $\varepsilon_{ijk}$  and  $\delta_{ijk}$  represent upper and lower bounds of gas supply of  $i$ th node in  $k$ th year of  $j$ th short horizon where

$$i \in \Psi_{jk}; \quad j = 1, 2, \dots, \lambda; \quad k = 1, 2, \dots, v.$$

The model also requires the forecasts of the demand for each demand node over the long horizon. The actual demands follow specific time series and are continuous functions of time. But for simplicity, each year is split into a number of periods, say  $\tau$  periods, and the model tries to satisfy the forecasted average demand of each period. The  $l$ th period of each year starts at time  $t_{l-1}$  and ends at  $t_l$ , where we set  $t_0 = 0$  and  $t_\tau = 1$ . We denote the forecasted average demand of  $i$ th node in  $l$ th period of  $k$ th year of  $j$ th short horizon by  $\theta_{ijkl}$ , where

$$i \in \Delta_{jk}; \quad j = 1, 2, \dots, \lambda; \quad k = 1, 2, \dots, v; \quad l = 1, 2, \dots, \tau.$$

The demand nodes not only consume a specific mass flow rate in each period, but also need the pressure of the gas to be within a range. Also, the supply nodes release the gas with a pressure that is bounded to a range. We denote the user-defined upper and lower bounds of the pressure demanded/supplied in  $i$ th node at  $l$ th period of  $k$ th year of  $j$ th short horizon by  $\eta_{ijkl}$  and  $\gamma_{ijkl}$ , respectively, where

$$i \in \Delta_{jk} \cup \Psi_{jk}; \quad j = 1, 2, \dots, \lambda; \quad k = 1, 2, \dots, v; \\ l = 1, 2, \dots, \tau.$$

#### 2.3.2. Parameters of network components

The sets of available compressor station entrance and exit nodes at short horizon  $j$  are denoted by  $I_j$  and  $O_j$ , respectively. We also define the set  $H_j$  as the union of  $I_j$  and  $O_j$  where

$$j = 0, 1, \dots, \lambda.$$

Transshipment nodes as the potential locations of joint of a number of pipes help the model find optimal pipeline routes with the goal of least total net present cost. The modeler selects a set of points inside the study area for each short horizon where by the new pipes in the development plans may join at these points. But these potential points are not necessarily the joint place of pipes because the optimal routes obtained by the model may pass through specific transshipment nodes and ignore the others due to cost issues. A transshipment node is called “active” if it is the joint location of pipes; otherwise, it is named “inactive”. We denote the set of potential transshipment nodes for short horizon  $j$  with  $T_j$  which is a combination of active and inactive transshipment nodes. The set of active transshipment nodes in the available pipeline network at  $j$ th short horizon is also denoted by  $A_j$  where

$$j = 0, 1, \dots, \lambda.$$

Two types of edges were defined for the network of the model. We suppose that the number of different pipe edges is limited. Specifically, there are  $\rho$  different types of pipeline that could be used in the network. These pipes which have unique type codes may differ in technical specifications such as diameter, thickness, etc. Similarly, we suppose that there are  $\sigma$  different types of compressor stations. Again, these compressor stations may differ in technical specifications like the number of compressors, fuel, etc. and have unique type codes.

The set of pipe edges in the available network of short horizon  $j$  is denoted by  $P_j$ . The members of this set are vectors  $(i, i')$  where  $i < i'$  and  $i, i'$  are 2 adjacent nodes through a pipe. If more than 1 pipe link  $i, i'$  (parallel pipe edges in the network), the set of pipe edges includes vector  $(i, i')$  as many as the number of parallel pipes between the corresponding nodes. Similarly, we define  $K_j$  as the set of compressor station edges containing vectors  $(i, i')$  where  $i < i'$  and  $i, i'$  are 2 adjacent nodes through a compressor station. Again we have

$$j = 0, 1, \dots, \lambda.$$

We also define  $\Gamma_{ii'j}$  as the set of pipe types between  $i$ , and  $i'$  in available network of short horizon  $j$ . Clearly, if only one pipe links  $i, i'$  for short horizon  $j$ ,  $\Gamma_{ii'j}$  has just one member which is the type code of the pipe. We have

$$(i, i') \in P_j; \quad j = 0, 1, \dots, \lambda.$$

Similarly, the compressor station type between  $i$ , and  $i'$  in available network of short horizon  $j$  is denoted by  $\mu_{ii'j}$  where

$$(i, i') \in K_j; \quad j = 0, 1, \dots, \lambda.$$

#### 2.4. Integrated model

The integrated strategic planning model is an optimization framework, which involves an objective function and a number of constraints. This optimization model is math-

ematically presented in this section after defining decision variables of the planning problem.

##### 2.4.1. Decision variables

Development plans are nothing but the type and location of new pipes and compressor stations in the network. So we define one set of decision variables for pipes and another set for compressor stations.

Any two nodes that are not adjacent with a compressor station edge may be planned to be linked with one or more pipes in a development plan for a short horizon. Let us denote the type code of  $n$ th new pipe between nodes  $i, i'$  in development plan for  $j$ th short horizon by  $X_{ii'jn}$  (note that more than 1 pipe may be installed between 2 points) where

$$i < i'; \quad i, i' \in \Psi_{j,v} \cup \Delta_{j,v} \cup T_j \cup H_{j-1} \\ (i, i') \notin K_{j-1}; \quad j = 1, 2, \dots, \lambda; \quad n = 1, 2, \dots, \xi,$$

where  $\xi$  is the maximum number of new pipes between any 2 nodes in a development plan; this user-defined parameter limits the set of pipe decision variable. If no pipe is planned between 2 nodes in a development plan, the corresponding decision variable takes zero. We have

$$X_{ii'jn} \in \{0, 1, 2, \dots, \rho\}.$$

We assume that the new compressor stations are located in the middle of some pipe edges. This means that the optimization algorithm of the model selects some pipe edges and split each in to 3 edges; 2 pipe edges and one compressor station edge between them. In order to define the new decision variable, we need to define the updated pipe set  $P_{j-1}^*$  as

$$P_{j-1}^* = P_{j-1} \cup \{(i, i') | i < i'; \quad i, i' \in \Psi_{j,v} \cup \Delta_{j,v} \cup T_j \cup H_{j-1}; \\ (i, i') \notin K_{j-1}; \quad (\sum_{n=1}^{\xi} X_{ii'jn}) \neq 0\} \quad (1)$$

The set  $P_{j-1}^*$  contains the pipe edges in available network of short horizon  $j - 1$  and new pipes that are planned in the development plan of short horizon  $j$ .

