Optimization

ISSN: 0233-1934 (Print) 1029-4945 (Online) Journal homepage: http://www.tandfonline.com/loi/gopt20

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**To cite this article:** Stephan Dempe (2003) Annotated Bibliography on Bilevel Programming and Mathematical Programs with Equilibrium Constraints, Optimization, 52:3, 333-359, DOI: 10.1080/0233193031000149894

**To link to this article:** https://doi.org/10.1080/0233193031000149894

Published online: 27 Oct 2010.

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*Optimization*
Vol. 52, No. 3, June 2003, pp. 333–359

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# ANNOTATED BIBLIOGRAPHY ON BILEVEL PROGRAMMING AND MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS

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*(Received 27 February 2003; In final form 12 May 2003)*

In this bibliography main directions of research as well as main fields of applications of bilevel programming problems and mathematical programs with equilibrium constraints are summarized. Focus is also on the difficulties arising from nonuniqueness of lower-level optimal solutions and on optimality conditions.

*Keywords:* Bilevel programming; Mathematical programming with equilibrium constraints; Set-valued optimization; Applications; Optimality conditions

*Mathematics Subject Classifications 2000:* 90C30; 91A65; 49K40

## 1 INTRODUCTION

Bilevel programming problems are hierarchical optimization problems in the sense that their constraints are defined in part by a second parametric optimization problem. Let this second problem be defined first as follows:

$$\min_x \{f(x, y)\colon g(x, y) \le 0, h(x, y) = 0\}, \tag{1}$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$, $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$, $g(x, y) = (g_1(x, y), \ldots, g_p(x, y))^\top$, $h(x, y) = h_1(x, y), \ldots, h_q(x, y))^\top$. This problem will also be referred to as the *lower level* or the *follower's problem*. Let $\Psi(y)$ denote the solution set of problem (1) for fixed $y \in \mathbb{R}^m$.

Then, the bilevel programming problem can be stated as

$$\text{``}\min_y\text{''}\{F(x(y), y)\colon G(x(y), y) \le 0,\ H(x(y), y) = 0,\ x(y) \in \Psi(y)\}. \tag{2}$$

This problem is the *bilevel programming problem* or the *leader's problem*. Here, $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^k$, $H : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^l$. This problem is generally

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ISSN 0233-1934 print: ISSN 1029-4945 online © 2003 Taylor & Francis Ltd
DOI: 10.1080/0233193031000149894

a nonconvex and nondifferentiable optimization problem with implicity determined objective and constraint functions.

The aim of the bilevel programming problem is to select that parameter vector $y$ describing the "environmental data" for the lower level problem which is the optimal one in the sense that the function $F$ is minimized subject to some inequality and equality constraints. The quotation marks in (2) have been used to express certain ambiguity in the formulation of the problem in the case of the existence of nonunique lower level optimal solutions. The *Stackelberg game* is a problem of mathematical game theory identical to the bilevel programming problem.

Using the necessary optimality conditions and presupposing validity of a certain regularity assumption the bilevel programming problem can be replaced with

$$\begin{aligned}\min_{x,y,\lambda,\mu} \{F(x,y)\}\colon\ & G(x,y) \leq 0, H(x,y) = 0, \nabla_x L(x,y,\lambda,\mu) = 0,\\ & g(x,y) \leq 0, h(x,y) = 0, \lambda^\top g(x,y) = 0, \lambda \geq 0\}.\end{aligned} \tag{3}$$

Both problems are equivalent provided that the lower level problem (1) is a convex one with a unique optimal solution and validity of a regularity assumption for all parameter values. This reformulation illustrates that the bilevel programming problem is a nonconvex optimization problem. Using this approach it can be locally transformed into a smooth optimization problem in the vicinity of a feasible solution provided the lower level problem is a convex one for which the sufficient optimality condition of second order together with linear independence and strict complementarity assumptions are satisfied there. Clearly, these assumptions are very restrictive and can be rarely satisfied. If these assumptions are not satisfied, this transformation is not possible since the set of active constraints can change. The usual constraint qualification for nonlinear optimization problems (linear independence or Mangasarian–Fromowitz constraint qualifications) are violated at every feasible point of (3). We will come back to this later on in Section 6.

