SP-145



X SEPOPE

 $\begin{array}{l} 21 \ a \ 25 \ de \ maio \ de \ 2006 \\ May - 21^{st} \ to \ 25^{th} - 2006 \end{array}$

FLORIANÓPOLIS (SC) – BRASIL

X SIMPÓSIO DE ESPECIALISTAS EM PLANEJAMENTO DA OPERAÇÃO E EXPANSÃO ELÉTRICA X SYMPOSIUM OF SPECIALISTS IN ELECTRIC OPERATIONAL AND EXPANSION PLANNING

INTEGRATED ELECTRICITY-GAS OPERATIONS PLANNING IN HYDROTHERMAL SYSTEMS

B.Bezerra, R.Kelman, L.A.Barroso, B.Flach, M.L.Latorre, N.Campodónico and M.V.F.Pereira

Power System Research/Mercados de Energia

Brazil

SUMMARY

The integration of natural gas and electricity sectors has increased sharply in the last decade as a consequence of combined cycle natural gas thermal power plants. In some countries such as Brazil, gas-fired generation has been a major factor in the overall growth of natural gas consumption.

Brazil's National System Operator dispatches these gas-fired plants (along with other thermal sources such as coal, oil and nuclear) in conjunction with the country's hydroelectric plants using a stochastic dual dynamic programming (SDDP) scheme. The SDDP algorithm determines the optimal hydro-to-thermal energy production ratio based on the expected benefit of reducing thermal plant generation over a large number of hydrological scenarios, along a planning horizon of five years. This means that the optimal scheduling decision today depends on assumptions about future load growth and future entrance of new generation capacity. However, the hydrothermal scheduling model does not take into account the possibility of future fuel supply constraints, either in production or in transportation. The assumption of fuel supply adequacy is felt to be reasonable for the more mature markets such as coal and oil. However, due to the fast growth of the natural gas market, it is possible that demand outpaces supply and/or transportation investments. A first indication that gas-related constraints could be relevant took place in January 2004, when 800 MW of combined-cycle generation (out of a total capacity of 1200 MW) could not be dispatched due to constraints in pipeline capacity.

The objective of this work is present a methodology for representing the natural gas supply, demand and transportation network in the stochastic hydrothermal power scheduling model. Gas demand in each node is given by the sum of non-power gas consumption forecasts plus gas consumption factors for the gas-fired power plants; gas production in each node is represented as minimum and maximum production levels, depending for example if the gas field is associated with oil production. Finally, fuel transportation is modeled both through pipelines and through LNG.

The application of the integrated electricity-gas scheduling model is illustrated in case studies with realistic configurations of the 90 GW Brazilian system.

KEYWORDS

Natural gas industry, Hydroelectric-thermal power generation, stochastic dual dynamic programming.

R. Kelman and L.A.Barroso are also with COPPE/UFRJ, Rio de Janeiro, Brazil.

B. Bezerra is also with PUC-Rio, Rio de Janeiro, Brazil.

1. Introduction

The integration of natural gas and electricity sectors was intensified in the last decade as consequence of a widespread construction of new gas-fired power plants, both combined-cycle and single-cycle. Brazil has followed the trend by building over 7,000 MW of gas-fired generation in the last years. These gas-fired plants, along with other thermal sources such as coal, oil and nuclear, correspond to 15% of the country's installed capacity; the major source of power production being hydroelectric power. Both hydro and thermal plants are dispatched by the country's National System Operator (ONS, in Portuguese) with basis on a stochastic dual dynamic programming (SDDP) scheme. The SDDP algorithm determines the optimal hydro-to-thermal energy production ratio based on the expected benefit of reducing thermal plant generation over a large number of hydrological scenarios, along a planning horizon of five years. This means that the optimal scheduling decision today depends on assumptions about future load growth and future entrance of new generation capacity. However, the hydrothermal scheduling model does not take into account the possibility of future fuel supply constraints, either in production or in transportation. The assumption of fuel supply adequacy is felt to be reasonable for the more mature markets such as coal and oil. However, due to the fast growth of the natural gas market, it is possible that demand outpaces supply and/or transportation investments. A first indication that gas-related constraints could be relevant took place in January 2004, when 800 MW of combined-cycle generation (out of a total capacity of 1200 MW) could not be dispatched due to constraints in pipeline capacity.