We define decision variable  $Y_{ii'j}$  as the compressor station type that is planned to be installed in the middle of pipe edge  $(i, i')$  in the development plan of  $j$ th short horizon where

$$i < i'; \quad (i, i') \in P_{j-1}^*; \quad j = 1, 2, \dots, \lambda.$$

Another possibility for compressor station decision variable is the case that a previously installed compressor station is upgraded in a development plan. Therefore, we extend the definition of the decision variable  $Y_{ii'j}$  to the type code of upgraded compressor station that replaces with existent compressor station  $(i, i')$  in the development plan of  $j$ th short horizon where

$$i < i'; \quad (i, i') \in K_{j-1}; \quad j = 1, 2, \dots, \lambda.$$

Like decision variable  $X_{ii'jn}$ , we set  $Y_{ii'j} = 0$  if no new compressor station is planned to be installed between nodes  $i$  and  $i'$  in development plan of  $j$ th short horizon.

Any way, we have

$$Y_{i'j} \in \{0, 1, 2, \dots, \sigma\}.$$

### 2.4.2. Objective function

Various goals may be pursued in gas transmission pipelines network design and planning. Among these, cost, reliability, and responsiveness are probably the most important and evident. But in addition to these objectives, environmental and social criteria must also be considered in the planning model to ensure a sustainable development. Our objective function here nominally includes cost-related objectives. Specifically, we use the net present worth (NPW) of operating and capital costs in long-run horizons. In Section 5, we discuss how various other objectives may be incorporated into the cost-based objective function of the model.

Considering a cash flow of costs and possibly revenues related to installation and operation of the network, we calculate the NPW at the beginning of long horizon as the objective function to be minimized:

$$\min \text{NPW} = \sum_{j=0}^{\lambda} \sum_{k=1}^v \frac{\alpha_{jk} + \beta_{jk}^*}{(1 + \chi)^{(j-1)v+k-1}},$$

where  $\chi$  is the effective annual interest rate and  $\alpha_{jk}$  and  $\beta_{jk}^*$  are, respectively, capital and optimal operating costs at  $k$ th year of  $j$ th short horizon, so that

$$j = 0, 1, \dots, \lambda - 1; \quad k = 1, 2, \dots, v,$$

We assume that the capital and operation costs are occurred at the beginning of the year.

The capital costs occurred in each short horizon are functions of the variables of development plan for the subsequent short horizon (remember that the development plan for a short horizon is implemented in its previous short horizon). Specifically, the two decision variables sets defined earlier, pipeline and compressor station variables, are the only variables that affect the capital cost function. In contrast, this is not the case for operating costs because there are 2 other sets of variables along with pipeline and compressor station variables that could alter the values of operating cost. These new variables as the decision variables of the so-called “Submodel” are pressure of the gas in nodes and the mass flow rate in pipes. Given a network structure for satisfying the demand of consumers for a period, different combinations of pressure and mass flow rate may be found that could be technically feasible; hence, we define  $\beta_{jk}^*$  as the total minimum (optimum) operating cost of all periods of  $k$ th year of  $j$ th short horizon if the combination of pressure and mass flow rate variables for all periods of the corresponding year take their optimal values that lead to least operating costs.

We also set

$$\alpha_{\lambda,k} = 0; \quad \beta_{0,k} = 0; \quad k = 1, 2, \dots, v.$$

**2.4.2.1. Capital costs.** We assume that the capital cost of installing a compressor station varies for different short horizons and is a function of the compressor station type. Let us denote this capital cost of replacing compressor station type  $a$  with compressor station type  $a'$  at  $j$ th short horizon with  $\alpha_{aa'j}^c$ . If  $a' = 0$ , no compressor station already exists in the installation location to be upgraded and  $\alpha_{aa'j}^c$  is just the capital cost of a new compressor station type  $a$  at  $j$ th short horizon. We assume that the capital cost of a pipeline is a function of pipe type and locations of end nodes; this cost varies for different short horizons as well. Let us denote the capital cost of installing pipeline type  $b$  between nodes  $i$  and  $i'$  at  $j$ th short horizon with  $\alpha_{bii'j}^p$ . For these parameters,

$$a = 1, 2, \dots, \sigma; \quad a' = 0, 1, \dots, \sigma; \quad a = a'; \\ b = 1, 2, \dots, \rho; \quad j = 0, 1, \dots, \lambda - 1.$$

Now we calculate the total capital cost  $\alpha_{jk}$  as

$$\alpha_{jk} = \sum_{\{(i,i') | (i,i') \in (K_{j+1} - K_j)\}} v_k^c \times \alpha_{\mu_i, i', j+1, 0, j}^c \\ + \sum_{\{(i,i') | (i,i') \in (K_{j+1} \cap K_j); \quad \mu_i, i', j+1 \neq \mu_i, i', j\}} v_k^c \times \alpha_{\mu_i, i', j+1, \mu_i, i', j, j}^c \\ + \sum_{\{(i,i') | (i,i') \in (P_j^* - P_j)\}} \sum_{m=1}^{\zeta} v_k^p \times \alpha_{X_{i,i', j+1, m, i, i', j}}^p \\ j = 0, 1, \dots, \lambda - 1; \quad k = 1, 2, \dots, v,$$

where  $v_k^c$  and  $v_k^p$  are, respectively, the percent of capital cost of installing compressor stations and pipelines for each short horizon which occur in  $k$ th year. Of course, we must have

$$\sum_{k=1}^v v_k^c = 1, \quad \sum_{k=1}^v v_k^p = 1.$$

**2.4.2.2. Operating costs.** Generally, operating cost for the pipeline network are those costs that are not classified as capital costs. We only consider two major operating costs in the model:

- The operating cost of compressor stations.
- The supply cost of natural gas to the network.

Compressor stations as the hearts of the network consume energy to increase gas pressure in the network. These stations have some other operating costs such as maintenance, labor, and overhead.

The gas is supplied to the network by different sources including refineries and importation from outside of the study area. Given the forecast of the consumption in demand nodes for a period, one may find different combination of gas supply from supply nodes to satisfy the demands. But, the sales cost of gas from the supply nodes to the network could be different for different supply nodes. So the combination of gas supply could affect the costs of model. Therefore, we also define the operating cost of gas supply.