Problem (3) is a special case of a *Mathematical Program with Equilibrium Constraints* (MPEC)

$$\min_x \{F(z)\colon G(z) \leq 0, H(z) \leq 0, G(z)^\top H(z) = 0, g(z) \leq 0, h(z) = 0\}. \tag{4}$$

Here, $z=(x,y)$ and $F:\mathbb{R}^{m+n}\to\mathbb{R}, G,H:\mathbb{R}^{m+n}\to\mathbb{R}^t$, $g:\mathbb{R}^{m+n}\to\mathbb{R}^p$, $h:\mathbb{R}^{m+n}\to\mathbb{R}^q$. This problem clearly reflects the optimistic position of bilevel programming (see Section 2) but it is investigated in a more general formulation without using uniqueness assumptions for (parts of) the variables and also without assuming that (part of) the constraints are related to (parametric) optimization problems.

Closely related to bilevel programming problems are also *set-valued optimization problems* e.g. of the kind

$$\text{``}\min_y\text{''}\{\mathcal{F}(y)\colon y \in X\}, \tag{5}$$

where $\mathcal{F}: X \to 2^{\mathbb{R}^p}$ is a point-to-set mapping sending $y \in X \subseteq \mathbb{R}^n$ to a subset of $\mathbb{R}^p$. To see this assume that the functions $G$, $H$ do not depend on $x$, the solution set of the

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system $\{y\colon G(y) \leq 0,\ H(y) = 0\}$ is identified with the set $X$, and $\mathcal{F}(y)$ corresponds to the set of all possible upper level objective function values

$$\mathcal{F}(y) := \bigcup_{x \in \Psi(y)} F(x, y).$$

Thus, problem (2) is transformed into (5). Here we will restrict our considerations to optimization problems with set-valued objective functions and will not consider problems with constraints in the form of e.g. set inclusions.

Since the first formulation of a bilevel programming problem in an economical context in the monograph [405] the bilevel programming problems as well as the mathematical programs with equilibrium constraints have been intensively investigated by many researchers. The results can be found in the monographs [31,117,290,376] on bilevel programming, the monographs [267,324] on mathematical programs with equilibrium constraints as well as in the edited volumes [14,294].

An edited volume on the related set-valued optimization problems is [82]. [192] is a survey on that topic. Earlier annotated bibliographies on the present topic can be found in [214,312,410].

Several master's theses [359,361,367,370,407] have been devoted to this topic. The number of related PhD theses is large. The interested reader is referred to [3,38,48] and [105,128,299,318,319] as well as [357,362,382,389,413,425]. Early formulations of bilevel programming problems can be found in the papers [217,229,265,301,348,379].

The most comprehensive overview over the historical development of bilevel programming can be found in the monograph [31]. This book is also very helpful as an introduction into bilevel programming.

As long as it is maintained, an up-to-date bibliography of this topic in bibtex format can be found in the internet on the page[1]

http://www.mathe.tu-freiberg.de/~dempe/Artikel/

## 2 OPTIMISTIC *VERSUS* PESSIMISTIC OR WEAK *VERSUS* STRONG SOLUTIONS OF THE BILEVEL PROBLEM

As it can be seen in the formulation (2) of the bilevel programming problem, the notion of an optimal solution of this problem is all but obvious if the lower level problem (1) has a nonunique (globally) optimal solution for at least one value of the parameter $y$. Assume for simplicity of the formulation that the upper level constraints do not depend on the follower's choice. Then, at least two ways out of this unpleasing situation can be found in the literature: First the so-called *optimistic* or *weak formulation*

$$\min_y \{\varphi_0(y)\colon G(y) \leq 0, H(y) = 0\},$$

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where

$$\varphi_0(y) = \min_x \{F(x(y), y) \colon x(y) \in \Psi(y)\}$$

and $\Psi(y)$ denotes the solution set mapping of (1) and, second, the *pessimistic* or *strong formulation*

$$\min_y \{\varphi_p(y) \colon G(y) \leq 0,\ H(y) = 0\},$$

where

$$\varphi_p(y) = \max_x \{F(x(y), y) \colon x(y) \in \Psi(y)\}.$$

The functions $\varphi_o(y)$ and $\varphi_p(y)$ are in general discontinuous, nondifferentiable and nonconvex. Moreover, they are only implicitely determined, which makes their minimization demanding.

