Applied Energy 147 (2015) 536–555

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Review

Optimization problems in natural gas transportation systems: A state-of-the-art review

Roger Z. Ríos-Mercado[a,*], Conrado Borraz-Sánchez[b]

[a] Universidad Autónoma de Nuevo León (UANL), Graduate Program in Systems Engineering, AP 111-F, Cd. Universitaria, San Nicolás de los Garza, NL 66450, Mexico
[b] Energy and Infrastructure Analysis Group, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

HIGHLIGHTS

- A review on optimization of natural gas transportation systems is presented.
- Gathering, transmission, and distribution systems are reviewed.
- Steady-state and transient models are analyzed.
- The most promising research challenges are highlighted.

ARTICLE INFO

Article history:
Received 17 September 2014
Received in revised form 27 January 2015
Accepted 2 March 2015

Keywords:
Natural gas transportation
Operations research
Pipeline optimization
Line-packing problem
Pooling problem
Fuel cost minimization problem

ABSTRACT

This paper provides a review on the most relevant research works conducted to solve natural gas transportation problems via pipeline systems. The literature reveals three major groups of gas pipeline systems, namely gathering, transmission, and distribution systems. In this work, we aim at presenting a detailed discussion of the efforts made in optimizing natural gas transmission lines.

There is certainly a vast amount of research done over the past few years on many decision-making problems in the natural gas industry and, specifically, in pipeline network optimization. In this work, we present a state-of-the-art survey focusing on specific categories that include short-term basis storage (line-packing problems), gas quality satisfaction (pooling problems), and compressor station modeling (fuel cost minimization problems). We discuss both steady-state and transient optimization models highlighting the modeling aspects and the most relevant solution approaches known to date.

Although the literature on natural gas transmission system problems is quite extensive, this is, to the best of our knowledge, the first comprehensive review or survey covering this specific research area on natural gas transmission from an operations research perspective. The paper includes a discussion of the most important and promising research areas in this field. Hence, this paper can serve as a useful tool to gain insight into the evolution of the many real-life applications and most recent advances in solution methodologies arising from this exciting and challenging research area of decision-making problems.

Contents

* Corresponding author. Tel.: +52 81 8329 4000x1634.
E-mail addresses: roger.rios@uanl.edu.mx (R.Z. Ríos-Mercado), conradob@lanl.gov (C. Borraz-Sánchez).

http://dx.doi.org/10.1016/j.apenergy.2015.03.017
0306-2619/

1. Introduction

Natural gas [1] is an essential energy source for the future. Its manifold benefits include low greenhouse gas emissions and relatively reduced capital costs, which make its position competitive in most of the sectors among other energy sources, particularly for new power generation facilities. Global projections in natural gas reserve levels are also a clear indication of the increasingly important role that natural gas will play to support growth in markets through 2035. Fig. 1 shows global projections in natural gas reserves by geographic regions, where the largest concentrations are observed in Eurasia and the Middle East.

The performance of natural gas as a primary energy source is highly representative within three specific natural gas end-use consumption sectors, namely, (a) the residential/commercial, (b) industrial and (c) electric generation sectors. Fig. 2 shows world projections of energy consumption by end-use sector and fuel through 2035. Electric power sector projections are shown separately in Fig. 3, with the addition of nuclear power projections. These figures reveal that most of natural gas consumption is concentrated in the (b)- and (c)-sectors, accounting for 87% of the total world natural gas consumption, with an average growth of 1.7% and 2.0% per year, respectively, through 2035.

Natural gas consumption can significantly be affected by short term factors, such as weather, fuel switching and price/market variability. However, it is the long term demand factors that reflect the basic trends for natural gas use into the future. For example, the most likely important long term driver of natural gas demand in the (a)-sector is heating applications (see Fig. 4). The percentage increase in the number of new households using natural gas for heating over the next 20 years is expected to provide a strong driver for residential natural gas demand.

In the (b)-sector, two long-term driving forces may be observed due to the movement away from energy-intensive manufacturing processes, namely: the increased energy efficiency of equipment and processes, and the shift to the manufacture of goods that require less energy input. Although these factors lead to modest increases in energy demand, the trend is expected to hold into the future.

In the (c)-sector, the long-term factor is primarily attributable to natural gas-fired combined cycle generation plants, which require relatively low capital investments and provide emission reductions from using natural gas as opposed to other fossil fuels. The U.S. Energy Information Administration (EIA) expects 60% of new electric generation capacity built by 2035 will be natural gas combined-cycle or combustion turbine generation. Readers

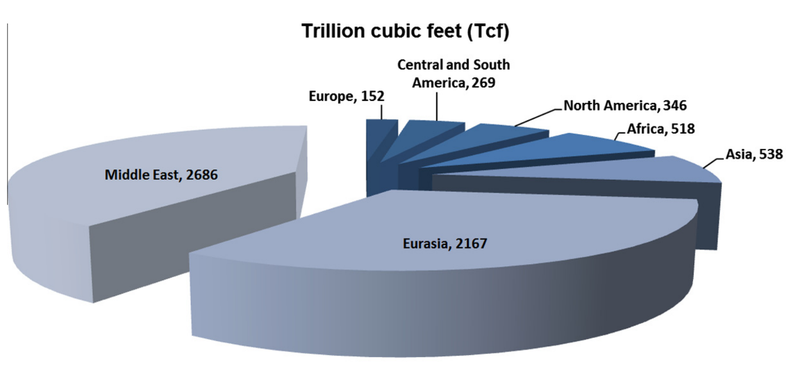

Fig. 1. Projected world natural gas reserves by geographic region.