Given a fixed network structure for a period, both operating cost defined above are functions of steady-state mass flow rate in the pipes and gas pressure in the nodes. In fact, whenever we have a fixed structure for the network to satisfy the demand of some consumers, we may find different combination of mass flow rate and pressure variables for different parts of the network that can satisfy demands. These different combinations of mass flow rates and pressures could have different operating costs. Hence, there must be an optimization model within our integrated model to find the best combination of mass flow rates and pressures in the network for a given network structure. We call this new optimization model as “submodel”. The objective function of this submodel is minimization of the operating cost. This submodel is subject to many technical constraints.

The procedure that is used in our model is that whenever we have a fixed network structure for a period, we run the submodel to find least possible operating cost for the given network, then this cost is plugged into the objective function of our integrated model.

As we will see in Section 2.4.3, some constraints of the integrated model are also affected by the best feasible values of mass flow rates and pressure variables of the network. So, we postpone discussing about more details of the submodel problem until Section 2.5.

#### 2.4.3. Constraints

There are 4 classes of constraints for the integrated model as follows (in Section 5, we discuss how various other constraints including some environmental and social limitations may be incorporated into model):

*Class 1:* domain of variables.

*Class 2:* network structure constraints.

*Class 3:* financial budget constraints.

*Class 4:* submodel constraints.

We informally provided the first-class constraints when we defined the decision variables of integrated model. The first set of decision variables ( $X_{i'jn}$ ) could only take these values:

$$X_{i'jn} \in \{0, 1, 2, \dots, \rho\},$$

$$i < i'; \quad i, i' \in \Psi_{j,v} \cup \Delta_{j,v} \cup T_j \cup H_{j-1};$$

$$(i, i') \notin K_{j-1}; \quad j = 1, 2, \dots, \lambda; \quad n = 1, 2, \dots, \xi.$$

Similarly, for the second set of decision variables ( $Y_{i'j}$ ),

$$Y_{i'j} \in \{0, 1, 2, \dots, \sigma\},$$

$$i < i'; \quad (i, i') \in P_{j-1}^*; \quad j = 1, 2, \dots, \lambda.$$

The second class of constraints refers to the feasible structures of the network. Applying these constraints, the model searches best structures of the network among

logically acceptable options. Three types of this class of constraints are as follows:

1. There must be at least one path between any demand node (with a positive demand) and at least one of the supply nodes; otherwise there is no way that the given network structure could satisfy the consumption of the demand nodes. Many algorithms exist in graph theory to check the existence of these paths (see Ore, 1963; Harary, 1969).
2. The structure of the network must not have transshipment nodes that are adjacent to exactly one edge. In other words, the number of pipes or compressor stations connected by a transshipment node must be zero or more than 2.
3. The transshipment nodes cannot be the joints place of any combination of pipes. Technically, it may be infeasible to join specific types of pipes together. These constraints must be defined with more details based on the type of available pipelines and engineering designs.

In the 3rd class of constraints, the total financial budget available in each year of the long horizon is assumed to be limited. If this budget for the  $k$ th year of  $j$ th short horizon is denoted by  $B_{jk}$ ,

$$\alpha_{jk} + \beta_{jk}^* \leq B_{jk},$$

$$j = 0, 1, 2, \dots, \lambda; \quad k = 1, 2, \dots, v.$$

The 4th class of constraints is basically the constraints of the submodel introduced in Section 2.4.2.2. In the perspective of integrated model, these constraints are generally defined to make sure that the network structures under study can meet the demand of consumers. We discuss about the details of submodel constraints in Section 2.5.3.

#### 2.5. Control optimization submodel

In Section 2.4, we discussed about the necessity of having a submodel in the heart of the integrated model to find the least possible operating cost and check some of the constraints of integrated model. The submodel which is called steady-state simulation of pipeline network in the literature is essentially the optimization model of mass flow rates and pressure in the network subject to some technical constraints and with operating costs as objective function.

The submodel problem differs from traditional network flow problems in some fundamental aspects (Ríos Mercado et al., 2006). First, in addition to the flow variables for each edge, which in this case represent mass flow rates, a pressure variable is defined at every node. Second, besides the mass balance constraints, there exist two other types of major constraints: (i) a nonlinear equality constraint on each pipe, which represents the relationship between the pressure drop and the flow and (ii) a nonlinear nonconvex set, which represents the feasible operating limits for pressure and flow within each compressor station. The objective function is

given by a nonlinear function of flow rates and pressures. In the real world, these types of instances are very large both in terms of the number of decision variables and the number of constraints, and very complex due to the presence of nonlinearity and nonconvexity in both the set of feasible solutions and the objective function.

The submodel must be run for each period of each year of any given network structure of a short horizon. So, we develop the submodel concepts for an arbitrary period, say  $l$ th period of  $k$ th year of  $j$ th short horizon.

2.5.1. Decision variables of submodel

The submodel has 2 sets of decision variables. The first set is the gas pressure in all nodes of the network. Let us denote the pressure in  $i$ th node of  $l$ th period of  $k$ th year of  $j$ th planning horizon by  $W_{ijkl}$  where

$$i \in \Psi_{jk} \cup \Delta_{jk} \cup T_j \cup H_j.$$

We also denote the mass flow rate in  $h$ th pipe between nodes  $i$  and  $i'$  at  $l$ th period of  $k$ th year of  $j$ th short horizon by  $V_{hi'i'jkl}$  where

$$h = 1, 2, \dots, \vartheta_{ii'j}; \quad (i, i') \text{ or } (i', i) \in P_j,$$

where  $\vartheta_{ii'j}$  is the number of pipes between nodes  $i$  and  $i'$  at  $j$ th short horizon. If the flow of gas in the pipes between  $i$  and  $i'$  at  $l$ th period of  $k$ th year of  $j$ th short horizon is from node  $i$  to node  $i'$ ,  $V_{hi'i'jkl}$  is positive; otherwise, it takes a negative value to indicate that the gas flow direction is from node  $i'$  to node  $i$ .

2.5.2. Objective function of submodel

The objective function of submodel is the operating costs of the network. It includes the operating cost of all periods within a year:

$$\beta_{jk} = \sum_{l=1}^{\tau} \varpi_{jkl},$$

where  $\varpi_{jkl}$  is the operating cost paid at  $l$ th period of  $k$ th year of  $j$ th short horizon. Since we model each period independently, the annual least operating cost is the sum of least periodic operating costs:

$$\beta_{jk}^* = \sum_{l=1}^{\tau} \varpi_{jkl}^*,$$

where  $\varpi_{jkl}^*$  is the optimum (least) operating cost paid at  $l$ th period of  $k$ th year of  $j$ th short horizon.