To the author's opinion, recommendable papers introducing the challenges of the pessimistic formulation are [261,263]. For an introduction to the optimistic formulation [117] can be useful. The number of papers investigating optimistic and pessimistic formulations is large. In the following, we try to cite them together with a classification with respect to the questions discussed in the papers.

Different solution concepts in the case when the upper level constraints do also depend on the follower's selection are suggested in the paper [191]. In most papers the optimistic formulation is (implicitly) used and all reformulations of the bilevel problem as optimization problem with equilibrium or variational inequality constraints are possible only in that case. An early formulation of the pessimistic approach can be found in [229]. Both formulations have been compared in [261].

Conditions guaranteeing the existence of an optimal solution are very different in both cases. They are formulated in [263] for the pessimistic and in [173,239,241,244] for the optimistic formulations. Problems which fail to have an optimal solution can be found in [30,263]. Methods to transform the linear bilevel programming problem with nonunique lower level optimal solutions into a linear bilevel problem with unique lower level solutions are given in [54,420].

Bearing in mind the effort needed for computing all globally optimal solutions of a nonconvex optimization problem, an alternative notion of an optimal solution of the bilevel programming problem in the case when the lower level problem is a nonconvex optimization problem is given in the paper [414].

The investigation of the behavior of perturbed problems is important for the construction of solution algorithms as well as for the interpretation of an optimal solution. In the optimistic formulation such results can be found in the papers [240,274] showing e.g. stability of the bilevel problem provided that the set of $\varepsilon$-optimal solutions of the lower level problem is used instead of $\Psi(y)$. Similar results in the pessimistic formulation are presented in [252,255,256], as well as in the papers [258,260,273]. All these results are based on very general topological assumptions for the problem definition. Conditions guaranteeing strong stability of solutions to the mathematical programs with equilibrium constraints can be found in the paper [360].

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Some of the papers aim to suggest different regularization approaches as for instance adding quadratic strongly convex functions to the lower level objective function to solve both the optimistic and the pessimistic problems [259,298].

The optimistic and pessimistic solution concepts in set-valued optimization can be found in [168,169]. Other solution concepts can be found in the paper [193].

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## 3 RELATIONS TO OTHER OPTIMIZATION PROBLEMS AND COMPLEXITY

Using necessary (and sufficient) optimality conditions or the optimal value function of the lower level problem, the bilevel programming problem can be reformulated as a one-level optimization problem. But there are also more direct relations between (2) and different well-known optimization problems. Problem (2) is directly linked to multiobjective optimization problems in [155,292], to mixed 0–1 linear optimization problems in [19,20,146], and the relations between bilevel programming and generalized semi-infinite optimization are highlighted in [387]. Moreover, the relations between MPEC and two-stage stochastic problems with recourse have been investigated in [338].

Since some of these problems are $\mathcal{NP}$-hard it is clear that also the bilevel programming problem has this property. The first result in this direction is given in [197]. The linear bilevel problem has been shown to be $\mathcal{NP}$-hard in [30,41,57] and to be strongly $\mathcal{NP}$-hard in the papers [122,171]. This implies also the nonexistence of fully polynomial approximation schemes for the bilevel programming problem [122,171]. Polynomially solvable special cases of the bilevel problem can be found in [223,341]. Moreover, if the number of variables of the lower-level decision maker is fixed, then the linear bilevel programming problem is also polynomially solvable with respect to the remaining data [251]. While investigating a standardization problem, [159] simultaneously verified that her problem itself is $\mathcal{NP}$-hard and identified conditions under which instances of the problem are polynomially solvable.

To the author's opinion a good paper introducing into the complexity issue of bilevel programming is [122].

## 4 OPTIMALITY CONDITIONS

Optimality conditions are one of the central topics in optimization theory. For bilevel programming problems different approaches have been given in [117]. For mathematical programming problems with equilibrium constraints, the paper [360] is recommended.