The objective of this work is present a methodology for representing the natural gas supply, demand and transportation network in the stochastic hydrothermal power scheduling model. Gas demand in each node is given by the sum of non-power gas consumption forecasts plus gas consumption factors for the gas-fired power plants; gas production in each node is represented as minimum and maximum production levels, depending for example if the gas field is associated with oil production. Finally, fuel transportation is modeled both through pipelines and through LNG.

This work is organized as follows: Section 2 presents an overview of the electricity and gas sectors in Brazil. Section 3 describes and motivates the main issues in the energy-gas integration in the country. Sections 4 presents a procedure that has been developed for assessing the feasibility of the schedules of the gas-fired power plants. Section 5 presents the integrated representation of the electricity-gas sectors in a hydrothermal scheduling model and Section 6 concludes.

2. Overview of Electricity and Gas Sectors

Brazil is the largest electricity market in South America, accounting for 40% of the continent's energy consumption. As mentioned in the Introduction, the country is hydro-dominated: 85% of the 90 GW installed capacity and more than 90% of the electricity production (44 average GW) comes from hydropower. Thermal generation includes nuclear, coal, diesel, biomass and, more recently, natural gas plants. The country is fully interconnected at the bulk power level by a 80,000 km meshed high-voltage transmission network, shown in Figure 1. The direct international interconnections are the back-to-back links with Argentina (2,200 MW) and smaller interconnections with Uruguay and Venezuela.

On the natural gas side, Brazil has proven gas reserves of 320 bcm [1,2]. The country also has a natural gas production¹ of about 27 MMm^3 /day available to the market, mostly associated with the exploration of oil. Since 1999 up to 30 MMm^3 /day of imported natural gas has been flowing into the country through





¹ This number excludes reinjection, E&P consumption and flares & losses

pipelines from Bolivia and Argentina. In 2003 a discovery of a large offshore natural gas field (Santos field), capable of more than doubling the country's reserves, was announced.



figure 2 – Natural gas network (source ANP & PSR)

In contrast with Argentina and Chile, Brazil's gas market is relatively undeveloped. One of the reasons is that there is no market for space heating, which is an important factor in the other countries.

Figure 2 shows the gas pipelines and the areas of exploration and production. There are three separate systems: the largest comprises the South and Southeast regions; coastal cities from the Northeast form the country's second natural gas system; the third system is in the Amazon region.

Finally, a Natural Gas law which regulates pipeline access and other topics is currently being discussed in Congress.

3. Electricity-natural gas integration issues

As mentioned previously, Brazil has 7000 MW of gas-fired plants. Their potential gas consumption is quite significant: if dispatched simultaneously, the gas-fired plants would use 35 MMm³/day of gas, about the same amount as the entire "non-power" gas demand. Also as mentioned previously, the thermal plants' dispatch depends on the hydrological conditions: if the system is "wet", the entire electricity load

can be met with hydro generation alone.

In other words, power-related gas consumption is both large and stochastic. This creates a complex problem for investment decisions in new gas fields and in new pipelines, which may be either excessive or insufficient, depending on hydrological conditions. Although take or pay contracts can alleviate part of the financial uncertainty, a mismatch between gas supply and demand can have significant consequences for power scheduling. One example of this mismatch happened in January 2004, when a shortage of hydropower in the Northeast of Brazil made ONS command the dispatch of 1,200 MW of gas-fired plants of the region and only a third of this (400 MW) was delivered due to gas production and transportation constraints. This episode showed the need for greater coordination between the electricity and the natural gas sectors' operations planning.

4. Probabilistic evaluation of gas-fired plant schedules

We initially developed a probabilistic model for evaluating whether the sum of gas consumption requirements resulting from the hydrothermal dispatch and of "non-power" gas consumption forecasts

could be adequately supplied by the existing and planned gas fields and pipeline network. Figure 3 shows the information flow. The upper shaded area shows the first step of the process: the use of a hydrothermal tool based on Stochastic Dual Dynamic Programming - SDDP [7,8] that dispatches the power system for a given electric supply xdemand configuration. The result of interest is a set of power generation scenarios for each gas-fired power plant stage in each and simulated hydrological scenario for the study horizon. From these results and the



Figure 3 – data flow procedure

consumption rates of each plant, the projection of the gas consumption for power is immediately obtained. The simulation is carried out for a set of hydrological scenarios, yielding a corresponding set of natural gas consumption scenarios. The shaded area in the lower part of Figure 3 represents the scheduling of the gas sector and verifies the "feasibility" of these scenarios under the gas sector point of view.

