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Multi-Type, Multi-Zone Facility Location

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The placement of facilities according to spatial and/or geographic requirements is a popular problem within the domain of location science. Objectives that are typically considered in this class of problems include dispersion, median, center, and covering objectives—and are generally defined in terms of distance or service-related criteria. With few exceptions, the existing models in the literature for these problems only accommodate one type of facility. Furthermore, the literature on these problems does not allow for the possibility of multiple placement zones within which facilities may be placed. Due to the unique placement requirements of different facility types—such as suitable terrain that may be considered for placement and specific placement objectives for each facility type—it is expected that different suitable placement zones for each facility type, or groups of facility types, may differ. In this article, we introduce a novel mathematical treatment for multi-type, multi-zone facility location problems. We derive multi-type, multi-zone extensions to the classical integer-linear programming formulations involving dispersion, centering and maximal covering. The complexity of these formulations leads us to follow a heuristic solution approach, for which a novel multi-type, multi-zone variation of the non-dominated sorting genetic algorithm-II algorithm is proposed and employed to solve practical examples of multi-type, multi-zone facility location problems.

Introduction

In the facility location literature, the most popular classes of facility location problems are consistently formulated as median, center, dispersion, and covering problems (Owen and Daskin 1998; ReVelle and Eiselt 2005; Arabani and Farahani 2012; Farahani, Asgari, and Heidari 2012). In respect of distance-based optimization, median and center problems are typically concerned with minimizing the distances between facilities and their demand points, while dispersion problems aim to maximize inter-facility distances. Covering problems, on the other hand, typically aim to maximize service provided to a set of demand points by a number of facilities.

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According to Curtin and Church (2006), only one model from a diverse range of dispersion models involves a combination of different types of facilities, namely the regional energy facility location model of Church and Cohon (1976). In this model, two different types of facilities—fossil fuel power plants and nuclear power plants—were located simultaneously in a region where a number of objectives, including dispersion, were optimized. Curtin and Church (2006) proceeded to review the location science literature on the location of dispersed facilities and provided a detailed description of potential multi-type dispersion applications. Most importantly, they presented a family of multi-type discrete dispersion models that build on the existing foundation of discrete dispersion in location science. As opposed to dispersion problems in which inter-facility distances are considered, center and median problems involve minimizing the separation distances between facilities and demand points. A search in the literature for multi-type median or center models, however, returns no results or references to such possibilities.

Covering problems traditionally aim to locate facilities in such a manner that customers are optimally served according to some criterion based on the distances between the facilities and the customers (Farahani, Asgari, and Heidari 2012). In the context of this article, the customers are considered points on the terrain surface that require "service" from the facilities in terms of certain criteria—for example, cellular network coverage required from transmitters in regions of high cellular demand, or visibility required by critical zones from surveillance cameras. One of the most commonly found and practically implemented covering problems encountered in the literature is the maximal covering location problem (MCLP), in which the aim is to maximize cover of demand, given a fixed number of facilities that are available for placement. Covering models, such as the MCLP, traditionally involve the placement of a single facility type only. Multi-type MCLP solution processes have, however, recently been demonstrated by Bao et al. (2015) and Heyns and Van Vuuren (2015) for real-world bi-objective observation camera placement problems. Shillington and Tong (2011) have also demonstrated the solution process of the MCLP concerned with the siting of two facility types—internet gateways and mesh routers—for the purpose of wireless mesh network cover.

The formal conceptualization of multi-type dispersion by Curtin and Church (2006) included the potential requirement that not all of the facility types incorporated in a facility location model have to be placed in one designated placement zone (henceforth simply referred to as a zone). This is due to the unique placement requirements of different facility types—such as suitable ground surface type that may be considered and type-specific placement objectives—and it is therefore expected that suitable zones for different facility types may often differ. Zonal regulations/requirements may also impose type-specific placement limitations. These potential requirements were not mentioned by Curtin and Church (2006) and no prior reference to such possibilities are available in the literature, although recently it has been practically illustrated by Heyns and Van Vuuren (2015) for a real-world MCLP. The novel concept of multi-type, multi-zone (MTZ) facility location is therefore introduced here.

Planners and managers are unlikely to use only one of the above-mentioned facility location models as the sole criterion for planning a network of facilities as location optimization often involves the pursuit of multiple conflicting objectives (Kim, Murray, and Xiao 2008; Maliszewski, Kuby, and Horner 2012; Kwong et al. 2014; Heyns and Van Vuuren 2015; Bao et al. 2015). Multi-objective optimization solution processes are adopted in such instances to provide decision makers with a set of trade-off solution alternatives (instead of a single optimal solution only) from which a suitable solution may be chosen subjectively post-optimization

(Zitzler, Deb, and Thiele 2000; Kim, Murray, and Xiao 2008; Maliszewski, Kuby, and Horner 2012; Heyns and Van Vuuren 2016a). The models considered in this article are known to be computationally difficult to solve and have been shown to be NP-hard (Nagy 1994; Ravi, Rosenkrantz, and Tayi 1994; Owen and Daskin 1998; Alp, Erkut, and Drezner 2003; Murray 2003; Pisinger 2006; Drezner 2012; Bao et al. 2015). Techniques such as intelligent constraint elimination may reduce the computational complexity of solving these problems, resulting in the situation where only smaller single-objective problem instances may be solved exactly—within realistic computation times—by state-of-the-art optimization software such as CPLEX (Curtin and Church 2006). The inclusion of multiple instances of these problems in the model (either as objectives or constraints) compounds the computational complexity of the solution process and it follows that large real-world multi-objective problems are typically impossible to solve exactly within realistic computation times (Kim, Rana, and Wise 2004; Curtin and Church 2006; Murray et al. 2007). This is certainly the case for real-world facility location problems that involve the placement of observation/detection equipment with multiple covering objectives (Murray 2003; Kim, Rana, and Wise 2004; Heyns and Van Vuuren 2015 2016a). In such instances, approximate algorithms are employed instead of exact solution methodologies to provide a set of approximately optimal solution alternatives within a reasonable amount of time (Zitzler, Deb, and Thiele 2000; Deb et al. 2002; Kim, Rana, and Wise 2004, 2008; Bao et al. 2015; Heyns and Van Vuuren 2015, 2016a).[1] In this article, we introduce a novel MTZ implementation of the popular multi-objective approximate solution methodology which is based on the non-dominated sorting genetic algorithm-II (NSGA-II) (Deb et al. 2002) and is capable of solving problem instances involving multiple objectives.

The article opens with a review of the recent location science literature involving three classes of facility location models for which we develop novel MTZ integer-linear programming (ILP) formulations—dispersion, center, and MCLP models. The MTZ model formulations are derived in the third section, while the fourth section elucidates important multi-objective optimization concepts and approaches that lead to the development of a novel MTZ implementation of the NSGA-II. In the fifth section, the MTZ NSGA-II is used to solve a selection of single-objective and bi-objective examples of our novel MTZ formulations. The article closes in the sixth section with a brief conclusion and proposals for future work.

Review of facility location models

In this section we provide background context in respect of the facility location models considered in this article, namely dispersion models, center and median models, and the maximal covering location model.

Dispersion problems

Examples of facilities typically included in dispersion problems are those that may be considered for application in military defense, franchise location, transportation of hazardous materials, layout planning for explosive chemicals and telecommunication network design (Kuby 1987; Erkut and Neuman 1989; Erkut 1990; Daskin 1995; ReVelle and Eiselt 2005; Lei and Church 2015). An example of a typical dispersion problem in a military context is one where a decision maker may attempt to locate ammunition dumps in such a manner that the discovery and subsequent destruction of one such facility has as little impact as possible on the other facilities. An illustration of the notion of facility dispersion is provided in Fig. 1.

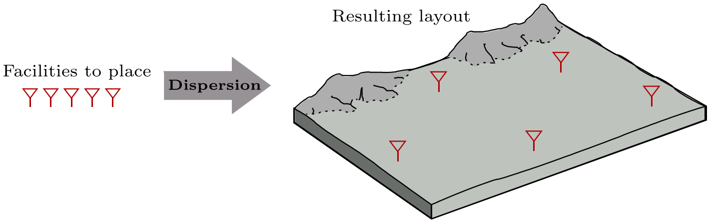


Figure 1. The notion of facility dispersion, in which the separation distances between facilities are maximized.

There exist four basic dispersion facility location criteria in the literature (Curtin and Church 2006). These are the *p*-dispersion (MaxMinMin) criterion (Shier 1977), the *p*-defense (MaxSumMin) criterion (Moon and Chaudhry 1984), the *p*-dispersion sum (MaxSumSum) criterion (Kuby 1987), and the *MaxMinSum* criterion (Erkut and Neuman 1991). As discussed in the introduction, the most recent advancement of *p*-dispersion problems in the context of this article is the multi-type dispersion models of Curtin and Church (2006). Here, a multi-type dispersion metric is conceptualized—this metric recognizes that the extent to which two facilities ought to be dispersed should depend on their types. The strength of facility interaction is modeled using the traditional separation distance adjusted according to a type-specific "repulsion" factor—a smaller repulsion measure reflects a stronger (friendlier) inter-facility interaction. A multi-type dispersion metric is then incorporated in four multi-type dispersion models based on the four basic dispersion constructs mentioned above. The other most recent advances in dispersion modeling come in the form of unified dispersion models (Lei and Church 2013, 2015). While these unified models are certainly a helpful addition to the dispersion literature, the multi-type dispersion models of Curtin and Church (2006) are considered essential for future facility location solution approaches. To narrow the scope of the work covered in this article, an MTZ extension is formulated for the multi-type *p*-dispersion (MaxMinMin) model of Curtin and Church (2006) only—in which *p* facilities are to be located with the aim of maximizing the minimum distance separating any two facilities.

Center and median problems

As opposed to dispersion problems, center and median problems involve minimizing the separation distances between facilities and demand points. In a traditional operations research context, problems that require such optimization include the location of facilities such as libraries, schools, or emergency service centers, to which close proximity of certain demand points is desirable (Owen and Daskin 1998). The notion of facility centering is illustrated in Fig. 2.