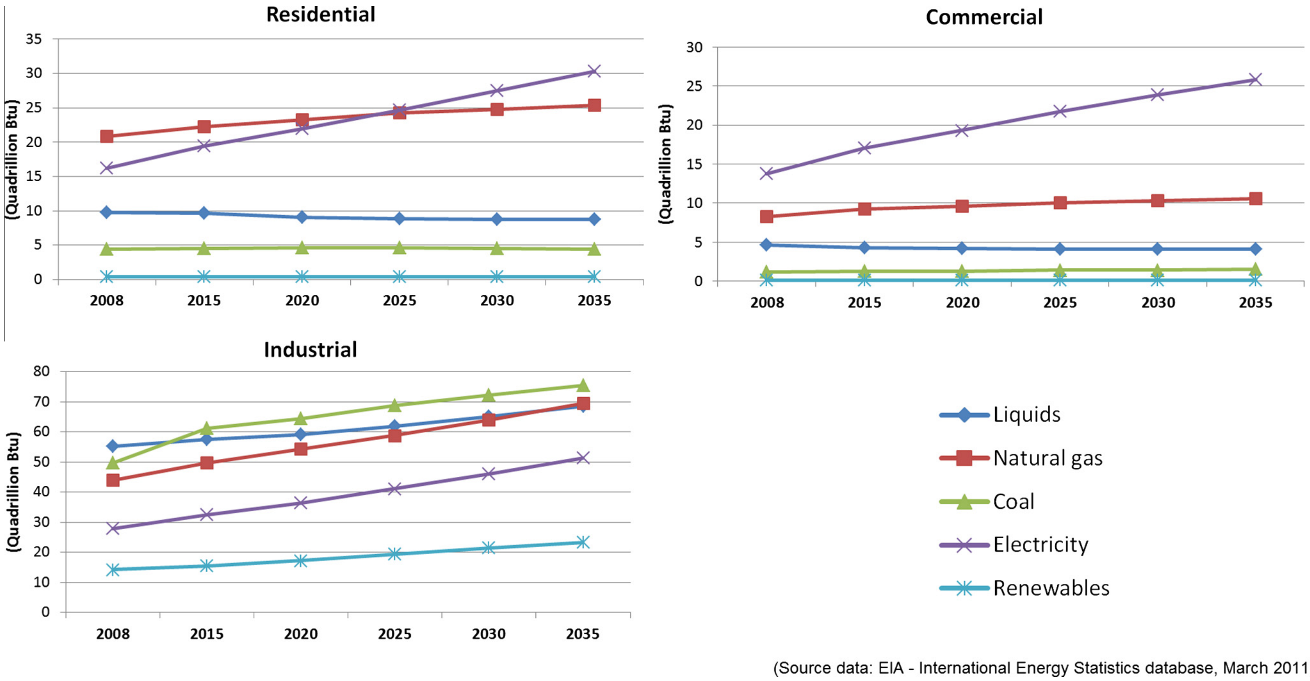

Fig. 2. Projected world energy consumption by end-use sector and fuel.

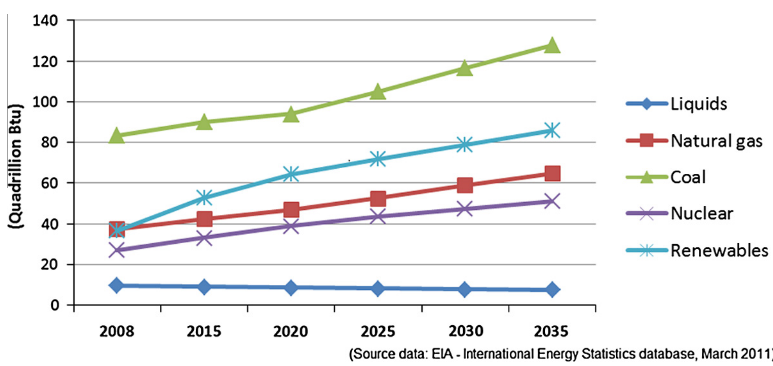

Fig. 3. Projected world energy consumption of the electric power sector.

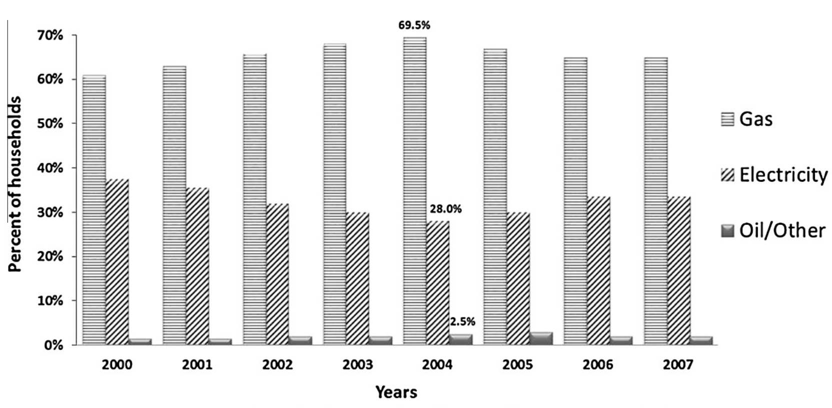

Fig. 4. New households by heating fuel type 2000–2007.

interested in prediction models for natural gas consumption are referred to, e.g., [2].

According to the EIA, in the U.S. for example, a 26.55 trillion cubic feet (Tcf) natural gas demand [3] is projected by 2035, which represents roughly a 16% increase over the demand levels observed in 2009. This projection is correlated to the energy demand expected in the U.S. in the decades ahead. For example, the energy demand projected by 2035 in the U.S. is roughly 6.0%, 23.4% and

25.8% for the (a)-, (b)- and (c)-sectors, respectively. Note that these sectors currently account for 21%, 27% and 16% of all natural gas consumption in the United States. Readers interested in a detailed analysis on global trends in natural gas demand, production and reserves are referred to [4].

The natural gas global tendencies may slightly differ from the different reports provided by one federal agency to another. Yet, the recurrent result across the studies is that natural gas consumption, production, reserves and dependencies will continue to steadily increase for the foreseeable future. Such increasing expectations may imply the need for more sophisticated optimization methods capable of handling larger and more complex projects in both national and international arenas.

The efficient and effective movement of natural gas from producing regions to consumption regions requires an extensive and elaborate transportation system. Such system consists of a complex network that includes pipelines, compressor stations, regulators, valves, city gates, among other components. In many instances, natural gas produced from a particular well must travel a long distance to reach its point of use, which may imply larger and more challenging pipeline systems. Should the natural gas being transported not be immediately required, it can be put into storage facilities for when it is needed.

We may distinguish three major types of pipelines along the transportation route, namely: the gathering system, the interstate pipeline transmission system, and the distribution system. Unlike the transmission system, which is characterized by long and large diameter pipelines operating at high pressure levels, the gathering and distribution systems consist of low pressure, small diameter pipelines. Should natural gas from a particular well have high sulfur and carbon dioxide contents (sour gas), a specialized sour gas gathering pipe must be installed to transport the raw gas from the wellhead to the processing plant.

The gas transport industry has changed during the last decades, and thus its models and needs. It has grown fast and spawned a gas marketing competition that varies from country to country. For example, in several countries, including USA, Canada, and Brazil, pipeline systems are fully-privatized, i.e., they are private company-owned and thus operated independently. Because of the deregulation started in the 80's, these pipeline companies are no longer the predominant owners of the gas being transported. They merely are in charge of the transportation stage and focus on the efficient operability of the gas system. In these scenarios, models such as the fuel cost minimization problem referred in Section 5 are of extreme importance for gas operators. This, however, is not exactly the case in most of the European countries where it is a common practice that the location, construction and operation of natural gas pipeline systems are regulated by federal and state regulations. For example, in Nordic countries like Norway and Denmark, the compressor stations located along the transmission lines are usually kept on to their maximum capacity for long periods of time, thus the fuel cost minimization models are either completely neglected or become a lesser matter. In such countries, because their natural gas is at large extent exported to neighboring countries with specific gas-quality requirements, the gas blending-type quality models presented in Section 4 are considered pertinent and totally applicable.

Moreover, the dynamic process of the natural gas transportation, which includes successively consistent commitments on a daily basis, encourages gas operators to make use of the gas line-packing models presented in Section 3. These models basically focus on a gas short-term planning storage along the transmission line as a strategy to meet customer demands. All of these needs certainly pose a great challenge to both the gas industry and the scientific community.

