TWO MILP MODELS FOR THE SSIST FLOWSHOP SEQUENCING PROBLEM

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Introduction

Recently, the authors described two new MILP (mixed-integer linear programming) models for the sequence-dependent setup time (SDST) flowshop [11]; and two general MILP models [10] which will each solve all of the following flowshop problems: (1) regular; (2) no wait (NIQ); (3) SDST; and (4) SDST/NIQ. Aldowaisan and Allahverdi [2] presented a heuristic for sequencing the ***two-machine*** no-wait flowshop wherein the setup times were separable from job processing times and were also sequence independent. Analyses of these three papers indicated that the authors' modeling approaches could be used to develop MILP models for the ***M-machine*** regular and no-wait flowshop problems with separable setup times, for either makespan or mean flowtime as minimizing objective functions.

The purpose of this paper is two-fold: (1) to present two new models for the regular flowshop problem with separable setup times; and (2) to compare these two new models with regard to problem size complexity and to required microcomputer solution times. The no-wait versions of these models will be presented in a paper of much broader scope that is destined for journal submission.

The Classical Flowshop Problem

The classical or regular flowshop problem consists of two main elements: (1) a group of M machines; and (2) a set of N jobs to be processed on this group of machines. Each of the N jobs has the same ordering of machines for its process sequence. Each job can be processed on one, and only one machine at a time (no job splitting); and each machine can process only one job at a time. Jobs may not pass each other in the processing. Each job is processed only once on each machine. Hence, the classical flowshop sequencing problem may be stated as follows: *Find that sequence* {out of the N! possible sequences} *of jobs which minimizes some performance measure which reflects appropriate processing costs at this machine center.*

Starting with Johnson's seminal paper [5], early researchers concentrated on ***makespan*** as the appropriate performance measure to minimize. Makespan, or total flow time, is that period of time required to completely process all N jobs on the series of M machines, one job following immediately behind its predecessor in the sequence. As early as Baker [4], arguments were made that a more appropriate measure of cost is ***mean flow time***, the average time jobs spend in the M-machine system waiting for processing and being processed on each of the M machines.

For the classical flowshop problem, job setup times were assumed to be minimal compared to the magnitude of the job processing times; or they were assumed to be sequence-independent and thus combined with job processing times to form a single MxN matrix of job parameters. Hence, the classical flowshop problem implicitly included the assumption that job setup times could not be separated from job processing times.

Extensions of the Classical Flowshop Problem

The No-Wait and Zero-Buffer Flowshop Problems. In the classical flowshop sequencing problem, queues of jobs are allowed at any of the M machines in the processing sequence. There are two variants of this problem wherein jobs are not allowed to form queues. First, with ***buffers*** (inter stage queues) of ***zero capacity***, a job i just finishing on machine r cannot advance to machine r+1 in the manufacturing sequence if machine r+1 is still processing job i's predecessor in the job sequence. Rather, job i must remain at machine r, thus temporarily denying machine r to job i's successor in the job sequence until such time as job i can advance to machine r+1.

Aldowaisan and Allahverdi [2] describe a slightly more restrictive scenario in which, once a job begins its processing on machine 1 of the production line, that job must continue without delay to be processed on each of the M machines in line. Not only are there no inter stage buffers to hold delayed jobs , but no job may wait on one machine until the subsequent machine in the line is free to begin processing on that job. Aldowaisan and Allahverdi refer to this as the ***no-wait*** flowshop problem. Earlier researchers, for example Stafford [8], Stafford and Tseng [10], and Wismer [13], called this the NIQ (no intermediate queues) flowshop problem. In this problem variant, jobs are held prior to machine 1 and launched only when they can be sequentially processed by all M machines without delays at any of the machines.

The SDST Flowshop Problem. In the classical flowshop problem, a job's setup time is assumed to be independent of that job's position in the sequence. That is, the setup time for job i on machine r is the same regardless of which job precedes job i in the sequence. This assumption does not hold in a variety of real-life flowshop environments. Rather, the setup time of job i on machine r is highly dependent on which job is processed just prior to job i in the sequence. This variant of the problem is called the ***SDST*** (sequence-dependent setup times) flowshop problem. Srikar and Ghosh [7] reported an MILP model for this problem in 1985. Stafford and Tseng [9] made corrections and enhancements to this model; and Rios-Mercado and Bard [6] showed that this enhanced model {hereafter referred to as the SGST SDST model} was superior to their own MILP model for the SDST flowshop. Tseng and Stafford [11] report two new MILP models for the SDST flowshop problem, and demonstrate that both of these models are far superior to the SGST SDST model with regard to computer solution times for a moderately large set of problems.