Center and median facility location criteria in the literature include the *p*-center criterion (Hakimi 1965) and the *p*-median criterion (Hakimi 1965). As in the case of the unification models for dispersion problems by Lei and Church (2013, 2015), unification models that encapsulate and extend *p*-center and *p*-median problems have also been proposed (Nickel and Puerto 1999, 2005; Marín et al. 2009; Lei and Church 2014). In contrast to dispersion models, however, a search in the literature for multi-type *p*-center or *p*-median models returned no results or references to such possibilities. Median and center models are closely related and offer similar practical implementation possibilities. Therefore, to narrow the scope of the work covered in this article, we propose a novel MTZ formulation involving the center model construct

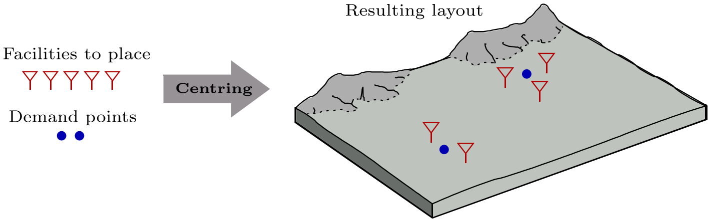


Figure 2. The notion of facility centering, in which the separation distances between facilities and their demand points are minimized.

only—in which *p* facilities are to be located with the aim of minimizing the maximum distance between any of the facilities and their demand points.

The maximal covering location problem

The MCLP is a natural extension of the location set covering problem (LSCP) initially proposed by Toregas et al. (1971). The objective in the LSCP is to minimize the number of facilities that are placed to service all members of a set of customers that require service. Solving the LSCP involves an undetermined and possibly impractically large number of facilities to be placed to satisfy the objective of total demand satisfaction—this approach is often considered financially and logistically impractical, in addition to being considered unnecessary.[2] The MCLP extension of the LSCP—first proposed by Church and ReVelle (1974)—makes more practical sense and is more generally adopted in real-world applications instead of the LSCP. The aim of the MCLP is to maximize demand satisfaction, given a fixed number of facilities that are available for placement.[3] The notion of maximal coverage is illustrated in Fig. 3.

In practical applications, the MCLP is most often included in problems concerned with the placement of transmission and surveillance/detection devices. In transmission problems, the aim is to locate transmitters in a wireless network, for example, for cellular communication or television broadcast purposes (Krzanowski and Raper 1999; Mathar and Niessen 2000; Gencer, Kizilkaya Aydogan, and Celik 2008; Shillington and Tong 2011; Marianov and Eiselt 2012; Schmidt-Dumont and Van Vuuren 2016). Examples of MCLPs involving facilities destined for surveillance purposes include watchtowers or cameras (Kim, Rana, and Wise 2004; Bao et al. 2015; Heyns and Van Vuuren 2015). Kim, Rana, and Wise (2004) conducted a well-cited study that considers the MCLP for general visibility coverage purposes with no specific facility type in mind, while Bao et al. (2015) solved a multi-type MCLP for forest fire monitoring purposes.

The MCLP also appears in military and related studies. In addition to the VHF/UHF radio jammer system application of Gencer, Kizilkaya Aydogan, and Celik (2008), the placement of

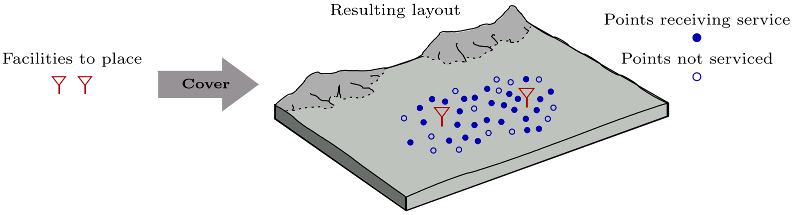


Figure 3. The notion of maximal cover, in which the objective is to maximize the number of demand points that receive service from a fixed number of facilities.

radar and weapon systems also exhibit objectives that may be considered within the MCLP paradigm. Whereas the previously discussed literature on transmitter and surveillance facilities involve the coverage of terrain surface/points of interest, the points that require service in a military context may be points along an enemy's potential approach path (Ghose, Prasad, and Guruprasad 1993; Tanergüclü et al. 2013) and the "service" is provided by radar or weapon systems in the form of detection or engagement efficiency. Tanergüclü et al. (2013) consider the simultaneous siting of radar and weapon systems in this context—although a closely related extension to the MCLP, the maximal expected covering location problem (Daskin 1983), is adopted instead.

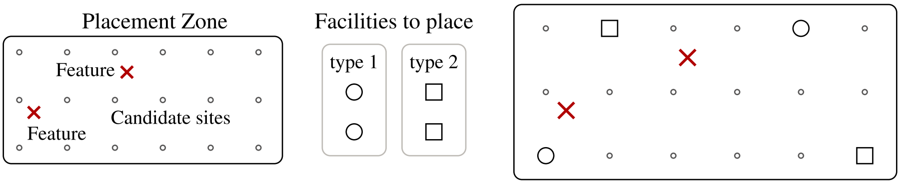
MTZ model formulations

In this section we elucidate the MTZ facility location concept, model the MTZ environment mathematically, and formulate MTZ ILP models for dispersion, center and MCLP criteria.

The MTZ facility location concept

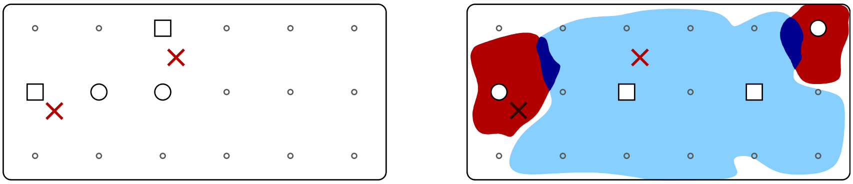
We introduce the notion of MTZ facility location by first discussing and illustrating the multi-type concept when applied to (1) dispersion, (2) center, and (3) maximal covering problems, as illustrated graphically in Fig. 4. A hypothetical weapon system placement scenario is considered throughout to illustrate the multi-type and MTZ concepts applied to the different models. The objectives in the scenario, however, are such that they may similarly be applied to other facilities, such as observation cameras and transmitters, in a different context.

In part (a) of the figure, there are two facility types which are distinguished by shape—two of each facility type—which require placement in a zone enveloping 18 candidate sites. Suppose that facilities of type 1 are short-range weapon systems, while facilities of type 2 are long-range weapon systems. In (b), a potential facility layout resulting from multi-type dispersion between all the facilities, irrespective of type, is shown. In a practical setting this may be a



(a) Placement zone and facilities

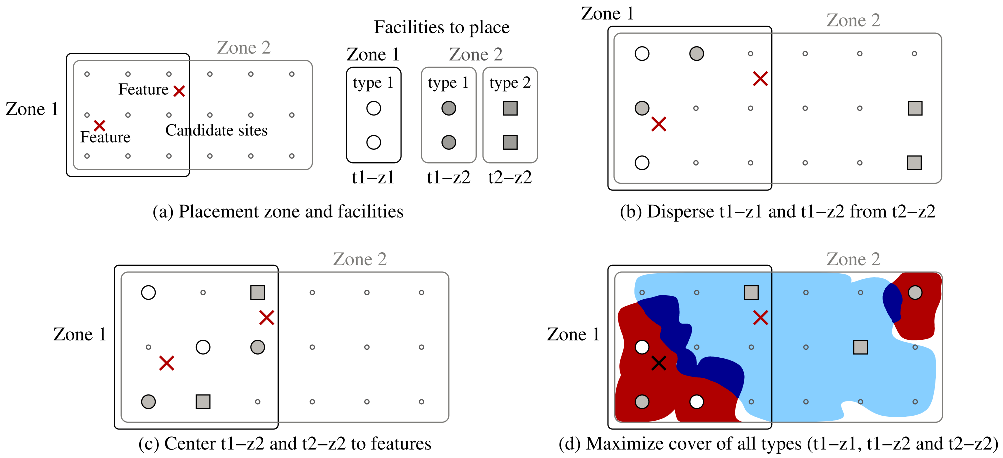
(b) Disperse all types from each other



(c) Center all types to features

(d) Maximize cover of all types

Figure 4. An illustration of multi-type facility location, given two types of facilities and a single zone in (a). Example layouts after having performed dispersion, centering, and covering perturbations for selected facility types are provided in (b)–(d).



(a) Placement zone and facilities (b) Disperse t1−z1 and t1−z2 from t2−z2

(c) Center t1−z2 and t2−z2 to features (d) Maximize cover of all types (t1−z1, t1−z2 and t2−z2)

Figure 5. An illustration of MTZ facility location in which type-zone pairs are employed, given two types of facilities and two zones in (a). Example layouts after having performed dispersion, centering, and covering perturbations for selected type-zone pairs are provided in (b)–(d).

security measure to avoid the simultaneous destruction of any two weapon systems. A potential layout that results from a center objective for the facilities with respect to two features is provided in Fig. 4c. This may, for example, be the siting of the weapon systems with consideration given to transportation distances from ammunition dumps (the features denoted by crosses in the figure)—assuming that the candidate sites are far enough from the ammunition dumps so that the placement of a facility at any of the sites would not jeopardize the safety of the dump. Finally, an example of a layout resulting from a maximal covering objective is provided in part (d) in the figure. Terrain cover achieved by the facilities—that is, the effective area within which the weapon systems are able to engage an enemy, subject to terrain interference and range capability—is indicated by red for facilities of type 1 and blue for facilities of type 2, with some overlapping cover.

The MTZ concept is introduced in Fig. 5, which builds on the illustration in Fig. 4. A new zone is specified in Fig. 5a, in addition to the original zone of Fig. 4. The facility types are again distinguished by shape, in addition to their colors indicating to which zone they “belong” and within which they are to be placed—white for zone 1 and gray for zone 2. Suppose that the new zone (zone 1) is of high strategic importance and two additional weapon systems of type 1 are specifically dedicated to this zone. Suppose that zone 2 and the facilities assigned to it are the same as in the previous example.