The field of Operations Research (OR) has taken a major role in the natural gas industry as a number of important and relevant problems in design, extraction, production, transportation, storage, distributing, and marketing, have been successfully tackled by OR models and techniques over the past 40 years. Zheng et al. [5] provide a recent survey on optimization models in the natural gas industry while focusing on three specific aspects: production, transportation, and market. The authors basically discuss a mathematical formulation of the underlying problem and provide a literature revision of the existing optimization techniques that solve it. Their study covers six general problems, namely the production scheduling problem, the maximal recovery problem, the network design problem, the fuel cost minimization problem, and the regulated and deregulated market problems.

Our goal in this paper is to discuss the most relevant research work that has been done in the natural gas transport industry from the operations research perspective. The paper covers works on three specific optimization areas posed by the gas transport industry when optimizing its transmission systems. This involves problems in short-term basis storage, pipeline resistance and gas quality satisfaction, and fuel cost minimization via pipeline transmission networks.

The paper is organized as follows. We provide next a more in-depth insight into the natural gas industry. Particularly, we emphasize the main components of natural gas transmission pipeline systems and highlight those stages that define the focus of the present work. Some concepts regarding the modeling and optimization of natural gas transportation problems used throughout the text are also discussed. In Section 3 we address the effective application of the optimization theory on the transport and storage of natural gas towards the contractual demands satisfaction. The section particularly discusses the existing, although very limited literature on the efficient transport and short-term basis storage of natural gas along transmission lines, also referred to as the line-packing problem. In Section 4 we deal with the pipeline resistance and gas quality problems in natural gas transportation systems. The research works discussed here are based on pipeline resistance studies, also referred to as the maximum flow capacity in a pipeline, and gas blending-type constraints to meet gas quality requirements, also referred to as the pooling problem. In Section 5 we discuss the problem of how to transport the gas through a pipeline network at minimum cost referred to as pipeline optimization. Final remarks and discussion on major challenges in the field are presented in Section 6.

2. Background: network properties and classification

Natural gas industry is a fast-growing infrastructure. It provides consumers with a virtually almost-free natural gas access in its front end. By simply turning on the main valves (taps) for delivering gas from a pipeline system, end users can make use of a wide list of home or industrial gas appliances. Nevertheless, the long trip that natural gas covers from the wellheads (as a raw material) to get to residential or businesses (as a clean and efficient source of energy, i.e., as we know it), entails a considerable number of complex tasks. Such tasks correspond to different transitional stages of natural gas that can be classified into two primary groups:

(a) Exploration, drilling, extraction, production and long-term storage of natural gas.
(b) Gathering, short-term storage, transportation and distribution of natural gas.

This classification obeys the key instrument used for the service to be achievable, namely *pipeline network systems*. Unlike group (a), long pipelines of various diameters are essentially required for the group (b) dynamics.

Note that long-term planning problems are typically subdivided into two separate stages, namely: long-term planning and expansion planning [6]. The first stage focuses on designing cost-efficient network configurations for several decades while considering product quality and uncertainties, which may require the use of stochastic models via multi-scenario, multi-stage recourse approaches [7]. At this stage, base load storage capacity may be also considered with yearly turn-over rates, i.e., injections during low-peak seasons (summer) and withdrawals during high-peak seasons (winter). The second stage focuses on finding cost-efficient strategies and identifying network sections for improvement while considering an existing network configuration and given load profiles. This last stage may take into account the outcome of the long-term planning.

Moreover, the 20th century witnessed the outbreak of business giants that captured most of the transitional stages involved with natural gas from the wellheads to end users. Yet the changing nature of the natural gas infrastructure called for new upgrades. Nowadays, small and medium size companies are transforming the natural gas industry worldwide. Unlike the former corporations, these companies derive profit by focusing their efforts on one or two specific transitional stages of natural gas. As a result, the monopolistic control of the natural gas industry observed back in the last century has been declining to some extent.

In this study, we focus primarily on works related to gas transport industry problems. Particularly, we discuss the most relevant works in the field of optimization that deal with short-term storage, quality, and compressor fuel cost of natural gas pipeline network systems. Hence, we provide next some insights into the gas transportation via pipelines.

2.1. Natural gas transportation via pipelines

The natural gas transportation is a crucial activity performed by the gas industry in which the gas has to be moved from one location to another. Several types of transportation means might be applied to transport the gas, yet it is well known that pipelines represent the most economical means to transport large quantities of natural gas. In addition, the advent of metallurgical improvements and welding techniques, coupled with the exponential increase of pipeline networks during the last decades all over the world, have made the gas transportation via pipelines more economically attractive.

Currently, pipelines are used both offshore and onshore, with a remarkable difference in terms of security and construction prices. Building pipeline systems under the sea is highly costly and technically demanding, a lot more than onshore. For example, according to *Gazprom*,[1] *the Nord Stream*[2] *(41 in) pipeline project* is expected to cost around € 14.8 billion [8], of which 40.5% [9] corresponds to the 965.7 km long onshore pipeline system on Russian and German territories, whereas the remaining 59.5% is destined to the 259.4 km long offshore section of the project. Hence, when financial, political or environmental issues arise, gas transportation operators look for different alternatives to perform this task. This includes tanker ships and flatboats, by which natural gas can be transported as LNG (liquefied natural gas), MLG (medium conditioned liquefied gas), or CNG (compressed natural gas). More detailed information on commercially applicable methods for natural gas storage and transport can be found in [10–13].

Note that the size of a gas network system may greatly vary from one country to another.

In the US, for example, a large gas network system may encompass several hundreds of pipelines (adding up to several hundreds of thousands of miles) and tens of compressors stations strategically distributed along the transmission lines. However, from the market standpoint, these large networks are typically divided into sub-networks and assigned to specific gas operators who work diligently and jointly with each other to meet all the gas contracts. On the contrary, the natural gas transmission network in Belgium is composed of a relatively smaller number of pipelines (20–40) and compressor stations (4–8) when compared to those found in the US and Russia, for example.

While the size of a gas pipeline system definitely plays an important role when solving natural gas network flow problems, it is the network topology that really defines the complexity of the model, e.g., cyclic networks are extremely more difficult to solve than its (gun-barrel and tree-shaped) network counterparts. The current state of the art on natural gas transmission network problems in steady-state can efficiently handle large gas systems by, e.g., applying network reduction and decomposition techniques, or hybrid-heuristic algorithms (see Sections 4 and 5), most of them, however, with no guarantee of optimality, which enforces the scientific community to enhance the existing methods.

2.2. Technicalities of gas transmission network components

2.2.1. Pipelines

There are essentially three major types of pipelines (usually buried underground) along the transportation lines, ranging in size from 4 inches to 48 inches in diameter (100 to 1220 mm): gathering systems, transmission systems, and distribution systems. Gathering pipeline systems gather raw natural gas from production wells. Transmission pipeline systems transport natural gas thousands of miles across the world to bring natural gas from the pre-processing plants or storage facilities to distribution systems. Distribution pipeline systems can be found in communities and distribute natural gas to homes and businesses.