To formulate optimality conditions it is often necessary to use a one-level reformulation of the bilevel programming problem. A first attempt in this direction by replacing the lower level problem with an infinite number of constraints can be found in the paper [27], for a counterexample to a part of the results in this article see [92]. A second approach to formulate necessary and sufficient optimality conditions uses assumptions guaranteeing that the lower level problem has a unique strongly stable optimal solution [104,106] as well as [107,108,116,326]. In this case all the results known from nondifferentiable optimization can be used.

Necessary optimality conditions using a reformulation of the bilevel problem by the help of the optimal value function of the lower level problem can be found in [246,446,447,452,454,455], some of the results in [455] have been disproved in [113]. Duality theory applied to the lower level problem is used to derive a minimax problem for which then optimality conditions can be developed [272]. Necessary optimality conditions of Karush–Kuhn–Tucker type can be found in the paper [88]. A last attempt consists in the use of a penalty function approach [459] or of an exact penalty function [49]. The special case of a quadratic lower level problem is addressed in [411,422].

With the aim to develop necessary and sufficient optimality conditions for mathematical programs with equilibrium constraints different approaches have been used. Karush–Kuhn–Tucker type conditions can be found in [333,360], conditions based on differentiability conditions for strongly stable lower level solutions are derived in [227,329], and Karush–Kuhn–Tucker type conditions applying Mordukhovich's coderivative have been developed in [330,449–451]. Some of these conditions can be linked to optimality conditions for bilevel programming problems which has been done in [117].

Optimality conditions for set-valued optimization problems have been derived under different assumptions and using various differentiability tools. Due to the very special structure of bilevel programming problems these are sometimes too restrictive for bilevel programming. The use of a Farkas–Minkowski theorem of the alternative is demonstrated in the paper [189], generalized derivatives for set-valued maps are applied in [37,84,141] as well as in [165,194,262] and [351,444]. An explicit description of the set-valued objective function by finitely many functions is used in the paper [101].

## 5 SOLUTION ALGORITHMS

Algorithms solving bilevel programming problems can be subdivided into three classes: algorithms solving them globally, methods for computing locally optimal solutions respectively stationary points and heuristics.

The comprehensive insight into methods globally solving bilevel programming problems is given in [31]. For the use of methods of nondifferentiable optimization to mathematical programs with equilibrium constraints, the monograph [342] is recommended. With respect to the behavior of known algorithms for nonlinear optimization to mathematical programs with equilibrium constraints, the paper [139] is a first address.

Historically the first methods aimed to solve the problem globally. The monograph [31] describes a large number of such algorithms. For linear bilevel programming problems the fact that an optimal solution can be found at a vertex of the underlying polyhedron can be used. This results in vertex enumeration methods [72,77,321] and in the complementary pivoting algorithm for the linear bilevel problems [203,204] and for problems with quadratic lower level problems [205,408].

Barrier and penalty functions are used in [4,5], and in [163,164]. For more results see [191,248] as well as [373,374,377,419]. For linear lower level problems penalization of the duality gap results in an exact penalty function. This is shown in the papers [16,50,436], as well as [437]. A penalty function method for linear bilevel problems can be found in [8].

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In the $k$th best algorithm the computation of nonoptimal vertex solutions in the lower level problem is used to solve the bilevel problem globally. This is used e.g. in [53,288,289,426]. Branch-and-bound methods where the complementarity constraints of the Karush–Kuhn–Tucker reformulation of the lower level problem have been relaxed can be found in [29,32,34,129,171], in combination with algorithms for locally solving the bilevel problem in [249]. Algorithms being used on bicriteria optimization are given in [25,404] which have been shown to be not adequate in [429].

In [438] a cutting plane algorithm for linear bilevel programs is given. An algorithm for a special multilevel linear problem can be found in [423]. The linear bilevel programming problem has been attacked by transforming the Karush–Kuhn–Tucker reformulation into mixed–discrete linear constraints and solving the resulting problem [145,415].