As discussed in Section 2.4.2.2, we consider only the operating costs of compressor stations and gas supply to the network. If we denote the total operating costs of gas supply and compressor stations at  $l$ th period of  $k$ th year of  $j$ th short horizon by  $\varpi_{jkl}^s$  and  $\varpi_{jkl}^c$ , respectively, we have

$$\varpi_{jkl} = \varpi_{jkl}^c + \varpi_{jkl}^s.$$

The price of natural gas for each supplier could be a function of mass flow rate and pressure required. These price functions should be known in order to run the model. Let us denote the price function of one mass unit natural gas in  $i$ th supply node at  $l$ th period of  $k$ th year of  $j$ th short

horizon if requested mass flow rate out of the node and pressure are  $M$  and  $P$ , respectively, by  $\pi_{ijkl}^s(M, P)$ . Therefore, the total operating cost of gas supply is

$$\varpi_{jkl}^s = \sum_{i \in \Psi_{jk}} \left( \left( \sum_{i' \in \Theta(i)} \sum_{h=1}^{\vartheta_{ii'j}} V_{hi'i'jkl} \right) (t_l - t_{l-1}) \times \left( \pi_{ijkl}^s \left( \sum_{i' \in \Theta(i)} \sum_{h=1}^{\vartheta_{ii'j}} V_{hi'i'jkl}, W_{ijkl} \right) \right) \right),$$

$$\Theta(i) = \{i' | (i, i') \text{ or } (i', i) \in P_j\}.$$

We assume that the compressor stations use the natural gas as energy. The operating cost of a compressor station is classified into some fixed costs and variable costs. Let us denote Annual fixed operating cost of a compressor station of type  $T$  at  $k$ th year of  $j$ th short horizon by  $\pi_{jk}^f(T)$  and the price of one mass unit natural gas consumed in a compressor station at  $l$ th period of  $k$ th year of  $j$ th short horizon by  $\pi_{jkl}^c$ . Now, the operating cost of compressor stations could be defined as

$$\begin{aligned} \varpi_{jkl}^c &= \sum_{(i,i') \in K_j} \{(\pi_{jk}^f(\mu_{ii'j})) (t_l - t_{l-1}) \\ &+ (\pi_{jkl}^c) (\sum_{i'' \in \Theta_1(i)} \sum_{h=1}^{\vartheta_{ii''j}} V_{hi''ijkl} - \sum_{i''' \in \Theta_2(i')} \sum_{h=1}^{\vartheta_{i'i'''jkl})\}, \\ \Theta_1(i) &= \{i'' | (i, i'') \text{ or } (i'', i) \in P_j\}, \\ \Theta_2(i') &= \{i''' | (i', i''') \text{ or } (i''', i') \in P_j\}. \end{aligned}$$

2.5.3. Constraints of submodel

There are many classes of constraints which limit the values of the two decision variable sets defined for submodel. This section mathematically discusses about these constraints.

Class 1 (Domain of variables):

$$W_{ijkl} \geq 0 \quad \& \quad \text{real}$$

$$i \in \Psi_{jk} \cup \Delta_{jk} \cup T_j \cup H_j$$

$$V_{hi'i'jkl} \text{ is real}$$

$$(i, i') \text{ or } (i', i) \in P_j; \quad h = 1, 2, \dots, \vartheta_{ii'j}.$$

Class 2 (Pressure limits in demand and supply nodes):

$$\gamma_{ijkl} \leq W_{ijkl} \leq \eta_{ijkl}$$

$$i \in \Psi_{jk} \cup \Delta_{jk}.$$

Class 3 (Gas flow direction in the pipes):

$$\text{If } W_{ijkl} > W_{i'jkl} \Leftrightarrow V_{hi'i'jkl} > 0 \text{ for } h = 1, 2, \dots, \vartheta_{ii'j}$$

$$\text{otherwise if } W_{ijkl} < W_{i'jkl} \Leftrightarrow V_{hi'i'jkl} < 0 \text{ for } h = 1, 2, \dots, \vartheta_{ii'j}$$

$$(i, i') \text{ or } (i', i) \in K_j$$

and also

$$V_{hi'i'jkl} = -V_{hi'i'jkl}$$

$$(i, i') \text{ or } (i', i) \in P_j; \quad h = 1, 2, \dots, \vartheta_{ii'j}.$$



Class 4 (Entrance and exit nodes of compressor stations):

$$\begin{aligned} \Theta_1(i) &= \{i'' | (i, i'') \text{ or } (i'', i) \in P_j\}, \\ \Theta_2(i') &= \{i''' | (i', i''') \text{ or } (i''', i') \in P_j\}, \\ (i, i') &\in K_j; \quad i' \in I_j \end{aligned}$$

If  $W_{ijkl} < W_{i'jkl} \Leftrightarrow i \in I_j$  and  $i' \in O_j$ ,  
otherwise if  $W_{ijkl} \geq W_{i'jkl} \Leftrightarrow i \in O_j$  and  $i' \in I_j$ ,  
 $(i, i')$  or  $(i', i) \in K_j$ .

Class 5 (Mass conservation in nodes):

Class 6. Pressure drop

$$\begin{aligned} |W_{ijkl}^2 - W_{i'jkl}^2| &= \zeta_{hi'} V_{hi'jkl}^2, \\ (i, i') &\in P_j; \quad h = 1, 2, \dots, \vartheta_{i'j}, \end{aligned}$$

where  $\zeta_{hi'}$  is the pipe resistant of  $h$ th pipe between nodes  $i$  and  $i'$ .

Class 7. Operational constraints of compressor stations.

Compressor stations cannot increase any pressure to any pressure. These components may not be able to work with any incoming mass flow rate. Let us denote the set of feasible vectors of (incoming mass flow rate, inlet pressure, and outlet pressure) for a compressor station of type  $T$  by  $\Omega(T)$ . Then,

$$\begin{aligned} \left( \sum_{i'' \in \Theta(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi''ijkl}, W_{ijkl}, W_{i'jkl} \right) &\in \Omega(\mu_{i'j}), \\ \Theta(i) &= \{i'' | (i, i'') \text{ or } (i'', i) \in P_j\}, \\ (i, i') &\in K_j, \quad i \in I_j \end{aligned}$$

or equivalently,

$$\begin{aligned} \left( \sum_{i'' \in \Theta(i')} \sum_{h=1}^{\vartheta_{i'j}} V_{hi''i'jkl}, W_{i'jkl}, W_{ijkl} \right) &\in \Omega(\mu_{i'j}), \\ \Theta(i') &= \{i'' | (i', i'') \text{ or } (i'', i') \in P_j\}, \\ (i, i') &\in K_j, \quad i' \in I_j. \end{aligned}$$