The main differences among these pipeline systems are their physical properties (e.g., diameter, stiffness and material) and the specifications of their maximum and minimum upstream and downstream pressures. For instance, gathering and transmission lines are constructed from steel pipe, whereas distribution lines can be constructed from steel or modern plastic pipe. The flow lines in the gathering systems are composed of narrow pipelines typically buried 4 ft underground and working at a roughly 250 psi pressure. According to an Environmental Protection Agency (EPA) study [14], flow lines represent one of the largest sources of emissions in the gas industry due to methane leakage [15]. In contrast to transmission systems which locate compressor stations usually working at a pressure of roughly 200 psi to 1400 psi, distribution systems typically operate below their capacity and work at a pressure of approximately 0.5 psi up to 200 psi for safety reasons.

2.2.2. Compressor stations

Compressor stations, typically composed of several compressor units connected in series or in parallel, play a crucial role in the natural gas industry. A compressor unit is a device used to increase the pressure of natural gas by reducing its volume, thus providing the required propel force or boost to keep it moving along the line.

As important assets to the gas transport industry, compressor stations are strategically installed along the gas transmission lines to provide enough energy to natural gas for its transmission. More precisely, a compressor station is a large mechanical facility that receives the gas at pressures ranging from 200 psi to 600 psi, and compresses it back up to 1000 psi to 1400 psi. (As a reference, the typical vehicle tires work with compressed air at roughly 30

[1] *OAO Gazprom* (Open Joint Stock Company) is the Russian state-owned energy monopoly established as the largest extractor of natural gas in the world.

[2] *Nord Stream is a two-line gas submarine pipeline to link Russia and the European Union via the Baltic Sea.*

to 50 lb of pressure per square inch.) As a result, natural gas overcomes frictional losses and maintains required pressures to keep moving through the transportation line towards another compressor station or end users.

We can find several types of gas compressors units in the gas industry. Among the most frequently found are those compressors characterized by a centrifugal dynamic movement or by means of reciprocating positive displacements. The latter is a compressor in which the compressing element is a piston having a reciprocating motion in a cylinder. The decision regarding the selection of centrifugal or reciprocating compressor units requires a thorough analysis of operating conditions, hydraulic pipeline studies, emission requirements, and general lifecycle cost estimates [16,17]. In addition, the pressure limits and flow characteristics of the pipeline system also influence this selection [18,19]. Readers interested in the design and arrangement of compressor stations in natural gas transmission systems are referred to the works of Akhtar [18], Kurz [20], Mokhatab et al. [17], and Santos [21].

2.2.3. Valves and regulators

Valves and regulators are typical components installed for operational and safety reasons within a pipeline system. For example, by means of a valve, gas operators can restrict or direct the gas flow from one point to another. This is rather beneficial, among other circumstances, in order to perform scheduled maintenance or to satisfy demand requirements at specific points or to prevent loss of fluid by completely shutting down the gas flow through a specific pipeline section due to malfunctions. Valves are constructed of steel due to regulations and specifications imposed by the American Petroleum Institute (API), the American National Standards Institute (ANSI), and the American Gas Association (AGA) [22]. In real-practice, regulatory requirements [22] force the transporter to install mainline block valves at certain fixed spacing along the transmission line.

For several decades the scientific community in the field of gas pipeline optimization has been significantly challenged with the large number of complex issues arisen from this particular real-life decision making problem. Such aspects are thoroughly discussed in the following sections.

3. Short-term basis storage

Reasons for success in different arenas of the natural gas industry are due to both the efficient management of resources and equipment, as well as the effective implementation of the appropriate analytical strategies. The natural gas transport and storage industry is no exception. Because of the substantial increase in both natural gas demand and its reserves in recent decades, coupled with the expected promising growth in its production and distribution in the years ahead, the gas industry has become more aware of the need for a sustainable infrastructure that may lead to increases in revenue.

It is well known that natural gas can be stored and transported in its different states of matter [10]. However, the natural gas transportation means other than pipelines, e.g., truck, train, or ship, are usually not economically feasible [23]. For example, natural gas in its liquid form at approximately −163 °C (−260 °F), also known as liquefied natural gas or LNG [11,13], can be transported in cryogenic containers while its volume is increased about 1/600th the volume in its gaseous state. The liquefaction and re-gasification processes, as well as the specially designed cryogenic vessels (LNG carriers) or cryogenic tankers have shown to be very costly. However, LNG can become economically appealing for the gas transport industry when the distance over which natural gas is transported is significantly long. According to [13], shipping natural gas in its liquid form is more beneficial than transporting it in its gaseous state via onshore and offshore pipelines when the distance exceeds 700 miles and 2200 miles, respectively.

Natural gas in its gaseous form is still considered the most economical way to be stored and transported. In this section, we focus on those research works that address the effective application of the transport and storage of natural gas that leads to contractual demands satisfaction over a given planning horizon. More precisely, we discuss the existing, although very limited literature on the efficient transport and short-term basis storage of natural gas along transmission lines, also known as the line-packing problem. Readers interested in empirical models, theoretical foundations and applications of long-term basis storage of natural gas are referred to the works of Zwitserloot and Radloff [24], Neumann and Zachmann [25], Holland [26,27], and the references therein.

The line-packing problem in natural gas transmission pipeline systems basically entails the optimization of gas refill in pipelines in periods of low demand or sufficient capacity, and the gas withdrawals in periods of shortfall. This is done by, e.g., closing (or throttling) a downstream valve while upstream compressors continue sending gas into the pipeline, i.e., packing more gas in the pipelines by increasing the pressure. A more complete description of the line-packing problem is provided next.

3.1. The line-packing problem

Gas pipelines have proven to be the most suitable transportation means for the gas industry since the advent of metallurgical improvements and welding techniques after World War II. Since then, dependable and economic pipeline systems have become essential in preserving the continuous business growth of the gas transport industry in national and international arenas. Nevertheless, a common denominator in the transportation process is that a number of unpredictable or scheduled events do occur on a daily basis. Among these events we can find, e.g., the break down of flow capacities elsewhere in the system due to malfunctions, routine maintenance or inspection; failures in upstream process capacity; shortfall in downstream capacity; demand uncertainty; and high fluctuation in demand due to seasons (in the winter the demand is usually higher than in the summer). However, gas producers must be able to supply gas to their customers despite such difficulties.

As a strategy to some extent alleviate the consequences of those events, natural gas operators must take into account one key fact: Gas pipelines do not only serve as transportation links between producer and consumer, but they also represent potential storage units for safety stocks. That is, due to the compressible nature of dry gas, large reserves can be stored on a short-term basis inside the pipeline through a process called line packing. This is accomplished by injecting more gas into the pipelines during off peak times by increasing the gas pressure, and by withdrawing larger amounts of gas during periods of high demand when flow capacities elsewhere in the system break down. Hence, the problem of keeping a sufficient level of line-pack during a given planning horizon becomes critical to the gas transporter.