Global optimization algorithms using the power of d.c. programming are given in [6,400–403]. The computation of a good initiation of this process is investigated in [306]. The successive cone relaxation algorithm [392] approximates the convex hull of the feasible set of (2). For some traffic management problem the lower level problem has been replaced by a projection operator added to the upper level objective function in [336]. In the paper [167] the problem is solved by a combination of a branch-and-bound approach and a convex underestimation of nonconvex functions.

Bilevel programming problems are nonconvex optimization problems. Due to the inherent difficulties for solving such problems globally, many researchers focus their investigations on deriving descent methods for computing stationary solutions. For the linear bilevel programming problem this enables one to compute a locally optimal solution [103].

For nonlinear problems there are two main stationarity concepts used. The first one is coming from Lipschitz optimization and aims to compute a Clarke stationary point. Consider the simplest situation where the lower level optimal solution proves to be locally Lipschitz continuous and it is inserted into the upper level objective function, which is assumed to be differentiable. This results in the auxiliary problem

$$\mathcal{F}(y) := F(x(y), y) \to \min_y .$$

Then, a point $y^*$ is Clarke stationary provided that zero belongs to the Clarke generalized differential of the function $\mathcal{F}(y)$ at $y^*$:

$$0 \in \nabla_x F(x(y^*), y^*)\partial x(y^*) + \nabla_y F(x(y^*), y^*).$$

The stronger concept of the Bouligand stationarity uses instead of the Clarke generalized derivative the directional derivative of the function $\mathcal{F}(y)$ provided it exists and calls the point $y^*$ Bouligand stationary provided that

$$\mathcal{F}'(y; d) = \nabla_x F(x(y^*), y^*)x'(y^*; d) + \nabla_y F(x(y^*), y^*)\, d \geq 0 \quad \forall\, d,$$

where $x'(y^*; d)$ denotes the directional derivative of the (vector-valued) function $x(y)$ at $y^*$ in direction $d$. If the function $x(y)$ is directionally differentiable and locally Lipschitz continuous, Bouligand stationarity implies Clarke stationarity but not vice versa. If necessary, constraints can be invoked by moving to Lagrange functions.

The following algorithms for computing Clarke resp. Bouligand stationary points have been suggested: Bundle algorithms for computing Clarke-stationary points using sensitivity information for $x(y)$ [114,115,135,136,323]. Other algorithms being based on sensitivity information of $x(y)$ are given in [215,358]. All these papers essentially use that the lower level optimal solution is assumed to be uniquely determined. An algorithm for the bilevel programming problem with nonunique lower level solutions computing a Bouligand-stationary point can be found in the paper [121]. This uses a regularization approach and supposes that, in the vicinity of a computed solution, the lower level optimal solution is strongly stable in the sense of Kojima such that sensitivity information is available. Descent algorithms for stochastic bilevel problems are proposed [337]. An interior point algorithm solving the Karush–Kuhn–Tucker reformulation of the bilevel programming problem is given in [231]; a trust-region algorithm is formulated in [247].

Theoretically the violation of constraint qualifications can cause difficulties in the behavior of many nonlinear optimization algorithms. In contrast, a generally good behavior of many nonlinear programming algorithms (especially all those which converge to so-called AGP points) applied to the Karush–Kuhn–Tucker reformulation of bilevel programming problems in the presence of second-order sufficient optimality conditions and strong complementarity in the lower level problem is shown in [17].

Heuristic algorithms (including artificial intelligence based heuristics, simulated annealing and genetic algorithms) have found great interest in the last years especially in discrete optimization. Bilevel programming problems inherit many structural properties from discrete optimization at least if the lower level problem is replaced by its (under convexity and regularity assumptions equivalent) Karush–Kuhn–Tucker conditions. This motivated the attempts to use heuristic algorithms also for solving bilevel programming problems which has been done in the papers [15,148,150,151, 157,279]. For other results see [281,317] as well as [356,434].

Only a very few results are available in the moment with respect to algorithms solving mixed–discrete bilevel programming problems. Such algorithms are the following: branch-and-bound algorithms for exact and approximate solutions [130,300,433]. Algorithms being based on parametric linear discrete optimization can be found in the papers [120,195,196], and a cutting plane algorithm in [112]. In the paper [393], the $k$th best algorithm is applied to discrete bilevel problems. A penalty function approach for problems with only a small number of discrete variables is given in the paper [207].