The important notion of a type-zone pair is introduced in Fig. 5. A type-zone pair is a facility of a specific type belonging to a specific zone and is viewed as a single entity. For example, a facility of type 1 belonging to zone 1 is considered independently from a facility of type 1 belonging to zone 2, even though the facility types are the same. For simplicity in the remainder of this article, type-zone pairs are specified using abbreviations, for example, the type-zone pair of type 1 in zone 2 is expressed as t1-z2.

Returning to the scenario, suppose that facilities of type 2 are more likely to be attacked by the enemy, according to their resources and traditional protocol. For this reason it is desired that the distances between facilities of type 1 and type 2 are to be maximized to minimize the possibility of subsequent damage to facilities of type 1 in the event of an attack. In part (b) of Fig. 5, this is illustrated in terms of two groups of type-zone pairs which are to be dispersed from each other. The two type-zone pairs of the first group in question are t1–z1 and t1–z2, while the sole type-zone pair in the second group is t2–z2. As may be seen in the figure, the model attempts to disperse the facilities of the first group from the facilities of the second group. Suppose that the strategic importance of the two dedicated weapon systems of type 1 in zone 1 results in a situation where a large on-site supply of ammunition is available at all times, so that proximity from the ammunition dumps is not an important consideration for type-zone pair t1–z1. A potential layout that results from a center objective for the remaining group of type-zone pairs, t1–z2 and t2–z2, is provided with respect to the ammunition dumps in part (c) of the figure. Finally, an example of a layout resulting from a maximal covering objective for all type-zone pairs, as well as the cover that they achieve, is provided in part (d) of the figure, indicated by red for facilities of type 1 and blue for facilities of type 2, with some overlapping cover.

Mathematical modeling of the MTZ facility location environment

The modeling of facility location problems according to type-zone pairs offers dynamic problem solution possibilities. Zones are modeled in such a manner that they may comprise overlapping sections (i.e., mutual candidate sites) and a zone may be considered for the placement of multiple facility types. Facilities of a specific type may also be assigned for placement in more than one zone. In this section, the facility placement decision variables for the assignment of facilities within specified zones are modeled.

Denote the number of zones in a facility location problem by Z and, if the set of all candidate sites form a set

$\mathbb{S}$, denote the set of candidate site locations in zone $z \in \{1,\ldots,Z\}$ by $\mathbb{S}_z \subseteq \mathbb{S}$. Suppose $\mathbb{T}$ denotes the set of all facility types to be considered, and let $T=|\mathbb{T}|$. Furthermore, denote the number of facilities of type $t \in \{1,\ldots,T\}$ to be placed in zone z by n_z^t and consider the type-zone pair facility location decision variable

$$x_z^t(s)=\begin{cases} 1, & \text{if a facility of type}\, t\in\mathbb{T} \text{ is placed at location}\, s\in\mathbb{S}_z, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The facility location layout of Fig. 5b is illustrated again in Fig. 6. The candidate site sets of the zones are $\mathbb{S}_1=\{1,2,\ldots,9\}$ and $\mathbb{S}_2=\{1,2,\ldots,18\}$. The decision variables $x_z^t(s)$ that are equal to 1, indicating the placement of specific type-zone pairs at sites in $\mathbb{S}_1$ or $\mathbb{S}_2$, are also indicated in the figure.

On closer inspection of Fig. 6, some important observations may be made in respect of the placement decision variables related to the placement of facilities of type $t=1$ (circles). In such a candidate layout, the facility locations at sites 3 and 6 may be “swapped” to form a new layout which remains feasible in respect of the number of facilities of each type to be placed in each zone, n_z^t. Despite such a “swap” resulting in the same layout in terms of the location of facility types, the new layout may result in different objective function values. For example, suppose facilities of type 1 in zone 2 are evaluated according to their dispersion distance from another type-zone pair. If the “swap” is performed between the facilities at sites 3 and 6, different distance values are obtained for the new facility placement in zone 2, which may affect the objective function values achieved by the layout. Also, if a facility of

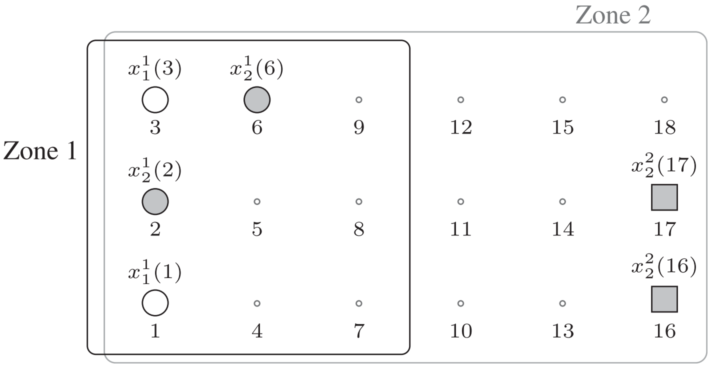


Figure 6. The decision variables $x_z^t(s)$ that are equal to 1 for the facility layout in Fig. 5b.

type 1 in zone 1 is assigned to zone 2, the layout will not be feasible according to the prespecified values of n_z^t, although the facility layout will remain precisely the same in terms of facility type locations. The decision in respect of the zone to which a facility should be assigned during the solution process lies with the methodology employed to solve the problem.

For the above-mentioned reasons, we include the constraints

$$\sum_{s\in\mathbb{S}_z}\sum_{t=1}^{T} x_z^t(s)=n_z^t, \tag{2}$$

for all $z \in \{1,\dots,Z\}$, in addition to requiring that

$$\sum_{z=1}^{Z}\sum_{t=1}^{T} x_z^t(s) \leq 1, \tag{3}$$

for all $s \in \mathbb{S}$, and that

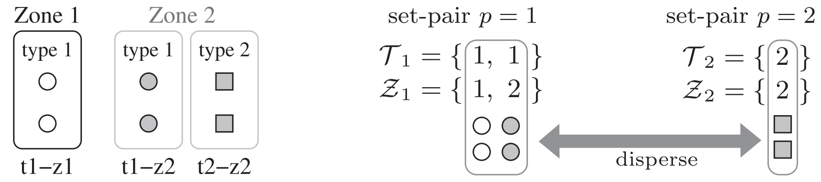
$$x_z^t(s) \in \{0,1\}, \tag{4}$$

for all $t \in \{1,\dots,T\}$, for all $z \in \{1,\dots,Z\}$, and for all $s \in \mathbb{S}$.

Constraint (2) ensures that the number of facilities of type t placed in zone z is n_z^t. Constraint (3) ensures that at most one facility is placed at any of the candidate sites in $\mathbb{S}$, while constraint (4) specifies a binary requirement on $x_z^t(s)$.

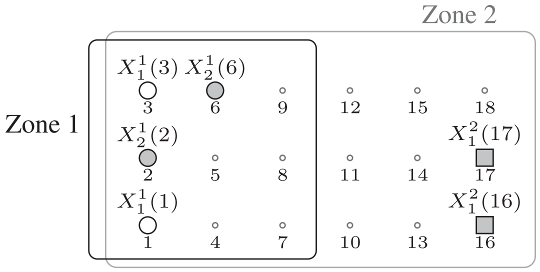
Mathematical formulation of MTZ facility location models

To accommodate type-zone pair facility placement, groups of type-zone pairs which are associated with each other have to be specified—for example, the group of selected type-zone pairs which are to be centered around the features in Fig. 5c. The facility types and zones forming the type-zone pairs that are included in such a group are entries in so-called set-pairs (a set-pair is a group of type-zone pairs). Suppose there exist P such set-pairs, and let $\mathcal{T}_p$ denote the set of facility types in set-pair $p \in \{1,\dots,P\}$ which is paired with the set of zones in p, denoted by $\mathcal{Z}_p$. Facilities in $\mathcal{T}_p$ and zones in $\mathcal{Z}_p$ are paired to form type-zone pairs according to their corresponding indices, so that facility type $\mathcal{T}_p(1)$ forms a type-zone pair with zone $\mathcal{Z}_p(1)$ and,



(a) Facility types and placement zones

(b) Set-pair specification



(c) TZP decision variables, $X_i^p(s)$, equal to 1

Figure 7. The notion of MTZ facility location when applied to dispersion between type-zone pairs grouped together in set-pairs.

generally stated, the i^{th} type-zone pair in set-pair p is formed by associating facility type $\mathcal{T}_p(i)$ with zone $\mathcal{Z}_p(i)$. We denote the number of type-zone pairs in set-pair p by I_p.

Returning to the dispersion layout of Fig. 5b, the notion of set-pairs is illustrated in Fig. 7. The selected set-pairs that are dispersed from each other are illustrated graphically in Fig. 7b. The sets $\mathcal{T}_1=\{1,1\}$ and $\mathcal{Z}_1=\{1,2\}$ correspond to the ordered type-zone pairs t1–z1 and t1–z2 in set-pair 1, while the sets $\mathcal{T}_2=\{2\}$ and $\mathcal{Z}_2=\{2\}$ correspond to the type-zone pair t2–z2 in set-pair 2. The layout resulting from the dispersion of type-zone pairs in set-pair 1 from type-zone pairs in set-pair 2 is shown in part (c) of the figure.

The decision space in MTZ facility location is governed by the placement of type-zone pair entities. For simplicity and specificity of mathematical notation, type-zone pair-specific decision variables may be denoted as special instances of the regular facility placement decision variable, $x_z^t(s)$. Denote a type-zone pair decision variable by

$$X_i^p(s)=x_{z_i}^{t_i}(s) \tag{5}$$

for the ith type-zone pair in set-pair p, where $t_i=\mathcal{T}_p(i)$ and $z_i=\mathcal{Z}_p(i)$. In general, this decision variable may be interpreted as

$$X_i^p(s)=\begin{cases} 1, & \text{if the } i\text{th type}-\text{zone pair in set}-\text{pair } p \text{ is placed at site } s\in\mathbb{S}_{z_i},\ z_i\in\mathcal{Z}_p, \\ 0, & \text{otherwise.} \end{cases}$$

The type-zone pair decision variables $X_i^p(s)$ that are equal to 1, indicating the placement of type-zone pairs in the set-pairs in part (b) of the figure, are indicated in Fig. 7c.