To conceptualize this problem, let us see the simplest example. Let us suppose that there is a unique transmission line between one producer and one costumer, and let us assume that the amount of gas required by the client during several consecutive periods can easily be satisfied with only 70% of the maximum capacity. An obvious solution is simply to send the required amount for the mentioned periods. However, let us assume that the demand increases up to 130% of maximum capacity for some subsequent period. Here, the producer cannot meet such requirement, thus leading to considerable economic losses. Hence, the strategic idea

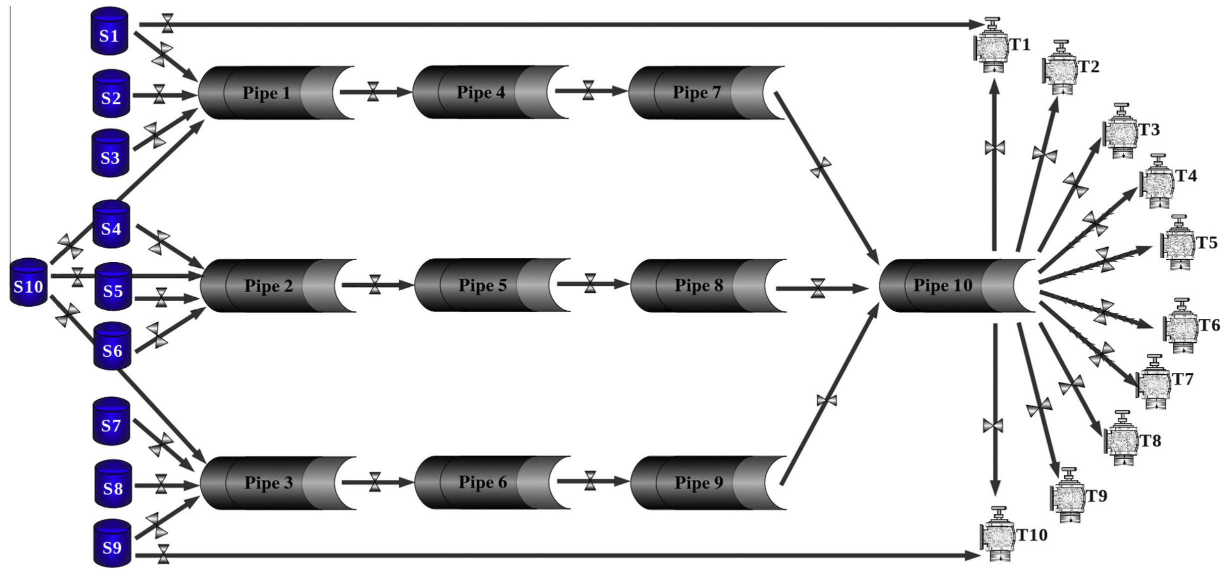

Fig. 5. Potential network elements for short-term storage.

would be to send for instance 100% of the maximum capacity, then consuming just the required demand in each period, and storing the remaining gas to satisfy future extraordinary requirements.

Fig. 5 shows a typical network instance that is composed of 10 source nodes, 10 sink nodes and 10 pipeline segments. As we can observe from the figure, a highly potential storage unit can be inferred by each pipeline in order to meet multiple gas demands during a given planning horizon.

3.2. Literature survey

Related works facing problems of seasonal peak demands can be traced back since 1970. However, the literature reveals that these works were more focused on solving this problem from a managerial perspective, rather than from an operative perspective. Welch et al. [28], for example, proposed to deal with this problem by using other fuels and optimizing a number of scheduled interruptions whenever the gas flow broke down. In addition, they showed that the availability of large industrial contracts was an important factor in containing the peak demand. More recently, Contesse et al. [23] conduct a study on the natural gas supply chain, in which they infer that the changes in the gas industry regulatory system have lead to several alternatives for absorbing demand fluctuations based on contractual strategies of, for example, the use of storage facilities. They mainly refer to two types of contracts: (a) a sale customer contract on a supply interruptible basis in which customers have their gas supply shortened during periods of peak demands in exchange for a lower price; and (b) the firm transportation contract, which allows shippers to reserve a portion of the pipeline's total delivery capacity for their own use.

From the mathematical programming perspective, some attempts, although few, have been made in the direction of mathematical planning models for the line-packing problem [29–33].

For instance, de Nevers and Day [29] examine the natural gas pipeline inventory from a mathematical perspective to match time-varying demands with supplies in an unsteady-state pipeline network system. Their study is based on two dimensionless parameters for the packing and drafting behavior. As a result, their model is capable of showing the limits of the line-packing and line-drafting for a single pipeline segment.

Carter and Rachford [30] discuss several control strategies to operate pipeline network systems through periods of fluctuating loads. Their study aims at finding an optimal schedule for the line-pack under uncertain demand assumptions. As a result, they provide a number of possible scenarios with specific schedules for modifying the set-point values of compressor stations.

Krishnaswami et al. [31] present a simulation approach for optimizing pressure units of compressor stations to meet a specific line-packing along transient, non-isothermal pipeline network systems. They first formulate an implicit finite difference model to provide a flow capacity analysis, and then propose a nonlinear programming model to minimize the average fuel consumption rate of each compressor station over a given planning horizon. The model is solved by applying a sequential unconstrained minimization technique based on a directed grid search method that solves the unconstrained subproblems. Due to the complexity of problem, their study is, however, limited to a linear (gun-barrel) pipeline network system with two compressor stations composed of three compressor units each.

Frimannslund and Haugland [32] follow the ideas presented in the work of Carter and Rachford [30], and propose a mathematical formulation to cope with line-packing levels for a pipeline network system in steady-state conditions. Their study is based on homogeneous gas batches, a concept introduced in [30]. The concept refers to the creation of a number of batches (gas packages) inside the pipelines for their future scheduled withdrawal. The "homogeneous" term in turn establishes that all gas batches are made of the same gas composition no matter when they are constructed, thus implying the assumption that all gas sources in the network provide gas of the same quality. Due to this assumption, no quality constraints on the transported and delivered gas was required. According to [32], a blending process between the batches inside the pipeline seems to be unrealistic unless a long lasting shortfall in downstream capacity takes place.

Borraz-Sánchez [33], motivated by the work of Frimannslund and Haugland [32], proposes and implements a mixed-integer nonlinear programming (MINLP) model and a global optimizer-based mathematical programming algorithm for solving large-scale natural gas transmission networks problems under steady-state assumptions. Unlike Frimannslund and Haugland's work, the key idea behind Borraz-Sánchez's MINLP model is to build up 'heterogeneous' batches (i.e., gas packages of possibly different composition) for a multiple-time period planning horizon. This strategy basically allows the model to account for gas sources that may provide gas of different quality, thus resulting in a more sophisticated model. An essential assumption of Borraz-Sánchez's work is to consider that no blending process among the batches takes place inside the pipelines [32], which is a rather common practice in the gas industry. Moreover, a fundamental part of Borraz-Sánchez's model is also its capability to keep track of energy content and gas quality to ensure that contract terms are met. The model assumes a specific gas quality at the sources (which may be determined by producers), and satisfies the gas quality imposed at the terminals. Here, several gas streams of different composition may be blend at junction points of the network in order to meet the quality requirements. The inherent problem in satisfying natural gas quality requirements, which directly introduces an NP-hard problem known as the pooling problem [34,35], is addressed in Section 4.