Mathematical programs with equilibrium constraints are the topic of two monographs. The monograph [324] completely investigates the application of the bundle algorithm, and [267] suggests (exact) penalty function methods and other procedures.

A global optimization algorithm for Mathematical Program with Equilibrium Constraints (MPECs) is the branch-and-bound method in [307].

As for bilevel problems, descent methods for MPECs aim at finding stationary solutions. A descent algorithm being based on sensitivity analysis for the unique lower level solution is given in [149]. Bundle algorithms for the computation of Clarke-stationary points are developed in the papers [218,219], as well as in the papers [220–222,325]. An interior point algorithm is developed in [267]. Other algorithms compute the stronger Bouligand-stationary points. These include trust-region methods of [285,365], smoothing methods using the Fischer–Burmeister function applied to the complementarity condition in [134,152,154], the path-following approach

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for MPECs in [364], a penalty method in [185] and piece-wise SQP methods [201,268,343,344]. The letter methods show superlinear convergence under appropriate assumptions. Piecewise SQP methods for problems with linear complementarity constraints can be found in [457]. A smooth penalization approach to MPECs converging under appropriate assumptions to Bouligand stationary points is presented in [184,185,188].

Penalty function methods for MPECs have been proposed in the papers [172,269, 277,286,331,365], as well as [448].

Many papers have been devoted to the question of how nonlinear programming problems behave when applied to MPECs. A generally good behavior of many such algorithms is reported in [139]. The behavior of interior point algorithms is the topic of [46]. They identify possible difficulties of such algorithms and suggest remedies. Smooth SQP methods have shown good behavior [200] and they proved to be superior with respect to local convergence in [140]. In [18] properties of MPECs have been derived guaranteeing a good behavior of elastic mode SQP algorithms.

The possible convergence of a penalty interior point method to a nonstationary point for MPECs is reported in [236].

The ways to regularize possibly unsolvable MPECs have been investigated in [276].

For the generation of test examples with special properties or known solutions see [66,67] and the papers [68,199] as well as [305]. Test problems for bilevel programming are contained in Chapter 9 of the book [144]. A large list of benchmark problems for MPECs can be found on Sven Leyffer's home page, presently under the URL

http://www-unix.mcs.anl.gov/~leyffer/MacMPEC/

## 6 THEORETICAL RESULTS

In this section focus is on some theoretical aspects. Most of the papers present also theoretical properties needed for the presented results. But, to the author's opinion, an insightful and recommendable description of the structural properties of mathematical programs with equilibrium problems (which also carry over to bilevel problems) can be found in the monograph [267]. These include also the theoretical investigation of parametric variational inequalities and the implicit function approach to MPECs.

Genericity aspects correspond to the question if some property can be assumed for real problems, i.e. if this property is satisfied for almost all problem instances. In the papers [210,242,388] it is shown that bilevel programming problems are generically well-posed and in [366] it is verified that the generalized linear independence constraint qualification is generically satisfied. Usual constraint qualifications of nonlinear programming, as the linear independence and the Mangasarian–Fromowitz constraint qualifications, are violated at all feasible points of the Karush–Kuhn–Tucker reformulation of a bilevel programming problem [360], the same has been shown for linear problems in [88]. But the weaker partial calmness condition can be satisfied [443]. Related is the sensitivity analysis of linear bilevel programming [198] and the behavior of solutions under perturbations in the case when the lower level optimal solution is unique [254]. Sensitivity analysis for MPECs with right-hand perturbations using Mordukhovich's coderivative has been done in [264]. More sensitivity results for MPECs can be found in [186,264,360].

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The structure of MPECs is highlighted in [267] and that of the feasible set in three- and multilevel linear problems in [26,45]. The problem with multiple leaders is addressed in [371]. A surprising result is the dependence of bilevel mathematical problems on irrelevant constraints [271]. This means that dropping some lower level constraints which are not active at an optimal solution $(x^*, y^*)$ of the bilevel programming problem can change the problem drastically such that $(x^*, y^*)$ will not remain optimal.