### 3. Optimization algorithm

#### 3.1. Limitations of choosing a solver

Although there are various optimization algorithms available in the literature of operations research, many of them are inapplicable to our model because the index range of decision variables of the integrated model has interdependencies. To clarify these interdependencies, let us revisit the definition of the decision variables of the integrated model. We denoted these variables by

$$(1) \quad X_{i'jn} \in \{0, 1, 2, \dots, \rho\};$$

$$\begin{aligned} i < i'; \quad i, i' &\in \Psi_{j,v} \cup \Delta_{j,v} \cup T_j \cup H_{j-1}; \\ (i, i') &\notin K_{j-1}; \quad j = 1, 2, \dots, \lambda; \quad n = 1, 2, \dots, \xi, \end{aligned}$$

$$(2) \quad Y_{i'j} \in \{0, 1, 2, \dots, \sigma\};$$

$$i < i'; \quad (i, i') \in P_{j-1}^* \cup K_{j-1}; \quad j = 1, 2, \dots, \lambda.$$

- Demand nodes

$$\sum_{i' \in \Theta(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'ijkl} = \theta_{ijk}, \quad i \in \Delta_{jk},$$

$$\Theta(i) = \{i' | (i, i') \text{ or } (i', i) \in P_j\}.$$

- Supply nodes

$$\varepsilon_{ijk} \leq \sum_{i' \in \Theta(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'ijkl} \leq \delta_{ijk}, \quad i \in \Psi_{jk},$$

$$\Theta(i) = \{i' | (i, i') \text{ or } (i', i) \in P_j\}.$$

- Transshipment nodes

$$\sum_{i \in \Theta(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'ijkl} = 0, \quad i \in A_j$$

$$\Theta(i) = \{i' | (i, i') \text{ or } (i', i) \in P_j\}$$

- Entrance and Exit nodes of compressor stations.

We assume that the natural gas consumption rate of a compressor station is a function of the incoming mass flow rate, pressure at entrance node and pressure at the outgoing node and the type of station. This function must be defined by the user of the model. Let us denote by  $\kappa(M, I, O, T)$  the mass flow rate function consumed by a compressor station of type  $T$  where the incoming mass flow rate of  $M$  with pressure  $I$  is delivered with an outgoing pressure of  $O$ . Then,

$$\begin{aligned} \kappa \left( \sum_{i'' \in \Theta_1(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi''ijkl}, W_{ijkl}, W_{i'jkl}, \mu_{i'j} \right) \\ = \sum_{i'' \in \Theta_1(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi''ijkl} - \sum_{i''' \in \Theta_2(i')} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'''jkl}, \end{aligned}$$

$$\Theta_1(i) = \{i'' | (i, i'') \text{ or } (i'', i) \in P_j\},$$

$$\Theta_2(i') = \{i''' | (i', i''') \text{ or } (i''', i') \in P_j\},$$

$$(i, i') \in K_j; \quad i \in I_j$$

and also

$$\begin{aligned} \kappa \left( \sum_{i''' \in \Theta_2(i')} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'''jkl}, W_{i'jkl}, W_{ijkl}, \mu_{i'j} \right) \\ = \sum_{i'' \in \Theta_1(i)} \sum_{h=1}^{\vartheta_{i'j}} V_{hi''ijkl} - \sum_{i''' \in \Theta_2(i')} \sum_{h=1}^{\vartheta_{i'j}} V_{hi'''jkl}, \end{aligned}$$

For  $j = 2, 3, \dots, \lambda$ , the sets  $\Psi_{j,d}, \Delta_{j,d}, H_{j-1}, P_{j-1}^*, K_{j-1}$  are known only if the values of decision variables for all previous short horizons are known; hence, since the index range of the decision variables of current short horizon depends on these sets, it means that the index range of decision variables of  $j$ th short horizon depends on the values of the decision variables of all previous short horizons.

Also, another type of interdependency between the decision variables exists. The set  $P_{j-1}^*$  depend on the values of  $X_{i'jn}$  (see Eq. (1)) as well as the values of decision variables of all previous short horizons which means the values of decision variable  $X_{i'jn}$  must be known in order to know the index range of the decision variable  $Y_{i'j}$ .

These interdependencies and the fact that the constraints and objective function of the model are highly nonlinear rule out the application of powerful mathematical programming approaches and direct use of the metaheuristic methods available in the optimization literature.

The key point here is that the optimization algorithm for the model; is not as important as using a robust and complete model because when a model plans the development of the network over a long horizon like 20 years, it does not really matter for the top managers and policy makers if the model optimization run time is 1 minute or 10 days! Most of the effort in the literature has diverted to finding fast solutions to the problem. Considering this fact and the limitations in choosing an optimization method due to interdependencies of the index range of the decision variables, we propose a heuristic random search method in the next section which sequentially and randomly assigns values to decision variables and evaluates the objective function in search of the optimum development plans.

### 3.2. Steps of the algorithm

We define a complete solution as a feasible combination of development plans which satisfies all constraints of the integrated model. In other words, a complete solution consists of the values of all decision variables for all short horizons. Also, a feasible development plan for a specific short horizon is simply called a “solution”. Therefore, a complete solution is a combination of solutions of all short horizons.

The optimization algorithm of the model has an iterative procedure which produces a number of complete solutions in each iteration and evaluates their objective functions. This iterative algorithm continues until one or more of the terminating conditions of the algorithm are satisfied. The best found complete solution at the end of the algorithm converges to global optimum if the number of iterations goes to infinity. As examples of the terminating conditions, one may use maximum number of iterations, maximum number of stalled iterations (consecutive iterations without any improvement in the objective function of the best found complete solution), etc.

The algorithm constructs a “solution tree” in each iteration which is a set of complete solutions. This tree is

formed as follows. First a feasible solution is created for the first short horizon (this solution is called the “trunk” of the tree or the “first level branch”; we explain later how a feasible solution may be created). Knowing the values of the decision variables of the first short horizon (trunk solution in the tree), the algorithm creates  $b_2$  different solutions for the second short horizon (these solutions are called “second-level branches”). The trunk solution plus any one of the second-level branches determine development plans of the first and second short horizons. In the next step, the algorithm creates  $b_3$  solutions for the third short horizon given the trunk solution plus each one of the second level branches. In a general step  $j$ , the algorithm creates  $b_j$  solution for short horizon  $j$  given any path from the trunk of the tree to the end of any of the  $(j-1)$  th-level branches. Using this procedure, a solution tree would have  $\prod_{j=2}^{\lambda} b_j$  complete solution where a complete solution is any path in the graph of the tree from the first-level branch (trunk) to any one of the  $\lambda$ th-level branch. Here,  $b_j : j = 2, 3, \dots, \lambda$  is a user-defined parameter.