More recently, Zavala [36] presents a stochastic model to solve the line-packing problem. The model also captures the network dynamics by discretizing the governing partial differential equations in time and space. Zavala considers a gas network with links comprising long pipelines and nodes consisting of junction points and compressors. The proposed model is a representation of a stochastic optimal control model that considers conservation and momentum equations, typical operational constrains, and uncertainty in demands. The author performs a degrees-of-freedom (DOF) analysis to verify the consistency of the model and uses the underlying results to derive consistent initial conditions and non-anticipativity constraints. In addition, the author also incorporates a risk metric into the objective function to mitigate cost variance and system volatility. The computational study demonstrates the benefits obtained with the stochastic formulation against the deterministic and robust counterparts.

4. Pipeline resistance and gas quality satisfaction

We start by distinguishing two disjoint research groups encompassing network flow problems. One group, which may be recalled as the classical group, defines constant arc capacities for transporting solid goods, whereas the second group, committed to optimizing fluid flows, defines non-constant arc capacities. In this section, we focus on those works contributed by the second group; particularly, optimization works that handle natural gas transmission pipeline systems under steady-state conditions.

Two major characteristics of steady-state network flow models are the strong dependence between the pipeline flow and the pressure drop along the transmission line, and the inclusion of pressure values as a state variable at interconnection points. Furthermore, the pressure values in the pipeline system are determined by the flow and pressure values of upstream network elements of the evaluated component. Consequently, more refined modeling techniques are required to compute the resistance of the pipelines.

4.1. Gas flow equation

The fundamental flow equation is based on derived solutions from partial differential equations. It has been universally accepted as the full statement to describe fluid flows under various boundary conditions. It is a mathematical derivation that describes the flow of fluids based on a physical principle and models of physical behavior such as the law of conservation of mass, Darcy's law, and equations of state [22].

Menon [22] establishes that the pipeline resistance, also referred to as the maximum flow capacity in a pipeline, is strongly dependent on the physical properties of pipelines and the composition of the gas. Several equations have been proposed during the last century to simulate compressible gas flows in long pipelines, including the Weymouth equation (developed in 1912), the Panhandle A equation (developed in 1940), and the Panhandle B equation (developed in 1956). These equations are developed from the fundamental energy equation for compressible flows, but each has a special representation of the friction factor to allow the equations to be solved analytically. In addition, they differ from each other by the method used to create them and the number of parameters used to define them. For low pressures and short pipeline, they may not be applicable. The works of Osiadacz [37], Crane [38], and Modisette [39] provide complete details of these equations.

Due to its simplicity and its accuracy when applied to gas flows at high pressures, coupled with the fact that it has been around the longest, the Weymouth equation is, however, the most-widely used to model flow capacities. The equation, which basically defines the relationship between the flow and the pressure drop through a horizontal pipeline segment, is given by:

$$x_{uv}^2 = W_{uv}(p_u^2 - p_v^2), \tag{1}$$

where x_{uv} is the mass flow rate through the horizontal pipeline segment $(u, v), p_u$ and p_v are the upstream and downstream pressure, respectively, and W_{uv}, referred to as the Weymouth factor, is a parameter that depends on gas and pipeline properties as given by

$$W_{uv} = \frac{d_{uv}^5}{K z_{uv} g_u T f_{uv} L_{uv}},$$

where z_{uv} is the compressibility of the flow in pipeline $(u, v), g_u$ is the specific gravity of the flow arriving at node u, T is the gas temperature, f_{uv} is the (Darcy-Weisbach) friction factor in pipeline $(u, v), L_{uv}$ is the length of pipeline $(u, v), d_{uv}$ the inside diameter of pipeline (u, v), and K is a global constant with value defined by the units used.

Note that (1) becomes more accurate when the variability of the gas compressibility and specific gravity is introduced into the calculation, which results in an alteration of the standard measure. This is briefly discussed next.

4.2. Gas compressibility and specific gravity estimates

The gas compressibility (z) factor can be considered as the deviation from ideal gas observed by the *ideal gas Law* Eq. (2). More formally, it is defined as the relative change in gas volume in response to a change in pressure and temperature. The importance of accurate estimates of this parameter is obvious from (1) and the definition of W_{uv}.

$$z = \frac{PV}{NkT}, \tag{2}$$

where z is the gas compressibility factor, P is the absolute pressure, V is the volume, N is the number of molecules, k is the *Boltzmann constant* (1.38066×10^{-23} J/K), and T is the absolute temperature.

The literature on gas metering reveals a number of diverse methods for approximating the z-factor, including experimental measurements, equations of state methods [40], empirical correlations [41], and regression analysis methods [42,43].

For instance, in Chapter 4 of Katz et al. [41] a graphical correlation for the z-factor as a function of pseudo-reduced temperature and pressure based on experimental data is presented. As a result, the Standing-Katz z-factor chart has been used to obtain natural gas compressibility factors for more than 40 years. Dranchuk and Abou-Kassem [40] used the equation of the state to fit the Standing-Katz data and extrapolated to higher reduced pressure. This was accomplished by a simple mathematical description of the Standing-Katz z-factor chart.

In addition, several equations have been introduced to compute the z-factor, including the CNGA method [44,45] developed by the

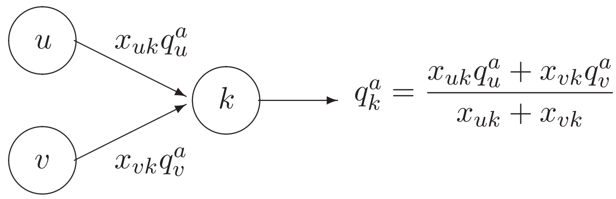

Fig. 6. Flow balance imposed by quality constraints.

California Natural Gas Association (CNGA), the AGA-NX19 method [46] developed by the American Gas Association (AGA), and the Dranchuk, Purvis, and Robinson (DPR) method [42] that uses the Benedict–Webb–Rubin equation of state [47] to correlate the Standing-Katz chart in order to approximate z as a function of (g, T, P). Note that all these methods have a domain where they are reasonably accurate, and may break down outside. A complete survey of these methods can be found in, e.g., [22].

Eq. (1) may also be highly influenced by the variation in specific gravity (g) values (dimensionless), i.e., the ratio between the density (mass per unit volume) of the actual gas and the density of air at the same temperature. Menon [22] provides a complete list of specific gravity values (ranging from 0.554 to 0.870) and other properties of various hydrocarbon gases.

Gas specific gravity values are typically estimated at interconnection points within a gas transmission network. The computations are based on a principle established for the pooling problem (i.e., quality constraints), which is discussed next.

4.3. The pooling problem

In the oil and natural gas industry, the pooling problem refers to the scenario in which a number of different sources in a network system provide gas or oil with different quality attributes (i.e., the product being transported is made of different composition) and which flow streams must be blended in a series of pools in order to meet given customer requirements.

As specified by Adhya et al. [48] and Foulds et al. [49], the mathematical model includes bilinear and nonconvex quadratic programming constraints, also referred to as quality constraints, that make it hard to solve. The nonlinearity appears in two types of constraints, namely the quality balance at pools or network elements, and quality bound at the terminals.

Theorem 1. *The pooling problem is NP-hard even in the case of single-layer of pools.*

Proof. A poly reduction from the 3-dimensional matching problem to the single-layered pooling problem. Interested readers in the complete proof are referred to [50].