Dispersion model

A dispersion model is formulated here to consider dispersion between type-zone pairs in separate set-pairs, using the multi-type model formulation of Curtin and Church (2006) as a guideline. The dispersion metric of Curtin and Church (2006) is omitted, since the question of whether it is to be applied between facility types or between set-pairs is not the focus in this article—such a metric may be integrated in future work. The objective in the set-pair dispersion model is to maximize the minimum distance determined between type-zone pairs in two separate set-pairs, p and q. The objective is to

$$\text{maximize } V \tag{6}$$

subject to

$$V \leq d(s,s^*)+M(2-X_i^p(s)-X_j^q(s^*)) \tag{7}$$

for all ordered type-zone pairs i in set-pair p, for all ordered type-zone pairs j in set-pair q, and for all $s \in \mathbb{S}_{z_i}$ and $s^* \in \mathbb{S}_{z_j}$. Here M denotes a large positive integer and the distance between candidate site locations s and s^* is denoted by $d(s,s^*)$. The formulation is completed with the obligatory constraints (2)–(4).

Constraint (7) forces the value of the objective function V to be at most the minimum of the distances determined between type-zone pairs in the respective set-pairs. If either (or both) of the two facility site locations under consideration does not contain a type-zone pair of the set-pairs in question, then the objective function value V has the simple requirement of being less than or equal to a very large number M. In contrast, when both potential facility sites under consideration are assigned a type-zone pair in the particular set-pairs, the term in (7) that contains the large number vanishes and V is constrained only by the distance between the type-zone pairs. Since a constraint set exists for every pair of potential facility sites in the set-pairs, V must be less than or equal to the minimum distance over any two type-zone pairs. The requirement of maximization in the objective function (6) ensures that a solution which maximizes the minimum distance is sought.

Center model

The set-pair center model aims to minimize the maximum distance between the type-zone pairs in set-pair p and a set of demand points, denoted by $\mathbb{K}$. Therefore, the objective is to

$$\text{minimize } V \tag{8}$$

subject to

$$V \geq d(s,k)X_i^p(s) \tag{9}$$

for all ordered type-zone pairs in set-pair p, for all $s \in \mathbb{S}_{z_p}$, and for all $k \in \mathbb{K}$. The distance between candidate site location s and demand point k is denoted by $d(s, k)$ in (9). The formulation is completed with the obligatory constraints (2)–(4).

Constraint (9) forces the value of the objective function V to be at least the maximum of the distances determined over all type-zone pair placements. If the facility site location evaluated by a constraint does not contain a type-zone pair in the set-pair under consideration, then the objective function value V has the simple requirement of being nonnegative. In contrast, when the potential facility site under consideration is assigned a type-zone pair in the set-pair under consideration, V is bounded from below only by the distance related to this set-pair.

Maximal covering model

The objective of the MCLP is to maximize coverage achieved by the type-zone pairs of set-pair p with respect to the set of demand points in $\mathbb{K}$. The MTZ MCLP model formulated for the purposes of this article is an extension to that presented by Church and ReVelle (1974).[4] Define

$$y^p(k)=\begin{cases} 1, & \text{if demand point } k \in \mathbb{K} \text{ is covered by at least one} \\ & \text{type-zone pair in set-pair } p \text{ from which cover is demanded,} \\ 0, & \text{otherwise.} \end{cases}$$

Furthermore, define the cover variable

$$c^{t_i}(s,k)=\begin{cases} 1, & \text{if demand point } k \in \mathbb{K} \text{ is covered by a} \\ & \text{facility of type } t_i \in \mathcal{T}_p \text{ sited at } s \in \mathbb{S}_{z_i},\ z_i \in \mathcal{Z}_p, \\ 0, & \text{otherwise.} \end{cases}$$

For the formulation of the objective function, a set is required that specifies which sites cover the points in $\mathbb{K}$. Let this set be denoted by

$$\mathbb{N}_i^p(k)=\{s|c^{t_i}(s,k)=1\}, \tag{10}$$

for all $s \in \mathbb{S}_{z_i}$ and for all type-zone pairs in set-pair p. The objective is then to

$$\text{maximize } V=\sum_{k\in\mathbb{K}} y^p(k) \tag{11}$$

subject to the constraints

$$\sum_{i=1}^{I_p} \sum_{s\in\mathbb{N}_i^p(k)} X_i^p(s) \ \geq \ y^p(k), \tag{12}$$

$$y^p(k) \in \{0,1\}, \tag{13}$$

for all $k \in \mathbb{K}$ and subject to the constraints (2)–(4).

The objective in (11) is to maximize coverage. The linking constraint (12) allows a demand point k to be covered ($y^p(k)=1$) only if one or more type-zone pairs are sited at locations in the corresponding sets $\mathbb{N}_i^p(k)$, while constraint (13) specifies a binary requirement on the auxiliary variables.

Multi-objective facility location optimization

This section is devoted to a discussion on typical approximate solution approaches that have been adopted to solve facility location problems. We also motivate our adoption of the NSGA-II solution approach for the purposes of MTZ facility location and describe our implementation of MTZ principles in this context.

Approximate solution approaches

Multi-objective facility location problems simultaneously seek to optimize a set of (often conflicting) objective functions by providing a set of trade-off solution alternatives instead of a single solution (Zitzler, Deb, and Thiele 2000; Knowles, Thiele, and Zitzler 2006). This set of trade-off solutions are called Pareto-optimal solutions and collectively form a Pareto-front in objective function space (Zitzler, Deb, and Thiele 2000). Pareto-optimal solutions are nondominated in the sense that no other solution yields an improvement in one objective without causing degradation to another (Cohon 1978). That is, no other solutions can dominate the solutions in the Pareto-front when all objectives are considered (Zitzler, Deb, and Thiele 2000).

As discussed in the introduction, solving multi-objective problems in pursuit of the exact Pareto-front may become computationally impractical (Owen and Daskin 1998; ReVelle and Eiselt 2005; Murray et al. 2007; Kim, Murray, and Xiao 2008; Heyns and Van Vuuren 2016a), especially in the context of the NP-hard facility location problems considered in this article (Nagy 1994; Ravi, Rosenkrantz, and Tayi 1994; Murray 2003; Pisinger 2006; Drezner 2012). Powerful metaheuristic optimization procedures are often employed in such instances to approximate the true set of Pareto-optimal solutions. Furthermore, the dynamic nature of these metaheuristics provide the opportunity to develop problem-specific artificial intelligence search strategies.[5] One approach toward solving multi-objective problems often followed is the weighted-sum method (Cohon 1978) in which solutions are the result of a weighted combination of criteria-related performance values aggregated into a single objective—by varying the criterion weights during multiple approximation runs, a Pareto-front approximation may be “traced” out. This approach is also followed to employ single-objective optimization software, such as CPLEX, to solve problems with multiple objectives. The weighted-sum approach has, however, been shown to hold numerous disadvantages, such as the laborious and sensitive iterative process of assigning suitable criterion weights (Das and Dennis 1997; Stanimirović, Zlatanović, and Petković 2011), the requirement of multiple approximation runs to trace out the Pareto-front approximately (Das and Dennis 1997; Hughes 2005; Kim, Murray, and Xiao 2008), and the possibility of obtaining biased and/or misleading results (Stewart 2007), especially in the case of non-convex problems. In some applications, however, a single-objective approach is followed simply because there is only one objective (Krzanowski and Raper 1999; Kim, Rana, and Wise 2004; Bao et al. 2015).

Evolutionary algorithms (EAs) are a popular choice for single-objective solution approaches—for example, for the purposes of visibility/transmission cover (Krzanowski and Raper 1999; Alp, Erkut, and Drezner 2003; Kim, Rana, and Wise 2004; Raisanen and Whitaker 2005; Tong, Murray, and Xiao 2009; Bao et al. 2015) or for wind turbine siting (Mosetti, Poloni, and Diviacco 1994; Grady, Hussaini, and Abdullah 2005; Emami and Noghreh 2010). Simulated annealing (Kirkpatrick, Gelatt, and Vecchi 1983) is another popular metaheuristic that is often employed for transmission and visibility cover optimization (Mathar and Niessen 2000; Kim, Rana, and Wise 2004; Schmidt-Dumont and Van Vuuren 2016). Facility location problems that involve the siting of multiple facility types in multiple zones are likely to involve more than one objective. A multi-objective EA (MOEA) solution approach is therefore adopted in this article. In contrast to the weighted-sum and single-objective approaches mentioned above, MOEAs provide a diverse set of trade-off solutions that approximate the Pareto-front during a single run (Fonseca and Fleming 1993; Zitzler, Deb, and Thiele 2000; Deb et al. 2002; Purshouse and Fleming 2003), and are known to achieve good results fast (Alp, Erkut, and Drezner 2003). EAs are a popular sub-class of the class of population-based metaheuristics—almost all population-based algorithms are, in fact, EAs (Deb 2004). EAs iteratively evolve a population of candidate solutions to an optimization problem based on natural principles (Beheshti and Shamsuddin 2013; Rajabalipour Cheshmehgaz, Haron, and Sharifi 2015): the basic premise involves an initial, randomly generated population of candidate solutions which undergoes multiple generations of carefully controlled evolution to arrive at a final generation of solutions that approximate the Pareto-optimal set of solutions (Deb et al. 2002; Rajabalipour Cheshmehgaz, Haron, and Sharifi 2015) of the problem. During each iteration, the current population generates an offspring population by mimicking natural selection and mutation processes.

Multi-objective facility location problems are typically solved by genetic algorithms (GAs), which are members of the sub-class of EAs—examples include radio network optimization in terms of coverage achieved (Meunier, Talbi, and Reininger 2000; Raisanen and Whitaker 2005; Schmidt-Dumont and Van Vuuren 2016), backup cover achieved for surveillance networks (Kim, Murray, and Xiao 2008; Heyns and Van Vuuren 2016a), and recently the wind turbine micro-siting problem (Kwong et al. 2014). In GAs, the offspring population comprises solutions that are represented as chromosome strings of decision variables. The offspring population results from the application of a crossover operator between solutions selected from the current population and these solutions are, in turn, subject to mutation. Once the offspring population has been computed, the current and offspring populations are combined and the best solutions in this combination are typically carried over to the next generation (Deb et al. 2002; Beheshti and Shamsuddin 2013; Rajabalipour Cheshmehgaz, Haron, and Sharifi 2015).