Separated Setup Times. Until recently, most researchers working with the regular and no-wait flowshop problems assumed that job setup times were either negligible in magnitude or that they were sequence-independent and thus could be added to the job processing times. Thus these setup times were implicitly assumed to be non-separable. The setup for job i on machine r could not begin until that job arrived at machine r after completing processing on machine r-1. Although this assumption simplified computations for many heuristic studies of flowshops, it also reduced the reality of these modeling procedures.

Yoshida and Hitomi [14] were among the first to investigate the flowshop wherein setup times were separated from processing times with their extension of Johnson's rule. In general, there is no reason to assume that the setup for job i on machine r cannot begin until job i has arrived at machine r. If job i's predecessor in the sequence has completed its processing on machine r, then the operator of this machine should be able to begin the setup for job i in anticipation that job i will be the next job to arrive for processing at that machine. There is no reason to let machine r stand idle for a time, and then do the job i setup after job i is free to be processed on machine r. The net effect of "starting job setups early" may be to reduce the total length of time (makespan) required to process the complete set of jobs.

Recent studies by Aldowaisan and Allahverdi [2], Allahverdi [3], and Aldowaisan [1] have presented heuristics for the regular and no-wait flowshops with setup times that are both separable and sequence-independent. Each of these three studies was limited to the two-machine, N-job flowshop. Except for a few very special cases, these heuristics could not guarantee optimal solutions for either makespan or mean flow time as measures of performance.

MILP Models for the SSIST Flowshop Problem

This section presents two different MILP models for the SSIST regular flowshop problem. Both models will solve this problem for any number of machines, M, and jobs, N. These models are derived from the recent work of Tseng and Stafford [11]. Both models can find optimal solutions for either job set makespan or mean flow time as performance measures. It is appropriate to first state the full set of assumptions of the SSIST flowshop problem.

The SSIST Flowshop Problem. The SSIST (separable, sequence-independent setup times) flowshop problem is essentially the regular flowshop problem described above with the added assumption that setup times are non-negligible in magnitude and are separable from the job processing times. Thus the setup for a job may commence on a machine in expectation of that job’s imminent arrival at the machine after that job’s immediate predecessor in the job sequence has completed its processing on this same machine.

SSIST Model I. This model is based on the recent TS/1 model of Tseng and Stafford [11], and on earlier works by Stafford [8] and Wagner [12]. It uses an assignment problem approach to identify the optimal solution job sequence and to insure that various model assumptions are met in this optimal sequence.

Model Parameters. Define **T** = $\{T_{rj}\}$ be an MxN matrix of processing times, T_{rj} being the processing time of job j on machine r. $\mathbf{T}_{r.}$ is the rth row of **T**, and $\mathbf{T}_{.j}$ is the jth column of **T**. The MxN matrix **S** = $\{S_{rj}\}$ represents the sequence-independent setup times, S_{rj} being the setup time of job j on machine r. $\mathbf{S}_{r.}$ is the rth row of **S**, and $\mathbf{S}_{.j}$ is the jth column of **S**. Let **C** be a Nx1 column vector of 1's and **R** be a 1xM row vector of 1's.

Model Variables. Define $\mathbf{Z} = \{Z_{ij}\}$ to be an NxN matrix of 0-1 integer variables. Further, let $\mathbf{Z}_{i.}$ be the ith row of $\mathbf{Z}$ and $\mathbf{Z}_{.j}$ be the jth column of $\mathbf{Z}$. Let $\mathbf{X} = \{X_{rj}\}$ be an MxN matrix of machine idle time variables, with X_{rj} being the idle time on machine r before the start of the job in position j in the sequence. $\mathbf{X}_{r.}$ Is the rth row of $\mathbf{X}$. And let $\mathbf{Y} = \{Y_{rj}\}$ be an MxN matrix of job idle time variables, with Y_{rj} representing the idle time of the job in position j in the sequence after finishing processing on machine r.

Model Constraints. The constraints and relationships of SSIST Model I for the regular flowshop may be written as:

$$\mathbf{Z}_{i.}\mathbf{C} = 1 \quad (i = 1,...,N) \qquad (1)$$

$$\mathbf{R}\,\mathbf{Z}_{.j} = 1 \qquad (j = 1,...,N) \qquad (2)$$

$$[\mathbf{S}_{r.} + \mathbf{T}_{r.} - \mathbf{S}_{r+1.}]\mathbf{Z}_{.j} - \mathbf{T}_{r+1.}\mathbf{Z}_{.j} + (X_{r,j+1} - X_{r+1,j+1}) + (Y_{r,j+1} - Y_{rj}) = 0 \qquad (r = 1,...,M-1;\ j = 1,...,N-1) \qquad (3)$$