4.1. Stochastic hydrothermal scheduling model

The objective of hydrothermal scheduling is to determine an operation strategy of a hydrothermal system that for each stage of the planning period produces generation targets for each plant. This strategy should minimize the expected value of the operation cost along the period, composed of fuel cost and penalties for failure of load supply [7]. Hydro plants are dispatched based on their marginal water values, which are computed by a multi-stage stochastic optimization methodology, Stochastic Dual Dynamic Programming (SDDP). The SDDP algorithm has been applied to the scheduling of large-scale power systems in more than thirty countries, including detailing modeling of system components and transmission networks [8]. However, as mentioned previously, the dispatch model did not consider the gas supply-transportation constraints. A simplified formulation of the one-stage problem solved in the SDDP recursion is shown next; further details can be found in [7-12].

4.1.1. Objective function

$$\alpha_{t} (v_{t}, a_{t-1}) = \operatorname{Min}\mathbf{Erro!} \mathbf{Erro!} c_{j} \times g_{tk}(j) + c_{\delta} \times \delta + \alpha_{t+1}(v_{t+1}, a_{t})$$
(1.1)

1	
where.	
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	

where.	
k	indexes load block in the stage
K	number of load blocks
j	indexes thermal plants
J	set of thermal plants
c _i	operating cost of plant j
$g_{tk}(j)$	Energy produced by thermal plant <i>j</i> (decision variable)
c_{δ}	generic representation of operating constraint violation cost
δ	violation amount (decision variable)
v_{t+1}	final storage vector in stage t (decision variable)
$a_{\rm t}$	lateral inflow vector in stage t

The Future Cost Function is expressed as a scalar variable subject to linear inequalities (Benders cuts).

s.t.
$$\alpha \geq w_{t}(p) + \mathbf{Erro!} \lambda_{tv}(i, p) \ v_{t+1}(i) + \mathbf{Erro!} \lambda_{ta}(i, p) \ a_{t}(i) \qquad p = 1, \dots, P$$
(1.2)

α	scalar variable that represents expected future operating cost
Р	indexes segments of the piecewise future cost function
$w_{t}(p)$	constant term of p^{th} segment
$\lambda_{tv}(i, p)$	plant <i>i</i> 's final storage coefficient in the p^{th} segment
$\lambda_{ta}(i, p)$	plant <i>i</i> 's lateral inflow coefficient in the p^{th} segment
p	number of segments in the piecewise future cost function

4.1.2. Water balance equations

$$v_{t+1}(i) = v_t(i) + a_t(i) - \varepsilon(v_t(i)) - \mathbf{Erro!}[u_{tk}(i) + s_{tk}(i)] + \mathbf{Erro!} \quad \mathbf{Erro!}[u_{tk}(m) + s_{tk}(m)]$$

for $i \in I$ (1.3)

where:iindexes hydro plantsIset of hydro plantsM(i)set of upstream plants immediately upstream of plant i $v_{t+1}(i)$ final storage of i in stage t (decision variable)

$v_{\rm t}(i)$	initial storage of <i>i</i> in stage <i>t</i>
$a_{\rm t}(i)$	lateral inflow to plant <i>i</i>
$\epsilon(v_{t}(i))$	evaporated volume from reservoir <i>i</i>
$u_{tk}(i)$	turbined outflow volume of plant <i>i</i> along stage <i>t</i> in load block <i>k</i> (decision variable)
$s_{\rm tk}(i)$	spilled outflow volume of plant <i>i</i> along stage <i>t</i> in load block <i>k</i> (decision variable)

4.1.3. Bounds on storage, turbined volumes and thermal generation variables

$v,(i) \leq v_t(i) \leq v,(i)$	for $i \in I$	(1.4)
_		

$$\begin{aligned} u_{tk}(i) &\leq u, \ _{t}(i) & \text{for } i \in I; \ k = 1, \dots, K \\ g_{-tk}(j) &\leq g_{tk}(j) \leq g, \ _{tk}(j) & \text{for } j \in J; \ \text{for } k = 1, \dots, K \end{aligned}$$
 (1.5)