In the context of MOEAs destined for facility location problems, the NSGA-II is a popular choice and has a good reputation, having achieved marked success in solving multi-objective facility location problems in the literature (Raisanen and Whitaker 2005; Kim, Murray, and Xiao 2008; Kwong et al. 2014; Yamani Douzi Sorkhabi et al. 2016). Similar single-objective implementations of GAs have also been used successfully to solve facility location problems (Mosetti, Poloni, and Diviacco 1994; Meunier, Talbi, and Reininger 2000; Kim, Rana, and Wise 2004; Grady, Hussaini, and Abdullah 2005; Emami and Noghreh 2010; Tran et al. 2013). We therefore adopt the NSGA-II as approximate solution technique in this article and our implementation of this technique is elaborated upon in the following sections.[6]

MTZ solution approach

We now consider suitable solution representation and the implementation of an MTZ NSGA-II.

Candidate solution representation

In our GA approach, a candidate solution is represented as a chromosome string of candidate facility location site numbers. The chromosome lists facility sites grouped according to zone sub-strings in numerical order from zone 1 to zone *Z*. Within each zone's grouping, the candidate facility site numbers correspond to the facility types placed within that zone and are listed in order. Not all facility types need to be included within a zone grouping, since facilities of a specific type may not be assigned to a specific zone. It may also be possible for a zone sub-string to include multiple entries of a specific facility type, depending on the number of facilities of that type assigned to the zone. A generic illustration of this chromosome representation scheme is provided in Fig. 8. The chromosome representation of a random facility layout is described in Fig. 9.

NSGA-II implementation

The NSGA-II performs evolution-inspired selection processes and modification operators on populations of solutions until a termination criterion is met (Deb et al. 2002)—in this article, the

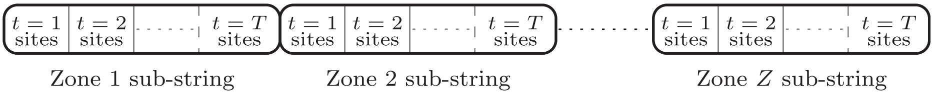
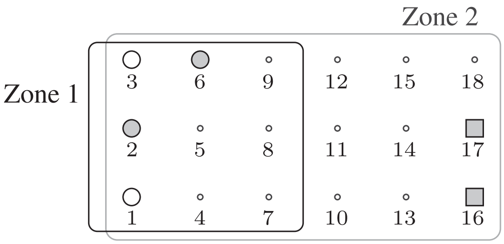


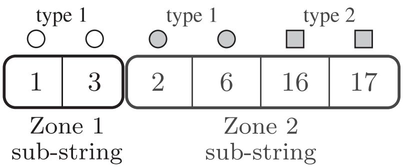
Figure 8. Chromosome string representation scheme employed for solving MTZ facility location problems, consisting of zone sub-strings and their constituent facility assignments according to facility types. The inclusion of each facility type and its number of site entries may vary between the zone sub-strings, according to problem-specific placement requirements.

termination criterion is when the solutions in successive approximate Pareto-fronts no longer change significantly over a fixed number of generations. An initial population of candidate solutions of size *N* is stochastically generated according to a uniform distribution. The NSGA-II then iteratively generates offspring populations that are typically also of size *N*—each offspring population is the result of carefully controlled evolutionary operators applied to a "mating pool" of solutions that are selected stochastically from the population of the previous generation (Deb et al. 2002). For the purposes of this article, we limit the rest of our discussion on the NSGA-II to its chromosome operators, specifically for the purposes of MTZ facility location. Thorough descriptions of the NSGA-II solution processes (including its solution sorting, ranking, and diversity preservation mechanisms) are available in the literature (Deb et al. 2002).

Tournament selection follows the creation of the "mating pool" during the process of offspring solution generation. Two typically superior parent solutions are selected stochastically from the mating pool according to the principles of solution dominance and crowding distance (Deb et al. 2002), after which crossover and mutation operators are applied to the two parents to generate two offspring solutions. To perform crossover, one or more crossover points are chosen along the chromosome representations of the parent solutions, at the same locations for both parents. The locations of these points may be chosen arbitrarily for each parent pair, or at fixed positions for all crossover operations (De Jong and Spears 1992). Each of the parent strings are "cut" at the crossover points and the resulting sub-strings are then interchanged alternately to create new chromosomes, so that the newly generated chromosome strings



(a) Facility layout



(b) Chromosome representation of solution in (a)

Figure 9. The MTZ chromosome representation for the solution in Fig. 6.

consist of ordered sub-strings that alternate in respect of the parent chromosomes from which the sub-strings are sourced. This type of crossover creates new site combinations as solutions, but does not alter the constituent sites that are present in the solutions. Since the parents that are selected for crossover typically perform well in respect of the objective functions, the offspring solutions inherit some of the strong properties of their parents, but at the same time also explore new solution combinations. Not all parent solutions chosen by tournament selection, however, undergo crossover. Instead, crossover is subject to a crossover probability.

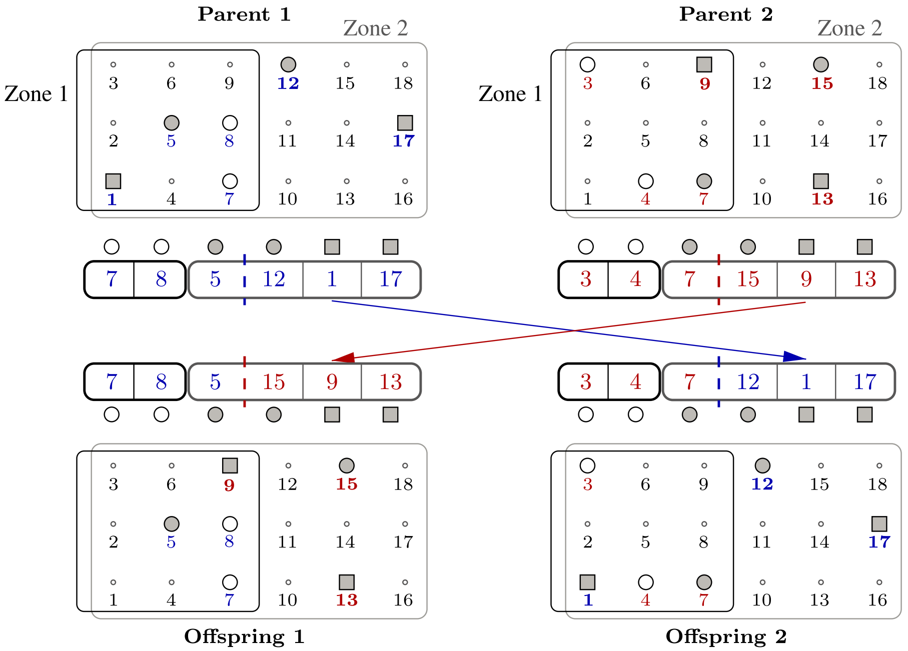
After the crossover stage, a final measure of solution modification, called mutation, is introduced which promotes solution diversity in the sense of introducing new sites into the location combinations of solutions, as opposed to merely exchanging these locations from an existing collection of sites. This is achieved by selecting a random site in the chromosome string and exchanging it for a randomly selected site from the zone that corresponds to the selected point. Mutation occurs for each solution in the post-crossover offspring population with a prespecified mutation probability.

Crossover and mutation operations may often result in infeasible solution combinations, for example, two facilities may be sited at the same location after the procedure. Such constraint violations are typically accommodated by one of two methods, namely the use of penalty functions or the use of repair algorithms (Coello Coello 2002; Coello Coello and Mezura Montes 2002; Bennet, Xiao, and Armstrong 2004; Xiao 2008; Kramer 2010). Penalty functions impose penalties on the objective function values associated with infeasible solutions (Coello Coello 2002; Coello Coello and Mezura Montes 2002), while repair algorithms attempt to “fix” infeasible solutions (Bennet, Xiao, and Armstrong 2004; Xiao 2008). A simpler method of constraint handling is used in our MTZ NSGA-II. The rejection method simply discards any infeasible solutions encountered during the search process. This may, however, make it difficult to establish a feasible population and often results in premature convergence (Kramer 2010). To address this shortcoming, our MTZ NSGA-II attempts multiple crossovers at randomly selected crossover points between two parents, until a crossover that results in at least one feasible offspring is found, or until a maximum number of crossover attempts have occurred, in which case the crossover process between the two parents is abandoned. In this article, a maximum of ten crossover attempts per parent pair are permitted. This approach is not followed in the mutation operator—that is, a solution selected to undergo mutation undergoes a single mutation attempt and the new candidate is accepted if it is feasible.

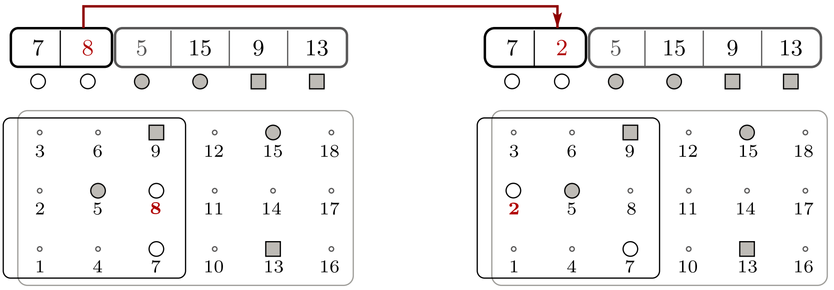
An illustration of the site locations and chromosome representations of two parent solutions that are selected to undergo crossover in an MTZ environment are shown in the top half of Fig. 10a. Suppose the crossover operator employs single-point crossover, randomly selected at the point indicated in the parent chromosome strings. The bottom part of Fig. 10a contains the resulting offspring solutions with respect to their string representations and site locations. In Fig. 10b, the first offspring solution of Fig. 10a undergoes mutation at a randomly selected point in its chromosome string. The site locations and chromosome representation of the resulting mutated solution are shown in the right half of Fig. 10b.