$$[\mathbf{S}_{r.} + \mathbf{T}_{r.} - \mathbf{S}_{r+1.}]\mathbf{Z}_{.1} + (X_{r1} - X_{r+1,1}) + Y_{r1} = 0 \quad (r = 1,...,M-1) \qquad (4)$$

$$C_{Mj} = \sum_{p=1}^{j}(\mathbf{T}_{M.} + \mathbf{S}_{M.})\mathbf{Z}_{.p} + \sum_{p=1}^{j} X_{Mp} \qquad (j = 1,...,N) \qquad (5)$$

$$C_{MAX} = C_{MN} = [\mathbf{T}_{M.} + \mathbf{S}_{M.} + \mathbf{X}_{M.}]\mathbf{C} = \text{constant} + \mathbf{X}_{M.}\mathbf{C} \qquad (6)$$

Equations 1and 2, identical to the classical linear assignment problem, represent 2N constraints which insure that (1) each job is assigned to one and only one position in the job sequence (Equation 1), and (2) each sequence position is filled with only one job (Equation 2). The (M-1)x(N-1) constraints represented by Equation 3 insure that: (1) a job does not start on a machine until it has finished processing on the previous machine; (2) a job does not start on a machine until its predecessor has completed processing on that machine; and (3) a job does not start on a machine until its setup on that machine has been completed. Tseng and Stafford [11] termed these JAML (job-adjacency-machine-linkage) constraints. Equation 4 represents (M-1) constraints which insure that the first job in the sequence gets started as early as possible on each of the M machines.

Equation 5 represents the relationships for measuring the completion time of each job on the last machine in the production system. Equation 6 is a relationship for measuring makespan, the span of time required to make all N jobs in the sequence. It is also redundant since it is an alternative version of the

Nth relationship from Equation 5.

Objective Functions. This model may optimize (*minimize*) either the classical performance measure, makespan, or the more recently popular measure, mean job completion time. These objective functions may be written as follows:

Makespan: Minimize: $C_{MAX} = C_{MN}$ (7)

and

Mean Flow Time: $$Minimize:\ \overline{C} \equiv \sum_{j=1}^{N} C_{Mj} \qquad (8)$$

SSIST Model II. This model is based on the SDST flowshop model of Srikar and Ghosh [7], which was adapted by Stafford and Tseng [9, 10] to fit the classical regular flowshop problem. It uses pairs of dichotomous constraints to identify the ordering of the N jobs within the processing sequence.

Model Parameters. The definitions of the MxN-sized matrices **T** = $\{T_{rj}\}$ (job processing times) and **S** = $\{S_{rj}\}$ (job setup times) are identical to the definitions used in SSIST Model I. At the same time, the row and column vectors from these matrices are not used in this model. In addition, P represents a "very large" number.

Model Variables. Define **C** = $\{C_{ri}\}$ to be an MxN matrix of job completion times, with C_{ri} representing the latest completion time of job i on machine r. Further, define E_r to be the earliest completion time of the last job on machine r. And lastly, define **D** = $\{D_{jk}\}$ to be the upper diagonal portion of an NxN matrix of binary integer variables, where $D_{jk} = 1$ if job j is scheduled anytime before job k in the processing sequence, and = 0 otherwise. (j = 1,...,N-1; k = 2,...,N; and k > j.)

Model Constraints. The constraints and relationships of SSIST Model II for the regular flowshop may be written as:

$$C_{1i} \geq S_{1i} + T_{1i} \qquad (i = 1,...,N) \qquad (9)$$

$$C_{ri} \geq C_{r-1,i} + T_{ri} \qquad (r = 2,...,M;\ i = 1,...,N) \qquad (10)$$

$$C_{rj} - C_{rk} + P[D_{jk}] \geq S_{rj} + T_{rj} \qquad (11)$$

$$C_{rk} - C_{rj} + P[1 - D_{jk}] \geq S_{rk} + T_{rk} \qquad (12)$$

$$E_M \geq C_{Mj} \qquad (j = 1,...,N) \qquad (13)$$

For Equations 11 and 12, r = 1,...,M; j = 1,...,N-1; k = 2,...,N; and k > j.

Equations 9 and 10 relate a job's completion time on a given machine to its completion times and processing times on all machines, insuring that each job is setup on machine 1 before it begins its processing on that machine. Equations 11 and 12 represent sets of dichotomous, paired constraints which insure that, for each pair of jobs considered, no matter which job is the predecessor anywhere in the processing sequence, the successor job can not begin processing on any of the machines until the predecessor job has completed processing on that machine, and the successor job has been set up on that machine. Equation 13 is simply a function (not a constraint) insuring that the job makespan is equal to the completion time of the last job processed on the last machine of the production system.