This problem can also be seen as an extension of the minimum cost flow problem on networks describing three sets of nodes: sources, pools and terminals, where the model imposes the quality constraints on the flow. These constraints can be stated as follows.

Let u and v be two network elements such that there is a link from u to v, and let x_{uv} be the flow variable through the link. Let q_u^a denote the quality a of the flow stream leaving element u, i.e., the relative content of some gas component like e.g. CO_2. For the flow stream q_v^a entering element v, the corresponding quality constraint is given by

$$q_v^a = \frac{\sum_{u \in N_v} x_{uv} q_u^a}{\sum_{u \in N_v} x_{uv}}, \qquad (3)$$

where N_v is the set of upstream elements that connect v with a direct link. □

The main challenge of the quality constraints is that their terms are products of two unknown variables and cannot be linearized. Fig. 6 shows the flow balance imposed by the quality constraint for the case where a network element k has two upstream neighbors, u and v. In the figure, we can observe that the resulting quality of the end products, after they might have been blended each other a certain number of times, depends on what sources they originated from, and in what proportion. Typically, an expected range in the quality of the product being transported is imposed at terminals.

Haverly [51] presents one of the pioneering works in this field, and since then the scientific community has put special attention into proposing optimization techniques, models and applications for the pooling problem. Aggarwal and Floudas [52] present a Bender's decomposition-based algorithm to search for global solutions to the pooling problem. Floudas and Visweswaran [53], Adhya et al. [48] and Almutairi and Elhedhli [54] pursue the goal of the previous work and apply Lagrangian relaxation techniques.

Three main formulations of the pooling problem, based on nonlinear programming, can be found in the literature [55,56]: (i) the p-formulation [51] which consists of flow and quality variables; (ii) the q-formulation [57] which replaces the quality variables of the p-formulation with variables representing flow proportions; and (iii) the pq-formulation [58] which adds RLT cuts to the q-formulation (RLT – Reformulation-linearization technique suggested by Sherali and Adams [59].) The pq-formulation dominates (i.e., is tighter than) both the q- and p-formulations, and the p-formulation is in turn dominated by the q-formulation. However, the pq-formulation cannot be applied to networks with multi-layered of pools. For this type of networks, dry gas pipeline transportation and multi-period inventory models are applied. Audet et al. [55] propose a branch-and-cut quadratic programming algorithm to solve the pooling problem, and study two mathematical models: a flow variables-based model and a model based on flow proportions entering pools. As a result, they propose a hybrid model based on the two tested models for general pooling problems. More recently, Alfaki and Haugland [50] propose a formulation based on source and terminal proportions (denoted the STP-formulation) that is stronger than the pq-formulation proposed by Tawarmalani and Sahinidis [58], and suggest a branching strategy for solving it. The STP-formulation basically combines the source and terminal flow proportions, and defines specific flow streams on diverse paths of the network.

The literature reveals a good selection of pooling problems published by Haverly [51], Ben-Tal et al. [57], Foulds et al. [49], and Adhya et al. [48].

Moreover, when uncertainty plays a key role in the design and operation of the pooling system, stochastic optimization models are applied. For instance, Li et al. [60,61] propose a duality-based decomposition method to guarantee finding an ϵ-optimal solution for the stochastic pooling problem. The method basically decomposes the stochastic nonconvex mixed-integer nonlinear program into a series of primal bounding subproblems by convexifying and underestimating the bilinear functions. Since the resulting master problem is typically hard to solve, they apply relaxation and dualization techniques to solve a sequence of primal bounding problems, feasibility problems and relaxed master problem. The sequence of subproblems are submitted to version 8.1.5 of the global optimizer BARON. BARON [62] is an implementation of a branch-and-bound algorithm where a convex relaxation of the submitted problem is solved in each node of the search tree.

4.4. Flow accurate estimates: related work

Since the last century, a substantial research work has been done in optimizing flow along arc capacity networks [63]. Among those works, the majority of those ones related to steady-state gas flow problems, have modeled the resistance of a pipeline as a function of state variables ever since the inception of gas pipeline optimization (around the middle of the 20th century). The work of Wong and Larson [64] when minimizing the total fuel cost incurred by compressor stations is a good example. They suggested to apply the well-known Weymouth equation [37] to compute the pipeline capacity. The same principle is followed by more sophisticated works, as those presented by Carter [65], Ríos-Mercado et al. [66], De Wolf and Smeers [67,68], Borraz-Sánchez and Ríos-Mercado [69], Bakhouya and De Wolf [70], Kalvelagen [71], and Borraz-Sánchez and Haugland [72].

Carter [65] propose a non-sequential dynamic technique that outperforms hybrid methods for cyclic networks and allows rapid turnaround of optimization runs for steady-state flow models. Ríos-Mercado et al. [66] propose a reduction technique for natural gas transmission network optimization problems that substantially decreases the size of the network without altering with its mathematical properties. De Wolf and Smeers [68] present a model to solve the problem of distributing gas at a minimum cost through a pipeline network under nonlinear flow-pressure relations constraints, material balance equations, and pressure bounds. The solution method is based on piecewise linear approximations of the nonlinear flow-pressure relations. Borraz-Sánchez and Ríos-Mercado [69], motivated by the work of Carter [65], propose a non-sequential dynamic programming algorithm for optimizing large-size cyclic network systems under steady-state assumptions.

Several works, such as those presented by O'Neill et al. [73], Wilson et al. [74] and De Wolf and Smeers in [67,68], propose various MINLP models to describe the operating settings of the compressor stations. Their models, however, integrate transportation functions with gas sale planning functions. O'Neill et al. [73] and Wilson et al. [74] apply a *Successive Linear Programming*, whereas De Wolf and Smeers [67,68] implement piecewise linear approximations solved by an extension of the Simplex algorithm [75]. Bakhouya and De Wolf [70] separate the integrated model proposed by previous works, and focus only on minimizing the total power consumption at compressor stations. They solve the problem by applying a two-phase method to Belgian and French gas transmission networks. Kalvelagen [71] proposes an improved model for the MINLP gas transportation problem and solves it using GAMS [76]. Borraz-Sánchez and Haugland [72] efficiently tackle the fuel cost minimization problem in steady-state natural gas transmission networks by proposing and implementing a dynamic programming-based tree decomposition algorithm.

All the cited works neglect the fact that the parameter in the Weymouth equation depends not only on pipeline characteristics, but also on thermodynamic and physical gas properties. This includes temperature, specific gravity (relative density) and compressibility (z-factor), which are assumed as universal constants in these works. In instances where the network elements show no or only modest variation in these properties, it may be valid to neglect their variability and to represent them by global constants. This does however not seem to be the case in all real-life instances.

Examples where the assumption is unrealistic exist. The pipeline network connecting wells on the Norwegian continental shelf with the European continent is supplied by gas from sources of relatively lean gas, situated in the North Sea, and sources located in e.g. the Haltenbank area. Since the latter area generally has richer gas, in the sense that it consists of components of higher specific gravity, the assumption of constant properties may be unrealistic. Also, gas compressibility depends on current temperature and pressure conditions, which also vary along the transmission line.