4.1.4. Load balance equation

Erro! $g_{tk}(i)$ + **Erro**! $g_{tk}(j) = D_{tk}$ for k = 1, ..., K (1.7)

4.2. Probabilistic Gas Scheduling Model

A gas network consists of supply nodes, where the gas is injected into the system; demand nodes where gas flows out of the system due to thermal power or non-thermal use; and intermediate nodes. A pipeline is represented by an arc linking the nodes. When modeling gas pipelines for short-term scheduling studies, the gas flow through pipelines depends on the pressure difference between the entry and exit nodes; also, nonlinear expressions relate flow limits with the pressure in the pipeline [see e.g. 3,4,5,6]. For the purposes of the present study – long-term planning, with a monthly step – a linear network flow model was felt to be adequate.

4.2.1. Gas Production and flow limits

Local production sources may be available at each node of the gas system. Operational constraints may impose daily minimum and maximum limits, represented by the following set of equations:

$$P_{t}(n) \le P_{t}(n) \le P_{t}(n) \qquad \text{for } n \in \mathbb{N}$$

$$(2.1)$$

where $P_t(n)$ is the gas production at node *n* (decision variable), stage *t* and the pair $\{P_{t}(n), P_{t}(n)\}$ is respectively the minimum and maximum production limits at node *n*, stage *t* represents the production curve of the gas field. Finally N is the set of gas nodes.

The nodes of the gas system are interconnected by pipelines. Each pipeline can be characterized by its maximum and minimum flow limits under equilibrium (steady state) conditions, originating the following constraints:

$$f, t(n,l) \le f_t(n,l) \le f, t(n,l) \qquad \text{for } n,l \in \mathbb{N}$$

$$(2.2)$$

where $f_t(n,l)$ is natural gas flow in the pipeline (decision variable) that connects nodes *n* and *l* and the pair $\{f, (n,l), f, (n,l)\}$ is respectively the minimum and maximum flow limit between nodes *n* and *l*.

4.2.2. Gas Balance equations

At each stage, the sum of the demands at each node must be equal to the sum of the supply – either locally produced or imported through the pipelines – and of the deficit – in case there is not enough natural gas to completely fulfill the demand. For each node of the gas system, we have:

 $P_{t}(n) + \mathbf{Erro!}[1-w_{t}(n,l)] f_{t}(l,n) - \mathbf{Erro!} f_{t}(n,l) + \mathbf{Erro!}\delta_{t}(k) + \mathbf{Erro!}\delta_{t}(j) = \mathbf{Erro!}d_{t}(k) + \mathbf{Erro!}\phi_{t}(j) g_{t}^{*}(j)$

for
$$n \in \mathbb{N}$$
 (2.3)

where $\Omega(n)$ is the set of nodes of the gas system connected to node n, T(n) is the set of thermal plants associated to node n of the gas system and D(n) is the set of non-thermoelectric demands at node n of the gas system (distribution companies, refineries, and others). The parameters are: $w_t(n,l)$ for the loss factor of the pipeline connecting nodes n and l and $\phi_t(j)$ for the gas consumption conversion factor for thermal plant j and $d_t(k)$ is the non-electric natural gas demand k. The generation of the gas-fired plant j, $g_t^*(j)$ is also known in this context, as it is obtained from the hydrothermal scheduling simulation.

The decision variables of the problem are: (i) scheduling of gas supply sources; (ii) scheduling of gas flows in the pipelines and (iii) deficits of natural gas for non-electrical demand k, $\delta_t(k)$ and the deficit of natural gas for thermal power plant j, $\delta_t'(j)$. They appear in the objective function associated with costs c_k and c_j' - the deficit cost for the natural gas non-electrical demand k and the electrical demand j, respectively.

4.2.3. Objective Function

The objective function is to minimize the natural gas rationings costs, thus:

Min Erro!
$$c_k \delta_t(k) +$$
Erro! $c_i \dot{\delta}_t(j)$

4.3. Case Study

In this section we apply the procedure shown in Figure 3 for a case study. The probabilistic evaluation scheme will be illustrated with basis on the (publicly available) power system configuration of the Brazilian Monthly Operations Plan ("PMO") for December 2005-December 2009. The stochastic operational policy for 2005/2009 was calculated (with five additional years as a buffer to prevent depletion at the end of the period). Monthly steps were used, with three demand blocks in each step. Once the hydrothermal operational policy was calculated, the system operation was simulated for a set

of hydrological scenarios, resulting in energy production schedules for each gas-fired power plant, for each month and for each hydrological scenario.