Objective Functions. This model may also optimize (*minimize*) either makespan or mean job completion time. These objective functions may be written as follows:

Makespan: Minimize: E_M (14)

and

Mean Flow Time: $$Minimize:\ \overline{C} \equiv \sum_{j=1}^{N} C_{Mj} \qquad (15)$$

Analysis and Discussion of the Models

Size Complexity of the Models. It is indicated throughout the flowshop sequencing literature that for M greater than two machines, all flowshop problems are NP-complete or NP-hard. Thus the size complexity of new models, as a function of M (number of machines) and N (number of jobs to process) is an important parameter to consider. The size complexity of the two new models presented in this paper is shown in Figure 1. Examination of the equations in Figure 1 indicates that SSIST Model I (assignment problem model) will always require more binary and total variables than SSIST Model II (dichotomous paired constraints model) for all values of M and N. At the same time, Model I will always require fewer constraints than Model II for all values of M and N except N = 1.

Similar Models in the Literature. The two new models presented in this paper are derived from earlier modeling efforts by the authors [10], and they are most similar to the alternative MILP models for the regular flowshop with non-separable setup times. SSIST Model I is akin to the WST regular flowshop model of the earlier paper, and the SSIST Model II is akin to the SGST regular flowshop model. For the regular flowshop problem, the SGST model averaged 1420 seconds of solution time for sixty problems

MILP Model	Model I	Model II
Variables		
Binary Integer	N^2	$(N^2 - N)/2$
Real	2MN	M(N+1)
Total	$N^2 + 2MN$	$(N^2 - N)/2 + M(N+1)$
Constraints	MN + 2N	$MN^2 + N$

†N = number of jobs in job set;
M = number of machines in production line

Figure 1. Size Complexity† of the MILP SSIST Models

solved, while the WST model averaged 6.56 seconds of solution time for these same problems. This yielded an average ratio of Time(SGST):Time(WST) of 216.5, with ratios ranging from 16.5 to 945.3 in the 12 cells of the experimental design. Based on these earlier results for the regular flowshop with non-separable setup times, it is hypothesized that SSIST Model I will require statistically significantly less computer solution time that will SSIST Model II for a similar set of flowshop problems.

Experimental Design. An experimental design identical to the authors' previous one [10] will be executed to investigate the hypothesis posed above. Four values of N (6, 7, 8, 9) and three values of M (5, 7, 9) will result in twelve cells of the experiment. Replications per cell will be five as with the earlier work. (This will be increased to 10, time allowing.) For each problem, the job processing times will be uniformly distributed, 1 to 100, while the setup times will be uniformly distributed, 1 to 25. All problems will be generated using a proprietary model generator, and all will be solved on a Pentium III 800 MHz microcomputer using Hyper LINDO 6.01. The performance measure to minimize will be job set makespan. (Time allowing, these same problems will also be solved to minimize mean job completion time.)

The minimum experimental design parameters (5 replications; both models; 12 cells; makespan) will be completed in time for inclusion in the meetings proceedings. If the larger parameters (10 replications; makespan and mean flow time) can be completed in time, they too will be included in the proceedings version of this paper. Otherwise, they will be presented in the meetings session.

The mean solution times for five problems in each cell, and the ratios of these mean times, are presented in Table 1. Clearly, Model I dominates Model II with regard to required computer solution times. The ratios of required mean solution times range from 11.4 (MxN = 7x9) to 1456.0 (5x9), with a mean ratio of 302.75 for the 60 problems solved by each model. These ratios follow the pattern found for MILP models for the SDST flowshop reported by Tseng and Stafford [11].

The material in this paper is but a portion of a much larger study of SSIST flowshop models currently being conducted by the authors. Also included in this larger study are models for the NSIST (non-separable, independent setup times) flowshop, and for the NIQ variants of both SSIST and NSIST flowshops. {NSIST is a new way of looking at the original Johnson description of the flowshop; it is also called the classical or regular flowshop.} It is anticipated that, by meeting time in November 2001, this study will have been submitted for journal review. If so, the general results of the complete study will be shared with meetings participants.

Machine	Job	Model I‡	Model II‡	Ratio (II:I)
5	6	0.262	8.038	30.679
	7	0.480	94.972	197.858
	8	0.888	709.320	798.784
	9	4.508	6563.674	1456.006
7	6	0.864	9.830	11.377
	7	1.776	70.620	39.764
	8	6.932	686.240	98.996
	9	18.600	10282.326	552.813
9	6	1.122	18.384	16.385
	7	2.770	131.372	47.427
	8	13.458	1371.746	101.928
	9	52.138	14652.038	281.024
Average		8.650	2883.213	302.753

‡Computer times given are in second.

Table 1. Computer Times for the Experimental Design Problems.

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