Implementation examples

This section is devoted to an illustration of the practical application of the MTZ models proposed in this article, as solved by the MTZ NSGA-II discussed in the previous section. The terrain, facilities and zones used throughout this section are introduced, after which the solutions



(a) Crossover between two parent solutions



(b) Mutation of offspring 1 in (a)

Figure 10. Site locations and chromosome string representations of (a) crossover and (b) mutation operations.

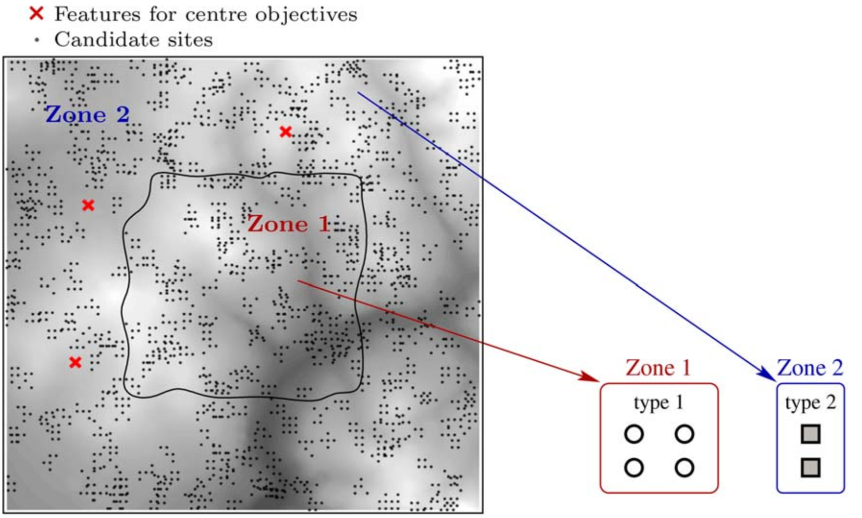
to three problem instances are described. The discussion closes with an investigation into the quality of the solutions returned by our MTZ NSGA-II, in the form of a comparison between exact and approximated results.

Terrain, facilities, and zones

A square section of terrain measuring 10 km south to north and 10 km west to east was selected in an area of the South African Western Cape, surrounding the town of Malmesbury. The relief exhibits gentle hills and multiple peaks, as illustrated in Fig. 11a in which elevation increases from dark to light shading.[7] Two disjoint zones are considered as indicated in Fig. 11a. For the purposes of the examples in this section we suppose that the zones are distinguished according to the notion of risk, where risk may be categorized according to the possibility of military attacks, wildfires or flash floods, among others. Zone 1 is categorized as being a high-risk area, while zone 2 is low-risk. The boundary that separates the two zones may, for example, be a security fence/perimeter or a fire break (e.g., a separation or obstacle that interferes with a wildfire's ability to cross from one area of vegetation to another).[8]

Suppose that there are two facility types available for placement and that they are used to monitor activity on the terrain surface, for example, watchtowers, observation cameras, or sound detection devices. Facilities of type 1 have an effective range limited to 2,500 m and a height of 3 m above the ground—this is important for cover computations, because of improved detection capability from the elevated viewpoint. Facilities of type 2 have an effective range that extends beyond the boundaries of the terrain surface in Fig. 11a—that is, when placed at any of the candidate site locations, type 1 facilities may achieve cover with respect to any point on the terrain surface, subject to terrain interference. The height of type 1 facilities is 5 m above the ground. We suppose that the number of facilities that are available for placement is limited—four of type 1 and two of type 2. Suppose that the superior detection capability and the taller structures of facilities of type 2 result in a situation where they are significantly more expensive than those of type 1 and are therefore limited to placement within the low-risk zone. The four facilities of type 2 may be placed anywhere in the high-risk zone, since they are more easily replaceable in the event of damage/attack, in addition to their short-range detection capability being considered more beneficial when applied to the high-risk zone and its immediate surrounds. The placement requirements of this scenario are illustrated in Fig. 11b. The type-zone pair markers used throughout the examples to denote site locations are also shown in Fig. 11b.

A number of candidate facility site locations were identified on the terrain surface—500 in zone 1 and 1,500 in zone 2, respectively, and illustrated as dots on the terrain surface in Fig. 11a. To arrive at the candidate sites within each zone, a number of sites were randomly selected from all the data-points included in high resolution raster (gridded) terrain elevation data that represent the entire terrain surface. Local "neighborhoods" were then populated around these



(a) Terrain in relief and placement zones

(b) Facilities to be placed

Figure 11. (a) Terrain, zones, features and candidate sites, and (b) facilities to be placed in the respective zones of the MTZ example problem instances.

selected candidate sites, by randomly selecting candidate sites within their nearby vicinities. The result is a set of candidate site locations that resembles real-world sets typically determined after performing terrain analysis to identify suitable site locations according to specific criteria, for example, degree of slope (Heyns and Van Vuuren 2015), terrain feature selection (Kim, Rana, and Wise 2004; Bao et al. 2015) and problem-specific search procedures (Heyns and Van Vuuren 2016a). Three “features” were manually selected for the purpose of center objectives and are indicated by crosses in Fig. 11a—none of these feature locations are candidate sites. In a real-world context, the features toward which the distances from facilities are to be minimized may be “safe zones” (e.g., bunkers, extraction points or fire safety zones). The features may alternatively be other points of interest, such as power grid access points for electricity supply or water sources, according to the context of the problem and facility requirements.

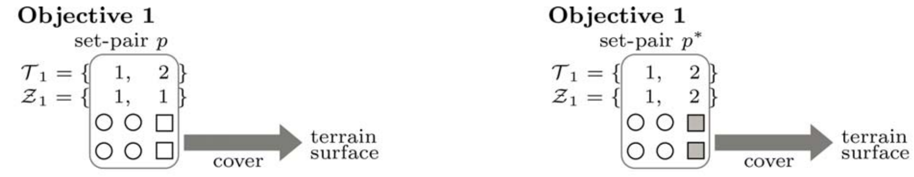
Example problem instances

Twenty Pareto-optimal approximation runs of the MTZ NSGA-II were performed and an attainment set of solutions (the globally best set of approximately Pareto-optimal solutions from all optimization runs) was identified for each of the three problem instances in this section. First, we solve a multi-type MCLP for single-zone and multi-zone instances and compare the results. In each of the two sections that follow, we extend the MTZ problem instance by adding center and dispersion objectives, respectively. Each example is accompanied by a graphical presentation of the results achieved and a brief discussion.

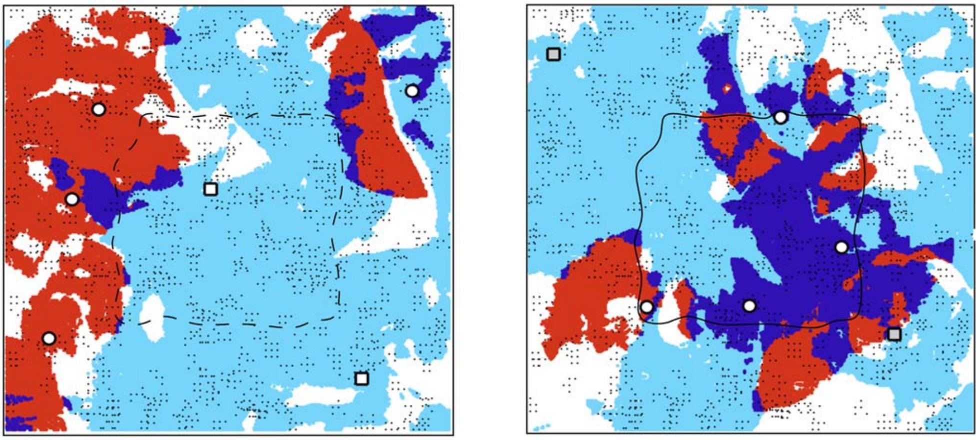
Multi-type MCLP problem instance with single and multiple zones

An MCLP with different zone specifications was solved to illustrate the effect of including multiple zones in facility location problems. First, the problem was solved by considering the candidate site locations of the two zones in Fig. 11a together as a single large zone, allowing the placement of all facilities in Fig. 11b within this zone. The same MCLP was then solved for the two zones and facility assignments as described in Fig. 11. The covering objective in both instances was to maximize the terrain surface area that receives cover from at least one of all the facilities. The objectives and set-pairs included in the models are described graphically in Fig. 12a, b for the single and multi-zone instances, respectively.

The best results obtained from the twenty approximation runs of our MTZ NSGA-II implementation for each problem instance are provided in Fig. 12c, d, respectively. These figures present terrain surface covered by at least one facility—red for type 1 only, light blue for type 2 only, and dark blue for overlapping cover from types 1 and 2. The site locations for the respective facility types are also presented in parts (c)–(d) of the figure. A significant difference in site locations for the two problem instances may be observed, illustrating the effect of multi-zone placement requirements. In the single-zone instance in part (c) of the figure, the facilities of type 1 that are destined for zone 1 in the multi-zone instance are all placed outside this zone (the zone is indicated by a dashed line in the figure for comparative purposes). Furthermore, facilities of type 2 that are destined for placement in zone 2 in the multi-zone instance are placed in both zones in the single-zone solution. The result in part (d) of the figure illustrates the zonal placement requirements of (b) in the figure being enforced by the MTZ NSGA-II, and the cover achieved by the solution is markedly different from that in (c). It is, however, surprising that the percentage of terrain cover achieved by at least one facility drops from



(a) Single-zone set-pair and objective (b) Multi-zone set-pair and objective



(c) Maximum cover solution (single-zone), 75.8% (d) Maximum cover solution (multi-zone), 72.9%

Figure 12. (a)–(b) Objectives and set-pairs considered in the MTZ MCLP problem instance with single and multiple zones. (c)–(d) Physical site locations and cover achieved for the best values returned with respect to the objectives. Terrain surface cover is indicated by red for type 1 only, light blue for type 2 only, and dark blue for overlapping cover from types 1 and 2. White areas are not covered.

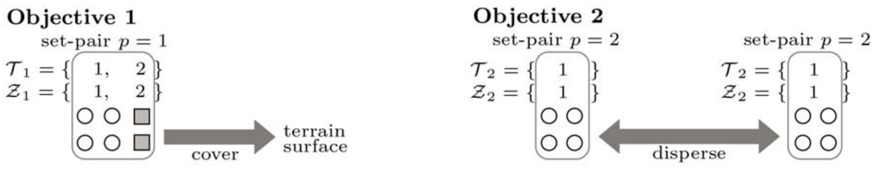
75.8% for the single-zone instance, to 72.9% for the multi-zone instance, which is only a small difference.