A feasible solution for a short horizon is created with a random search method. The algorithm first assigns values from set  $\{0, 1, 2, \dots, \rho\}$  to  $X_{i'jn}$  at random (with a limited maximum number of parallel pipes) and knowing the assigned values of  $X_{i'jn}$ , the algorithm, then, randomly assigns values from set  $\{0, 1, 2, \dots, \sigma\}$  to  $Y_{i'j}$ . Knowing the values of both decision variables, the constraints of the integrated model are checked to determine the feasibility of the solution comprised of the values of both decision variables. If all constraints are satisfied, a feasible solution has been produced. Verification of constraints of integrated model requires running the control optimization submodel for each period of each year of the corresponding short horizon. We use a genetic algorithm approach for finding (near) optimal solution of the submodel.

When a tree is constructed in an iteration, the objective function of each complete solution in the tree is evaluated using the objective function of the integrated model which consists of capital cost of the new physical components of the network to be installed in development plans plus the optimum operating cost of utilizing the network during the long-run horizon obtained by running the submodel. The best combination of development plans after a number of iterations of the optimization algorithm would be the created complete feasible solution with least objective function.

## 4. Case study

This section demonstrates the application of the model and designed heuristic optimization algorithm to a simplified case study.

### 4.1. Definitions and parameters

An arbitrary study area requires a strategic plan for development of an existing natural gas transmission

pipeline network in order to satisfy the demand of a number of major consumers from January 1, 2010 to December 31, 2020 ( $\omega = 10$ ). This 10-year long-run horizon is split into two 5-year short horizons ( $\lambda = 2$ ,  $\nu = 5$ ) and the goal is to find 2 development plans with least discounted operating and capital costs. We assume there is ample time before 2010 to implement the first development plan. Table 3 summarizes some parameters of the model of this case study. Fig. 1 shows the existing network before the development plans are implemented and Table 4 provides the location of the nodes in Fig. 1. Tables 5 and 6 show the type of pipelines and compressor stations in the available network, respectively.

We consider a 25km square grid in the map for the location of the transshipment nodes for both development plans. Also, another assumption is that no new demand or supply node would be added during the long-run horizon.

Table 7 shows the forecasted consumption rate of the demand nodes in each short-run horizon (for simplicity, we assumed one period in each year and similar consumption rate in each year). Table 8 shows the range of production at supply nodes over the long-run horizon. The supply nodes can provide natural gas with a pressure between 700 and 1050 psi. The demand nodes require a gas pressure with the same range. Also, the pressure of gas entered to a compressor station may not fall below 700psi and the maximum outlet pressure from a compressor station could not be more than 1050 psi.

Table 9 shows the price of one unit of natural gas over the long run horizon. For simplicity, we assume the incoming and outgoing mass flow rates of compressor stations are equal (this is a bit different from our assumption in the model that compressor stations consume a fraction of flowing natural gas). Consequently, we assume the operating cost (dollar) of a compressor station for a short horizon is calculated as follows:

$$\text{operating cost} = 5 \times 40,000 \times V_{in} \left( \frac{P_{out}}{P_{in}} \right)^{0.4/1.4},$$

where  $V_{in}$  is the daily volumetric incoming flow rate,  $P_{in}$  the inlet pressure and  $P_{out}$  the outlet pressure.

We also calculate the capital cost (dollar) of a compressor station throughout the long horizon using the

Table 3  
Parameters of the model

| Parameter | Value |
|-----------|-------|
| $\omega$  | 10    |
| $\nu$     | 5     |
| $\lambda$ | 2     |
| $\tau$    | 1     |
| $\rho$    | 14    |
| $\sigma$  | 1     |
| $\xi$     | 2     |
| $\chi$    | 0%    |

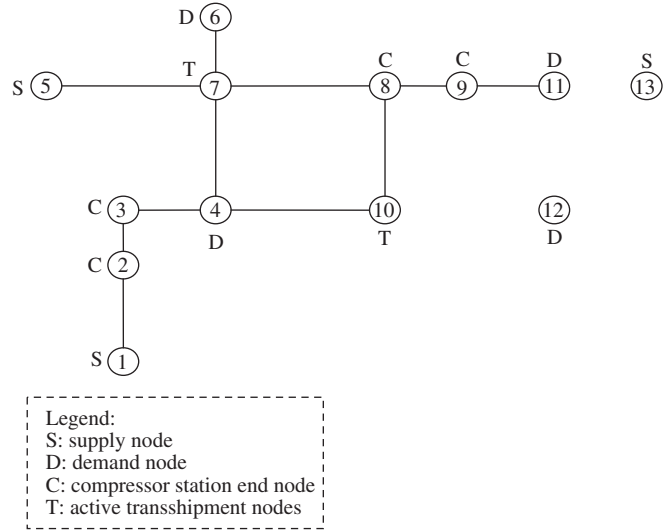


Fig. 1. The graph of the available network before implementation of the first development plan.

Table 4  
Location of the nodes of the available network before implementation of the first development plan

| Node | Location (km, km) |
|------|-------------------|
| 1    | (25,0)            |
| 2    | (25,100)          |
| 3    | (25,100)          |
| 4    | (50,100)          |
| 5    | (0,150)           |
| 6    | (50,175)          |
| 7    | (50,150)          |
| 8    | (75,150)          |
| 9    | (75,150)          |
| 10   | (75,100)          |
| 11   | (125,150)         |
| 12   | (125,100)         |
| 13   | (175,150)         |

Table 5  
Types of pipeline edges of the available network

| Edge   | Type |
|--------|------|
| (1,2)  | 8    |
| (3,4)  | 8    |
| (5,7)  | 10   |
| (6,7)  | 9    |
| (4,7)  | 8    |
| (7,8)  | 8    |
| (8,10) | 8    |
| (9,11) | 8    |
| (4,10) | 8    |

following formula:

$$\text{operating cost} = 485 \times 877 \times V_{in}.$$

Table 10 shows the capital cost of installing 100 km of pipelines in the study area throughout the long horizon (we assume installation location doesn't affect the cost).