Next, these energy production schedules were transformed into gas consumption schedules, though the use of efficiency factors for each power plant. Finally, these gas schedules were added to the "nonpower" gas consumption forecasts at the appropriate consumption nodes.



(2.4)

Figure 4 – South/Southeast gas network

Table I shows the gas supply projections, including production increase in local fields and imports. Figure 4 shows the pipeline network for the South-Southeast region. A similar procedure was applied for the Northeast network (remember that the gas networks are not integrated yet).

Finally, the "non-power" gas consumption was estimated for each sector (industrial, automotive, commercial, residential and co-generation), in addition to Petrobras (Brazil's oil and gas company) internal consumption in refineries and fertilizer plants. Figure 5 compares total supply and demand for the years of study. We see that the gas consumption from thermal plants is crucial for the demand x supply balance: if the thermal plants are not dispatched at all along the year (zero consumption of power-related gas), supply exceeds demand; at the other extreme, if the thermal plants are 100% dispatched along the year (base-loaded), supply cannot match demand. Given that the thermal plant

dispatch depends, as seen previously, on hydrological conditions and on the overall supply vs demand balance of the electricity sector, the question is then to assess the likelihood and severity of the gas supply shortfalls.

Table I – Gas supply projection available to market

(MMm3/day))	2006	2007	2008	2009
South/South	east				
	Campos	14.4	14.9	15.5	15.0
a + Lagosta	Merluz	1.2	1.9	1.9	1.9
	Gasbol	30.0	30.0	30.0	30.0
	TSB	0.0	0.0	0.0	0.0
	Santos	0.0	0.0	12.0	12.0
Total		45.6	46.8	59.4	58.9
Espírito San	to				
Total		4.4	6.6	10.0	10.0
Northeast					
Total		14.2	15.4	14.4	13.4
Brazil					
Total		64.2	68.8	83.8	82.3



Figure 5 – Gas supply x demand balance

Figure 6 shows the frequency of gas supply shortfalls in volumes higher than 5% of the gas-to-power demand. Figure 7 shows the cumulative duration curve of the gas volumes shortfall, expressed in average MW (assuming that the supply of "non power" demand has priority over the supply of power-related consumption). We see in Figure 6 that in 2007, 19% of the scenarios had shortfalls; in turn, Figure 7 shows that the severity of the shortfalls in concentrated in fewer scenarios, which is consistent with the skewed probability distribution of droughts ("wet" scenarios are more likely than dry scenarios).



Figure 6 – Gas deficit probability



5. Integrated electricity-gas modeling in hydro scheduling models

The previous study showed that the probability of dispatch failures of gas-fired plants due to fuel supply problems could be significant. Given that the hydrothermal dispatch model did not "know" about this possibility when calculating the water value of the hydroelectric plants, this means that the hydrothermal dispatch is not fully optimized: the system reservoirs will be depleted faster than expected, thus increasing the risks of energy deficits or of dispatching more expensive thermal plants such as fuel oil and diesel. One clear possibility for improving this situation is to incorporate the gas supply equations and constraints into the stochastic hydrothermal model, as described next.

5.1.1. Gas pipeline equations

The set of equations (2.1)-(2.3) is added to the one-stage presented problem formulation above. The only change lies in equation (2.3): thermal generation values $g_t^*(j)$ were known values in problem (2.1)-(2.4) and are decision variables $g_t(j)$ here. The modified equation becomes:

 $P_{t}(n) + \mathbf{Erro!}[1 - w_{t}(n,l)] f_{t}(l,n) - \mathbf{Erro!} f_{t}(n,l) + \mathbf{Erro!} \delta_{t}(k) + \mathbf{Erro!} \delta_{t}(j) - \mathbf{Erro!} \phi_{t}(j) g_{t}(j) = \mathbf{Erro!} d_{t}(k)$

for
$$n \in \mathbb{N}$$
 (3.1)

5.2. Case study

The integrated electricity-gas hydrothermal scheduling, composed by expressions (1.1)-(1.7), (2.1), (2.2) and (3.1), was applied for the same electricity-gas configuration and data of the previous analysis. Also as in the previous study, we gave more priority for the "non power" gas supply than for gas-fired generation, in case of fuel shortfalls.