The average time per approximation run was 5 m 43 s for the single-zone instance, while the multi-zone instance runs required 7 m 28 s on average.[9] The visibility computations were precomputed and stored to disk prior to the approximation runs, and so the average run time accounts for algorithmic computation and file-read time only.[10]

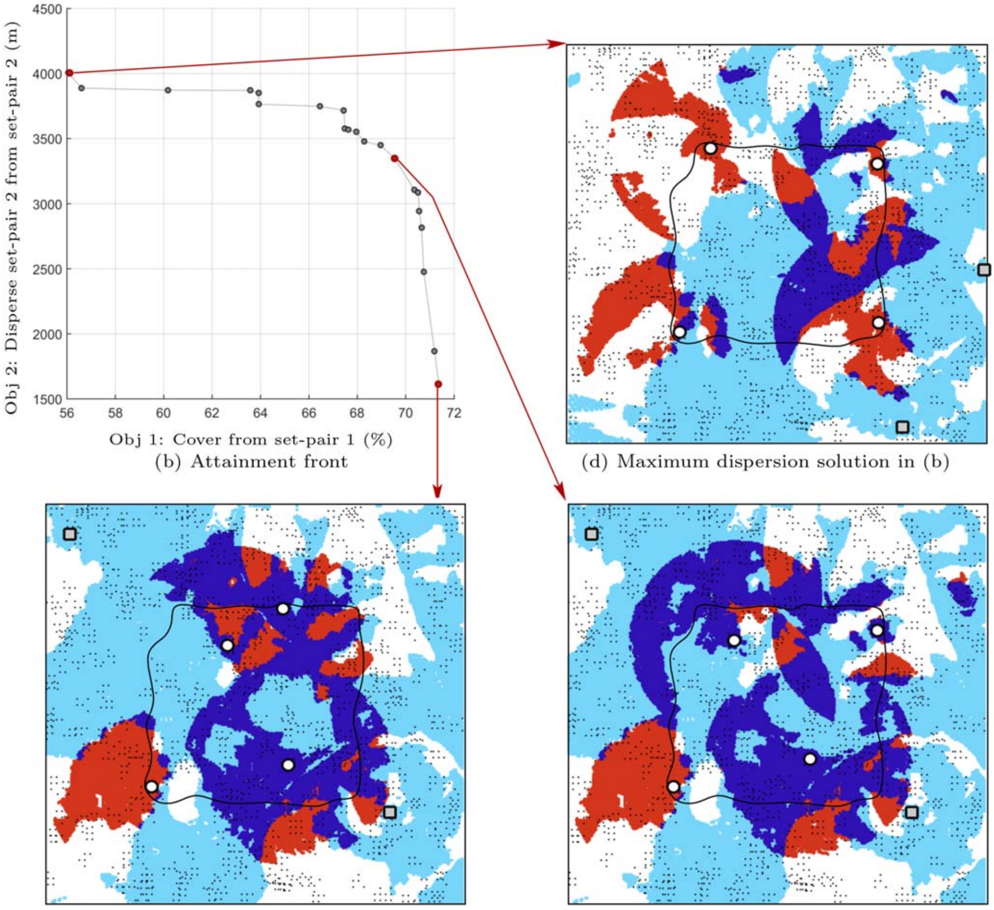
Bi-objective MTZ problem instance with MCLP and dispersion objectives

The second problem instance includes MCLP and dispersion objectives, as described graphically in Fig. 13a. The first objective is the same as in the previous example—to maximize the terrain surface area that is covered from at least one of all the facilities. The second objective is to disperse the facilities in the high-risk zone from each other, as this will reduce the possibility of simultaneous damage/attack to more than one facility in the zone.

The optimization results obtained by applying our MTZ NSGA-II implementation are provided in Fig. 13b–e. The average time per approximation run was 6 m 4 s. The attainment front achieved after twenty approximation runs is presented graphically in objective function space in Fig. 13b. A trade-off curve may be observed upon inspection of the attainment front, demonstrating the conflicting nature of the objectives. In parts (c)–(d) of the figure, the physical site



(a) Set-pairs and objectives



(b) Attainment front

(d) Maximum dispersion solution in (b)

(c) Maximum cover solution in (b)

(e) Another solution in (b)

Figure 13. (a) Objectives and set-pairs considered in the bi-objective MTZ problem instance with MCLP and dispersion objectives. (b) Attainment front obtained by twenty runs of the MTZ NSGA-II. (c)–(d) Physical site locations and cover achieved by the solutions in the attainment front in (b) that returned the best values with respect to the objectives. (e) Physical site locations and cover achieved by another solution from the attainment front in (a). Terrain surface cover is indicated by red for type 1 only, light blue for type 2 only, and dark blue for overlapping cover from types 1 and 2.

locations for the solutions from the attainment front that return the best values with respect to each of the objectives are shown. The physical site locations of an additional solution selected from the attainment front are also provided in part (e) of the figure. Fig. 13 encapsulates the type of information that would typically be provided to decision makers to facilitate a final decision with respect to the placement of facilities.

The physical site locations in Fig. 13c, d may not be practically desirable, because the best result obtained with respect to one objective is paired with the worst result obtained with respect to the other objective. Nevertheless, the placement of facilities in part (d) of the figure illustrate the effective dispersion search process of the MTZ NSGA-II obtained within the specified high-risk zone. The effect of the positions of the facilities in the high-risk zone on the positions of the facilities in the low-risk are evident, as the low-risk zone facilities attempt to compensate for the loss of cover that results from the dispersed layout of high-risk facilities. A "balanced" result is that of Fig. 13e, in the sense that good objective function values are achieved with respect to both objectives, without either objective function value being significantly less than the maximum values achieved. When deciding on which solution to implement, decision makers may typically consider more factors than just the objective function values and site locations—others factors may include subjective opinions, such as preferred areas to receive cover, cover patterns (type-cover and overlapping cover), and preference for certain site location combinations.

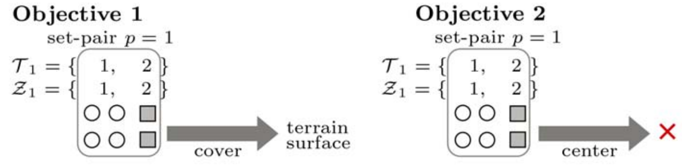
Bi-objective MTZ problem instance with MCLP and center objectives

The objectives in the final problem instance are those of MCLP and centering—the set-pairs and objectives are presented graphically in Fig. 14a. The first objective is to maximize the terrain surface area that is covered from at least one of any of the facilities, while the second objective is to center all facilities to the features. The center objective may, for example, be a result of the facilities requiring constant electricity supply, from which short distances to features such as energy grid access points are desirable.

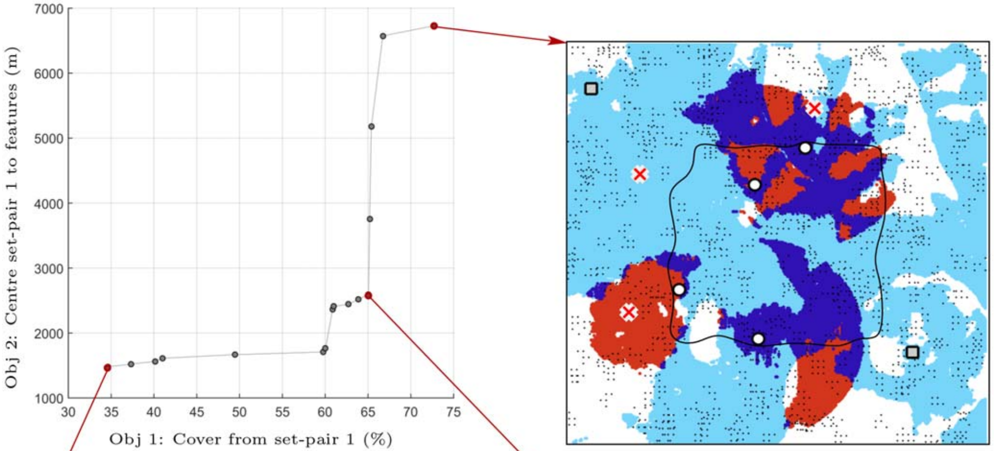
The results for the problem instance obtained by our MTZ NSGA-II implementation are provided in Fig. 14b–e, in the same format as in Fig. 13b–e. The average time per approximation run was 7 m 55 s. Again, the physical site locations in parts (c) and (d) in the figure may not be practically desirable because of the trade-off extremity of the objective values. The placement of facilities in part (d) of the figure illustrate the effective centering search process of the MTZ NSGA-II, while crucially maintaining the placement of facilities within the boundaries of the zones to which they belong. A "balanced" result is provided in Fig. 14e, with good objective function values achieved with respect to both objectives.

The quality of solutions returned by the MTZ NSGA-II

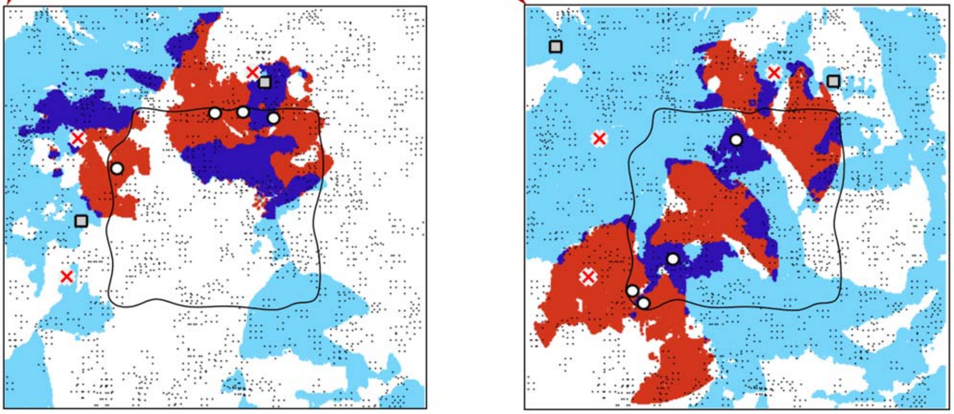
To investigate the approximation efficiency of our MTZ implementation of the NSGA-II, simplified versions of the bi-objective problem instances in the previous section were solved exactly and approximately and the results compared. The complexity of the problem instances was significantly reduced to determine the exact Pareto-fronts within a realistic computation time. This was achieved by extracting a small number of candidate sites from the existing ones in each zone—30 from zone 1 and 90 from zone 2—to be considered for facility placement. Furthermore, the facility placement requirements of the high-risk zone was reduced to only two facilities of type 1, as opposed to four in the previous examples. The reduced complexity candidates site locations and placement requirements are provided in Fig. 15a. The true Pareto-fronts of the problems instances were solved exactly by brute force (complete enumeration). The same problem instances were then solved by twenty successive runs of the MTZ NSGA-II and the results compared.