Table 6  
Types compressor station edges of the available network

| Edge  | Type |
|-------|------|
| (8,9) | 1    |
| (2,3) | 1    |

Table 7  
Forecasted consumption rates of the demand nodes

| Demand node | Consumption rate throughout the first short horizon (million m <sup>3</sup> ) | Consumption rate throughout the second short horizon (million m <sup>3</sup> ) |
|-------------|---|--|
| 4           | 7   | 11   |
| 6           | 20  | 22   |
| 11          | 12  | 13   |
| 12          | 9   | 18   |

Table 8  
Range of production rates of the supply nodes

| Supply node | Supply rate throughout the first short horizon (million m <sup>3</sup> ) |     | Supply rate throughout the second short horizon (million m <sup>3</sup> ) |     |
|-------------|--|-----|---|-----|
|             | Min  | Max | Min   | Max |
| 1           | 5  | 10  | 10  | 15  |
| 5           | 25   | 40  | 35  | 50  |
| 13          | 0  | 30  | 0   | 40  |

Table 9  
Price of natural gas

| Supply node | Forecasted price throughout the first short horizon (\$/million m <sup>3</sup> ) | Forecasted price throughout the second short horizon (\$/million m <sup>3</sup> ) |
|-------------|--|---|
| 1           | 30   | 33  |
| 5           | 36   | 42  |
| 13          | 45   | 49  |

4.2. Results

We used 1000 iterations of the optimization algorithm with  $b_2 = 3$ , which means that the algorithm has created 3000 complete solutions. The optimum complete solution found in the first 1000 iterations had very simple development plans which suggest these solutions:

- Optimum solution of the first short horizon (requirements of development plan 1):
  - Install a pipeline of type 7 between nodes 11 and 13 ( $X_{11,13,1,1} = 7$ ).
  - Install a pipeline of type 7 between nodes 11 and 12 ( $X_{11,12,1,1} = 7$ ).

- Optimum solution of the second short horizon (requirements of development plan 2):
  - Do not change the network.

Fig. 2 shows the network after implementing the first development plan (since the network does not change in development plan 2, the figure represents the available network during short horizon 2 as well).

The total operating cost during the long horizon for the optimal complete solution was \$14,020,969 and the capital cost was \$14,000,000. Since we set the discount rate equal to zero, the NPW (objective function) of the optimal complete solution is simply the sum of the operating and capital cost, \$28,020,969.

One of the possible reasons that the optimization algorithm recommended no change to be made in development plan 2 is that the discount rate was zero, which means paying capital cast late and saving cash at the beginning of the long horizon has no advantage.

5. Discussion

Development of natural gas transmission networks could affect different sectors such as economy, society, environment, and politics. So far, we discussed about the most evident issues in network development planning including technical, structural and economics of the pipeline network. In this section, we discuss how other issues such as environmental, social, safety, and political criteria could be incorporated into a typical network development project in general and our model in particular.

Some parts of these issues are beyond the scope of the model of this paper. The proposed model covers only the first phase of a typical network development project as outlined in Table 2 and for instance, many of the environmental concerns could be considered in the last phases of a network development project (installation and inspection) and are unrelated to our strategic planning model.

Table 10  
Capital cost of installing 100 km pipeline

| Pipeline type | Installation cost (million\$) |
|---------------|-------------------------------|
| 1             | 4                             |
| 2             | 6                             |
| 3             | 8                             |
| 4             | 10                            |
| 5             | 12                            |
| 6             | 16                            |
| 7             | 20                            |
| 8             | 24                            |
| 9             | 30                            |
| 10            | 36                            |
| 11            | 40                            |
| 12            | 42                            |
| 13            | 48                            |
| 14            | 56                            |

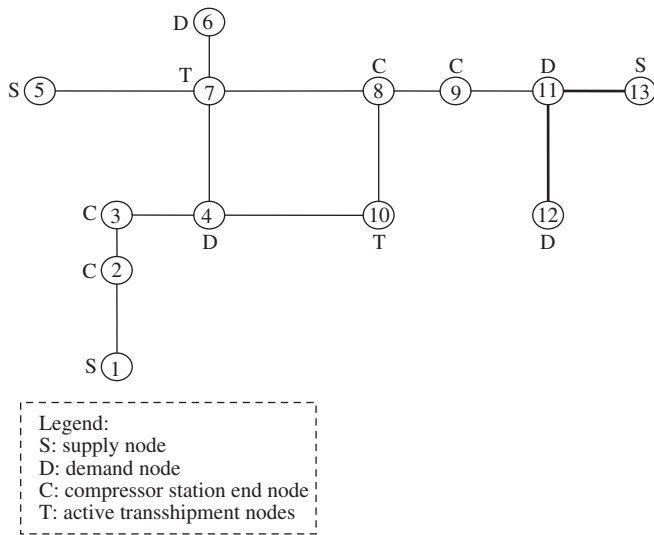


Fig. 2. The graph of the network after implementing optimal development plan 1.

Another key point is that some of the criteria whose effects are important on network development planning are satisfied when the network itself develops even if these criteria are not directly considered in the model (for example, see Sections 5.1). In multiobjective optimization language, such criteria are not in conflict with the objective function of the model and one need not worry about them.

Multiobjective optimization methods are useful only for conflicting criteria of network development planning. There are many types of multiobjective functions; two well-known types of them are (1)  $\min\{\max_{x \in F} f_1(x), f_2(x), \dots, f_k(x)\}$  (or similarly  $\max \min$ ) and (2)  $\min_{x \in F} f_1(x), f_2(x), \dots, f_k(x)$ , where  $x$  is the decision vector,  $F$  the set of feasible decision vectors, and  $f_i$  is the  $i$ th objective function. For the first type, one may add some constraints to the problem and reduce the objective function to a single-objective function (see Hillier and Liberman, 1997). The most widely used technique for the second type consists of reducing the multiobjective problem to a single objective one by means of the so-called “scalarization” procedure. Various scalarization procedures may be employed:

1. *Cost function method*: Each objective function is assigned a cost coefficient and then the function obtained by summing up all the objective functions scaled by their cost coefficients is optimized.
2. *Goal programming method*: An “ideal optimal value” for each objective function is chosen and then the Euclidean/absolute distance between the actual vector of objective functions and the vector made up of the ideal values is optimized.

The optimization algorithm described in Section 3 is basically a numerical optimization method, i.e. it only needs numerical values of objective function and does not

use the form of the objective function. Hence, this property along with the fact that the constraints of the model are not changed with different objective functions enable the model to have multiobjective functions if any of the above multiobjective techniques are employed to transform the multiobjective function to a single one.