Figure 8 shows the yearly short-run marginal cost (SRMC) of electricity (averaged over all months, load levels and hydrological scenarios) of the Southeast system for two situations: unrestricted gas supply and supply constraints. We see that the fuel supply constraints had an important effect on electricity costs. Figure 9 shows the distribution of yearly SRMC over the hydrological scenarios, again for the fuel-constrained and unconstrained cases. We see that fuel constraints did not affect electricity prices in most hydrological scenarios, which are "wet" and do not require thermal generation. However, they had a large impact on the remaining dry scenarios.



The impact of gas supply constraints on electricity prices could be alleviated by other measures, which can also be evaluated by the integrated gas-electricity scheduling model. One possibility is to transform the gas-fired plants into bi-fuel plants (the other fuel being diesel oil); the government has announced the implementation of this conversion in several of those plants. Another possibility, also being discussed, is to negotiate interruptible gas contracts with industry, which would switch to an alternative fuel or even decrease production in case the gas-fired plants were dispatched. These alternatives bring more flexibility to the electricity-gas market.

6. Conclusions

The vigorous growth of the natural gas market in hydro-dominated countries poses special challenges for planning and operations scheduling of both the electricity and gas sectors due to the substantial oscillation in power-related gas consumption when hydrological conditions vary from "wet" to "dry". In this paper, we examined two alternatives for coordinating these sectors. In the fist one, power dispatch assumes that there are no fuel constraints and produces a (stochastic) gas consumption schedule which is added to the "non power" gas consumption forecasts, all to be managed by the gas dispatch. In the second alternative, power and gas are dispatched jointly. It is shown that both alternatives can be modeled by stochastic optimization techniques, and their application is illustrated in case studies based on realist data from the Brazilian power system.

BIBLIOGRAPHY

- [1] L.A. Barroso, B. Flach, R. Kelman, B. Bezerra, S. Binato, J.M. Bressane, M.V. Pereira; "Integrated gas-electricity adequacy planning in Brazil: technical and economical aspects" Proceedings of the IEEE PES General Meeting, San Francisco, 2005.
- [2] IEA South American Gas Daring to tap the bounty, IEA Press, 2003

- [3] R.R. Mercado, "Natural Gas Pipeline Optimization", Handbook of Applied Optimization, Edited by P. M. Pardalos and M. G. C. Resende, Oxford university Press, 2002
- [4] D. Wolf and Y. Smeers. "The gas transmission problem solved by an extension of the simplex algorithm", Management Science, vol 46, no. 11, pp. 1454-1465, November, 2000
- [5] J. Munoz, N.J. Redondo, J.P. Ruiz, "Natural gas network modeling for power systems reliability studies", IEEE PES-Summer Meeting, Chicago, 2002
- [6] O.D. Mello, T. Ohishi, "Natural Gas Transmission for Thermoelectric Generation Problem", IX Simpósio de Especialistas em Planejamento da Operação e Expansão Elétrica, May, 2004
- [7] M.V.Pereira, N. Campodónico, and R. Kelman, "Long-term hydro scheduling based on stochastic models", Proceedings of EPSOM Conference, Zurich, 1998 Available at http://www.psr-inc.com
- [8] S. Granville, G. C. Oliveira, L. M. Thomé, N. Campodónico, M. L. Latorre, M. Pereira, L. A. Barroso, "Stochastic Optimization of Transmission Constrained and Large Scale Hydrothermal Systems in a Competitive Framework," Proceedings of the IEEE General Meeting, Toronto, 2003.
- [9] M.V.F. Pereira and L.M.V.G. Pinto, "Operation Planning of Large-Scale Hydrothermal Systems", Proceedings of the 8th PSCC, Helsinki, Finland, 1984.
- [10] M.V.F. Pereira and L.M.V.G. Pinto, "Stochastic Optimization of Multireservoir Hydroelectric System – a Decomposition Approach", Water Resource Research, Vol 21 No 6, 1985.
- [11] B.G. Gorenstin, N.M. Campodónico, J.P. Costa, M.V.F. Pereira, "Stochastic Optimization of a Hydrothermal System Including Network Constraints", IEEE Transactions on PAS, Vol. 7, No. 2, May 1992.
- [12] Application of Stochastic Dual DP and Extensions to Hydrothermal Scheduling PSR TR 012/99 available at http://www.psr-inc.com