The existence of a vertex optimal solution has been verified for the linear problem [27], for quasiconcave bilevel optimization problems [70], for problems with quadratic lower level problem [146], for problems with fractional lower level problems [71], and for problems with bottleneck objective functions [288,289]. Equivalence between solvability, existence of exact penalty functions and the existence of an optimal vertex for linear bilevel programming problems has been shown in [440].

For bilevel programming problems with nonunique lower level optimal solution it is important to investigate the question of how to define a solution [109]. Related are postoptimal considerations to find Pareto-optimal solutions of a corresponding bicriteria programming problem close to an optimal solution of the bilevel programming problem [381,431,432].

Also the significance of the order of the play [32] and the question if the leader's or the follower's position is the better one [398] have been investigated.

The existence of feasible solutions for MPECs with additional joint constraints has been the topic of [153].

Basic properties of discrete bilevel programming problems can be found in [412].

For set-valued optimization problems, Fenchel duality [225,383] and well-posedness [187] have been investigated. Super efficiency for set-valued vector optimization problems is discussed in the papers [158,237,347,424]. The existence of optimal solutions for such problems is shown in [83].

## 7 APPLICATIONS

Applications have been a stimulating factor for the development of bilevel programming and mathematical problems with equilibrium constraints. The number of papers presenting applications is growing rapidly. We present applications without reference to either bilevel programming or mathematical programs with equilibrium constraints.

Interesting applications of MPECs to engineering and economical problems are contained in [324], an overview of applications of bilevel programming is given in [284].

Applications in economics include the investigation of networks of oligopolies in [1,230], hierarchical structures of multidivisional organizations have been discussed in [24,79,213] and in the papers [216,372]. Focus is on resource allocation problems in the papers [133,322,352,353], as well as in [354,355,421].

Principal agency theory has been considered in a number of papers, including [21,62,110,166,211]. These problems are highlighted from a more economical point-of-view in the papers [226,339,345,346,348,380].

Interesting applications are the determination of optimal prices, as road tolls or prices for electricity [64,223,224,227,266], as well as [441]. Related are the determination of optimal tax credits for biofuel production in the papers [35,36,118], and the optimal penalization of too large or too small amounts of transported gas in [119,206].

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The aluminum production process has been analyzed in [314–316], and agricultural planning problems in [74]. Also a facility location problem [296], optimal software release policies [456] and defense problems [60] have been investigated.

A second large field of applications is in ecological problems. Here the problem of an optimal percentage of repaired products has been investigated in [111]; the question of subsidy options to reduce greenhouse-gas emissions [179] or pollution control policies [9] have been addressed topics. The disposal of hazardous waste is the topic of [7]. Also electric utility demand-side planning [181] belongs to this area.

Some applied problems are concerned with water networks as e.g. the problems in [12] and the surface and ground water related planning policies in [56]. In the PhD thesis [125] bilevel programming problems have been suggested for tuning water networks. The analysis of transmission networks has been done in [47,182].

A further field are the inverse problems where, given a point $x^*$ values of the parameters of a variational inequality [363,394] or an optimization problem [232,233] are looked for such that $x^*$ is a solution of the resulting problem for that parameter. A related problem is the optimization over the efficient set of a multiobjective optimization problem [399].

Many applications can be found within the optimization of transportation planning and the (re-)construction of transportation networks, including the highway design problem [42,43] and the network design problem in various papers, see [102,148,228,279], as well as [390,391,458]. Other transportation problems can be found in the papers [39,89,212,291,336,442].

Engineering applications can be found in [137,138,177,178,208,332]. For more applications in this field see [384,385]. The related optimal structural design problem has been investigated in the paper [91], and a multiload truss topology design problem in [44].

At a first glance more inner mathematical problems belong to game theory [132] and to the discrimination of point sets [275,283].

Bilevel programming problems and MPECs have been used to solve other problems as optimal control problems [131] and generalized semi-infinite optimization problems [386].

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## *Acknowledgments*

The author wants to thank many colleagues for their most valuable support in writing this bibliography. I am most thankful to Jacqueline Morgan, Jiri Outrata, Daniel Ralph and Stefan Scholtes.

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