In addition to the ability of the model to accept a completely new multiobjective function, we also believe that current cost-based single-objective function of the model has the potential to account for some other conflicting criteria if an appropriate cost function method briefly explained above is employed. For instance, one may measure environmental or safety criteria with “money” and add these cost to the cash flow of the current NPW objective function or select appropriate locations for transshipment nodes (we elaborate upon this approach in the subsequent sections).

### 5.1. Environmental concerns

Sustainable development requires protection of natural environment; and expansion of a natural gas transmission network could have significant impacts on environment. Some environmental criteria of network planning have clear conflicts with the economy of expansion in certain geographical areas, but some others do not.

Natural gas is a green fossil fuel which pollutes the environment less than the others. Because of its clean burning nature, the use of natural gas wherever possible, either in conjunction with other fossil fuels or instead of them can help reduce the emission of harmful pollutants. In this view, development of the network in our model even with its microeconomic goals improves environmental indicators.

On the other hand, preservation of trees and natural habitats is a concern that sometimes conflicts with economic objectives of the network development plans because an economically optimum plan may require cutting trees or trespassing protected habitats. The planner in this situation may be presented with two options:

1. Pay the cost of violating the environment (cutting trees, trespassing habitats, or any harmful interference in ecosystem); this cost may be paid to certain institutions (for example to obtain the right of way) or spent for compensating the impacts on environment by turning the environmental situation after implementing the plan to the position before that.
2. Money cannot do anything. Violation of the environment in the area is absolutely prohibited.

The capital cost of installing a pipeline in the objective function of the integrated model was a function of the location of the installation. For the first option above which let the planner pay the cost of installing pipes in an environmentally protected area, this cost could easily be considered in the capital cost of the pipeline passing the

protected area; and for the second option, no transshipment node is considered inside the protected area and an infinite cost of installation is assumed in the model which recommends the optimization algorithm not to trespass the absolutely protected area or pay an infinite cost.

### 5.2. *Social issues*

Social justice is based on the idea of a society which gives individuals and groups fair treatment and a just share of the benefits of society. Governments seek to provide a minimum level of income, service, or other support for disadvantaged peoples to improve indicators of social justice and welfare. Natural gas is one of these services that could be used as a tool to pursue certain social policies.

A common problem in developing countries is centralization of the country resources, income and welfare in limited metropolitan areas (such as Tehran, Khomeinishahr, Shahinshahr and Varnosfaderan State in Iran). This geographically unbalanced development causes many social and economical problems for people living far from these developed areas. These problems include migration to metropolitan areas, unemployment, and in general, poor standards of living and quality of life outside major cities. In the perspective of natural gas development planning, the governments may indirectly subsidize the physical development of the network in certain geographical areas to supply cheaper gas to rural and undeveloped areas and share the benefits of using this source of energy more fairly and improve social justice. These subsidies could be subtracted from the capital cost of installing pipelines and compressor stations for undeveloped areas in our model.

### 5.3. *Reliability and safety*

The integrity of the pipeline network and its related equipment is one of the industry's top concerns. The threat of catastrophic pipeline rupture, though extremely unlikely given the industry's safety precautions, hangs over the head of every pipeline executive and employee. Pipelines require regular patrol, inspection, and maintenance including internal cleaning and checking for signs of gas leaks.

Poor reliability and safety measures in a network could result in various failures and damages. Corrosion is a serious problem plaguing the industry. This is a sneaky enemy and unless it has caused obvious damages, corrosion is very difficult to detect and locate accurately. To minimize corrosion, pipeline companies install electrical devices called cathodic protection systems, which inhibit electrochemical reactions between the pipe and the surrounding materials. Mechanical damages also occur when heavy construction equipments dent the pipe, scrape off its coatings, gouge the metal or otherwise deform the pipe in some way. However, if the pipeline has to be shut

down for any reason, the production and receiving facilities and refineries often also have to be shut down because gas cannot be readily stored, except perhaps by increasing the pipeline pressure by some percentage. All these damages and failures involve cost.

Reliability and safety criteria could be incorporated into the cost-based objective function of the model if the modeler could measure the level of reliability in cost. The reliability cost has two parts. The first part is a fixed cost which is paid at the time of installing a physical component of the network. This cost should be considered in the capital cost of the component (pipeline or compressor station). The second part is the cost of inspections and safety measures after the physical components of the network are installed. Obviously, this part of the reliability cost is a part of the operating cost.

The development plans in the model are based on the forecasts of the consumption in demand nodes and production range in supply nodes. But, the actual production or consumption during the long horizon may deviate from forecasts. The network must be reliable in this situation to provide actual demanded natural gas of the consumers during the long-run horizon. The risk of poor reliability in this case is significant because inadequacy of supplied natural gas to certain consumers like large manufacturing industries or power plants could shut down these facilities for a longer period of time than a failure in the network. Also, when inadequate natural gas is supplied, consumers may use more polluting fuels and this problem leads to environmental damages. However, this problem could be addressed in the model by applying a margin of safety to the forecasts or accounting for the cost of probable inadequate supply of natural gas in the objective function.

### 5.4. *Politics*

Regional economic collaborations possess the power to engender as well as transform social and political discourse between countries. Energy ties play a more important role in the economics and politics of energy rich countries. Such regional and international energy relations sometimes serve the political goals of the countries. For instance, since the discovery of natural gas reserves in Iran's South Pars fields in Persian Gulf in 1988, the Iranian government began increasing efforts to promote higher gas exports abroad. The export of this energy resource could end political isolation of the country. With this aim, Iran is negotiating with India and China to export natural gas to these large regional consumers in the near future. In our model, each natural gas export terminal in the long-run horizon is a demand node; therefore, the export quantities at these nodes should be forecasted. The governments may subsidize installing physical components of network structure and reduce associated capital costs to encourage the export. Such policies could help political interests.



## 6. Conclusion

In this paper, we developed a new model for finding the optimal development plans over a long horizon. This nonlinear model accounts for various engineering and economic factors related to the problem. The objective function is essentially the NPW of all related operating and investment costs paid during the long-run study horizon. We study the feasibility of the development plans through different constraints. In addition to the planning model, a heuristic random search optimization algorithm was developed which finds (near) optimal solutions of the model. A case study was used to demonstrate the application of the model and optimization algorithm.

Although our model was originally designed to meet the planning difficulties in Iran with a noncompetitive gas market and monopoly decision-making environment, but we believe it could be applied to other cases as well. The major step in implementing the model is collecting the necessary information to feed into model. Accuracy of the data and forecast could improve the optimality of development plans suggested by running the model. We are working on implementing the model on Iran's network. We also intend to extend the applicability of the model by releasing some assumptions and introducing new cost elements in the objective function.

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