(a) Set-pairs and objectives



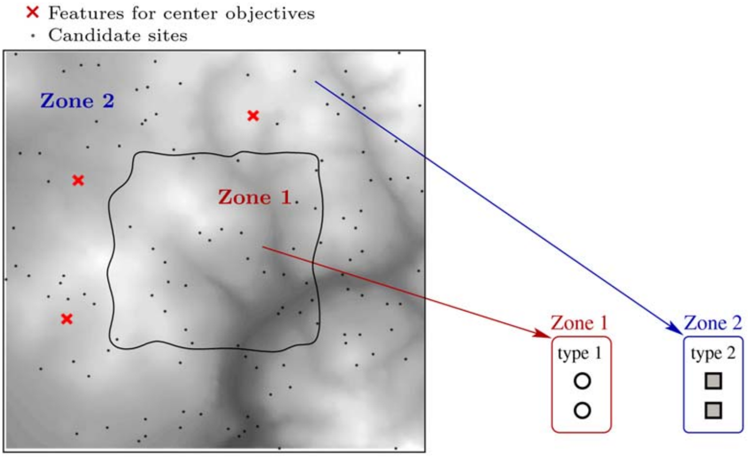
(b) Attainment front (c) Maximum cover solution in (b)



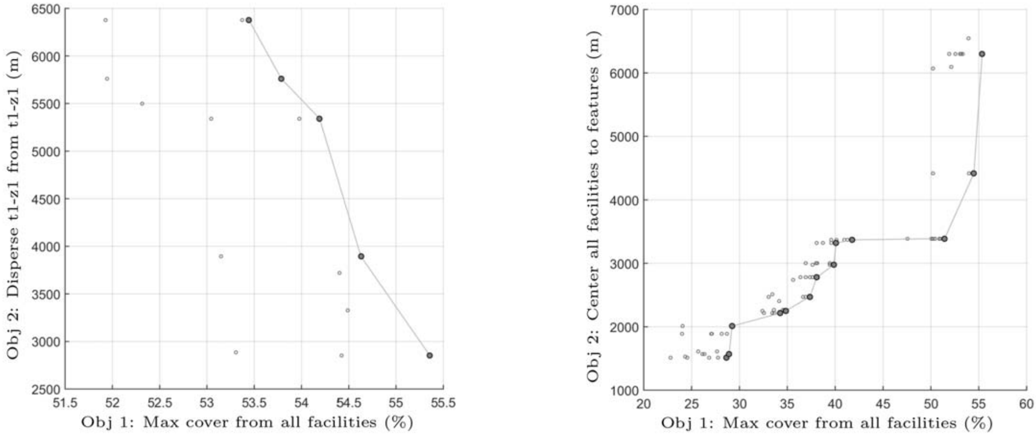
(d) Minimum centre distance solution in (b) (e) Another solution in (b)

Figure 14. (a) Objectives and set-pairs considered in the bi-objective MTZ problem instance with MCLP and center objectives. (b) Attainment front obtained by twenty runs of the MTZ NSGA-II. (c)–(d) Physical site locations and cover achieved by of the solutions in the attainment front in (b) that returned the best values with respect to the objectives. (e) Physical site locations and cover achieved by another solution from the attainment front in (a). Terrain surface cover is indicated by red for type 1 only, light blue for type 2 only, and dark blue for overlapping cover from types 1 and 2.

For the maximal cover-dispersion problem instance, the brute force approach took a total of 22 h to arrive at the true Pareto-front, which is indicated by the solid gray markers in Fig. 15b. The MTZ NSGA-II obtained all the nondominated solutions in the true Pareto-front after six runs, which took only 10 m 37 s. The solutions of the twenty Pareto-front approximations



(a) Reduced complexity candidate sites and placement requirements



(b) Cover-dispersion Pareto-front and approximations

(c) Cover-center Pareto-front and approximations

Figure 15. (a) Candidate sites and placement requirements considered in simplified versions of the bi-objective MTZ problem instances. (b) Exact Pareto-front and approximations obtained by twenty runs of the MTZ NSGA-II for the cover-dispersion problem instance. (c) Exact Pareto-front and approximations obtained by twenty runs of the MTZ NSGA-II for the cover-center problem instance.

are given by empty markers in Fig. 15b. The majority of these solutions occurred in more than one run and the approximated Pareto-fronts consisted of combinations of these solutions—seven of the twenty approximation runs returned the exact Pareto-front. This is a sign of consistently good approximation results. The average time per run was 1 m 39 s.

For the maximal cover-center problem instance, the brute force approach took a total of 22 h 20 m to arrive at the true Pareto-front, which is indicated by the solid gray markers in Fig. 15c. All nondominated solutions in the true Pareto-front were obtained by the MTZ NSGA-II after 12 runs, which took 19 m 44 s. The solutions of the twenty Pareto-front approximations are given by empty markers in Fig. 15c. Once again, the majority of these solutions occurred in more than one run. None of the twenty approximation runs, however, returned the exact

Pareto-front, although all twenty runs obtained at least one of the true nondominated solutions (typically more). The average time per run was 1 m 38 s.

Conclusion and ideas for future work

In this article we introduced the novel concept of MTZ facility location, in which multiple facility types and multiple zones are simultaneously considered in the solution process. We presented three novel MTZ ILP model formulations for dispersion, center and maximal cover facility location problems. To illustrate the implementation of the MTZ models, our novel MTZ NSGA-II solution approach was used to solve three problem instances, each involving different combinations of the MTZ model formulations (as objectives). The results obtained were presented graphically in the form of attainment fronts of nondominated solutions in objective function space, in addition to providing physical site locations of selected solutions from the attainment fronts. The attainment fronts and physical site locations illustrated the practical functioning of the MTZ models and solution methodology.

Future work may include the MTZ formulation and implementation of additional facility location models, such as the dispersion models not covered in this article (Moon and Chaudhry 1984; Kuby 1987; Erkut and Neuman 1991), the median model (Hakimi 1965), and other covering models (Toregas et al. 1971; Daskin 1983). Unified MTZ models—extensions to unified models such as those proposed by Nickel and Puerto (1999, 2005); Marín et al. (2009); Lei and Church (2014, 2015)—also merit investigation. In the context of the MCLP, multiple interest zones may be considered and included in the model for facility location problems in which service demand is not limited to a single area only, and where different interest zones may demand cover from different facility types or type-zone pairs (Heyns and Van Vuuren 2015; Heyns 2016).

Traditional dispersion models aim to maximize inter-facility distances, while traditional center models aim to minimize the distances between facilities and demand points. Alternative formulations should be investigated—for example, dispersion models that aim to maximize the distances between facilities and features and centering models that aim to minimize inter-facility distances. Such formulations may lead to novel facility location possibilities—especially in an MTZ context (Heyns 2016). Examples include the dispersion of transmitter devices to be placed as far as possible away from existing features that are sensitive to radio frequency interference. An example in the context of centering models is the placement of facilities which are desired to be close to each other—such as observation posts—to reduce inter-facility distances and response times.

While the NSGA-II returned good results in this article, other algorithms may be applied or modified to solve MTZ facility location problems. Swap-based algorithms such as the Teitz & Bart algorithm, for example, perform well with respect to distance-based objectives, while GAs are generally used to solve cover problems (Heyns 2016). Hybrid or problem-specific multi-objective MTZ algorithms and approaches (such as those proposed and investigated by Heyns [2016] and Heyns and Van Vuuren [2016a]) are considered to be beneficial future contributions to improving the effectiveness of complex real-world facility location solution processes.

Notes

1 Approximate algorithms are often able to identify the exact set of optimal solutions for multi-objective facility location problems (Kim, Murray, and Xiao 2008; Heyns and Van Vuuren 2016a,b).

2 See Bao et al. (2015) for an interesting comparison of the LSCP and MCLP alternatives implemented in the context of a surveillance camera siting problem, in which these considerations are practically illustrated.

3 A recent special issue of articles dedicated solely to the MCLP illustrates the remarkable significance of this model in the facility location literature (Murray and Church 2016).

4 A multi-type covering model has recently been formulated by Bao et al. (2015) for a bi-objective watchtower problem.

5 The multi-resolution approach proposed by Heyns and Van Vuuren (2016a) is one such method.

6 Our multi-objective implementation of the NSGA-II has been shown to identify all the nondominated solutions in the exact Pareto-front for small problem instances (Heyns and Van Vuuren 2016a,b). A similar NSGA-II implementation utilized by Kim, Murray, and Xiao (2008) also identified all the nondominated solutions in their problem.

7 Obtained from high resolution raster (gridded) terrain elevation data (Heyns and Van Vuuren 2016a).

8 The Western Cape region that envelops the example terrain is notorious for wildfires and the classification of terrain according to fire risk is therefore realistic—the corresponding author has served as a volunteer fire fighter during multiple wildfires in the region.

9 Determined on a personal laptop, using MATLAB R2015a on an Intel Core i5–7200U CPU, utilizing one of two 2.5GHz cores, 8 GB RAM, and running in the Windows 10 64-bit operating system.

10 Due to the large number of file-reads during the solution process, the file-read times were observed to take up to 50% of the average computation times stated in this article. Storing the files on an SSD is expected to reduce file-read times significantly.

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