The literature on optimization models for pipeline gas transportation does not seem to be very rich on models with variable specific gravity or compressibility, and most works focus on models for transient flow. Abbaspour and Chapman [77], for example, analyze non-isothermal transient flow of gas in natural gas pipeline. Their work is based on z-factor estimates as a function of pressure and temperature. They assume a steady-state heat flow between the gas in the pipeline and the surroundings. Chaczykowski [78] studies one-dimensional, non-isothermal gas flow model to simulate slow and fast fluid transients. Their work is based on unsteady heat transfer term in the energy equation. Simulations of two gas transmission pipeline networks were conducted to show that the unsteady heat transfer model hinders the gas temperature changes while considering the heat in the surroundings.

In steady-state flow models, Belyaev et al. [79] provide arguments on potential error causes in gas metering by studying a real-test case from the Russian Federation. Belyaev and Patrikeev [80] present a study on the influence of variations of the gas composition by using correction factors that depend on the density under standard conditions. Their work is based on readings of all the instruments involved in commercial operations during a certain period. Bahadori et al. [81] propose and develop a new method to account for difference in z-factor estimates between natural gases containing sour components and those ones characterized as sweet gases. Their work is based on two correlations for computing pseudo-critical pressure and temperature values as a function of the gas specific gravity. As a result, a simplified calculation method is introduced for quick estimations of z-factor values for sour natural gases.

Borraz-Sánchez and Haugland [44] study the effect caused by the variability of the specific gravity and compressibility of the gas on flow estimates in transmission pipeline systems. They extended previously suggested models by incorporating the variation in pipeline flow capacities with gas specific gravity and compressibility. Their work also applies the principle stated by the Weymouth equation to compute the resistance of the pipeline, and makes use of the California Natural Gas Association method [45], which depends on gas specific gravity and pressure values, to compute gas compressibility values in each pipeline of the network system. The variability of specific gravity is then estimated at junction points as the weighted average of specific gravities of entering flows. Due to the resulting nonconvex model, they propose a heuristic that iteratively solves a simpler model by means of a global optimizer. Their solution approach turns out to be very promising while providing exact solutions to many test instances and finding deviations less than 12% from optimality in the remaining cases.

5. Compressor station modeling

5.1. Introduction to the fuel cost minimization problem in natural gas pipeline systems

As natural gas pipeline systems have grown larger and more complex, the importance of optimal operation and planning of these facilities has increased. The investment costs and operation expenses of pipeline networks are so large that even small improvements in system utilization can involve substantial amounts of money.

The natural gas industry services include producing, moving, and selling gas. The main focus in this section is on the transportation of gas through a pipeline network. Moving gas is divided into two classes: transmission and distribution. Transmission of gas means moving a large volume of gas at high pressures over long distances from a gas source to distribution centers. In contrast, gas distribution is the process of routing gas to individual customers. For both transmission and distribution networks, the gas flows through various devices including pipes, regulators, valves, and compressors. In a transmission network, gas pressure is reduced due to friction with the pipe wall as the gas travels through the pipe. Some of this pressure is added back at compressor stations, which raises the pressure of the gas passing through them.

In a gas transmission network, the overall operating cost of the system is highly dependent upon the operating cost of the compressor stations in a network. A compressor station's operating cost, however, is generally measured by the fuel consumed at the compressor station. According to Luongo et al. [82], the operating cost of running the compressor stations represents between 25% and 50% of the total company's operating budget. Hence, the objective for a transmission network is to minimize the total fuel consumption of the compressor stations while satisfying specified delivery flow rates and minimum pressure requirements at the delivery terminals.

Depending on how the gas flow changes with respect to time, we distinguish between systems in steady state and transient state. A system is said to be in steady state when the values characterizing the flow of gas in the system are independent of time. In this case, the system constraints, particularly the ones describing the gas flow through the pipes, can be described using algebraic nonlinear equations. In contrast, transient analysis requires the use of partial differential equations (PDEs) to describe such relationships. This makes the problem considerably harder to solve from the optimization perspective. In fact, optimization of transient models is one of the most challenging ongoing research areas. In the case of transient optimization, variables of the system, such as pressures and flows, are functions of time.

The issue of how to design a pipeline network involves decisions on diameter and length of pipes, location of compressor stations, and network configuration. For works on optimal design of pipeline systems the reader is referred to the work of Babonneau et al. [83], Costa et al. [84], El-Shiekh [85], Marcoulaki et al. [86], Mariani et al. [87], Osiadacz and Górecki [88], Sanaye and Mahmoudimehr [89], Tsal et al. [90], and Zhou et al. [91]. Some authors have gone a step further by addressing the design and operation issues simultaneously. For instance, Üster and Dilaveroğlu [92] present a framework for designing, expanding an existing network while minimizing total investment and operating costs.

In this section, we survey the most significant work on both steady-state and transient gas transmission network problems, assuming an existing pipeline system, with the objective of minimizing the operational costs.

Gas transmission network problems differ from traditional network flow problems in some fundamental aspects. First, in addition to the flow variables for each arc, which in this case represent mass flow rates, a pressure variable is defined at every node. Second, besides the mass balance constraints, there exist two other types of constraints: (i) a nonlinear equality constraint on each pipe, which represents the relationship between the pressure drop and the flow; and (ii) a nonlinear nonconvex set which represents the feasible operating limits for pressure and flow within each compressor station. The objective function is given by a nonlinear function of flow rates and pressures. The problem is very difficult due to the presence of a nonconvex objective function and nonconvex feasible region.

5.2. Description of basic model

Let $G = (V, A)$ be a directed graph representing a natural gas transmission network, where V is the set of nodes representing interconnection points, and A is the set of arcs representing either pipelines or compressor stations. Let V_s and V_d be the set of supply and demand nodes, respectively. Let $A = A_p \cup A_c$ be partitioned into a set of pipeline arcs A_p and a set of compressor station arcs A_c. That is, $(u, v) \in A_c$ if and only if u and v are the input and output nodes of compressor station (u, v), respectively.

Two types of decision variables are defined: Let x_{uv} denote the mass flow rate at arc $(u, v) \in A$, and let p_u denote the gas pressure at node $u \in V$. The following parameters are assumed known: B_u is the net mass flow rate in node u, and P_u^L and P_u^U are the pressure limits (lower and upper) at node u. By convention, $B_u > 0$ ($B_u < 0$) if $u \in V_s$ ($u \in V_d$), and $B_u = 0$ otherwise.

The basic mathematical model of the minimum fuel cost problem (MFCP) is given by:

$$\min\ g(x,p) = \sum_{(u,v)\in A_c} g_{uv}(x_{uv}, p_u, p_v) \tag{4}$$

$$\text{subject to} \quad \sum_{v:(u,v)\in A} x_{uv} - \sum_{v:(v,u)\in A} x_{vu} = B_u \quad u \in V \tag{5}$$

$$(x_{uv}, p_u, p_v) \in D_{uv} \quad (u, v) \in A_c \tag{6}$$

$$x_{uv}^2 = R_{uv}(p_u^2 - p_v^2) \quad (u, v) \in A_p \tag{7}$$

$$p_u \in \left[P_u^L, P_u^U\right] \quad u \in V \tag{8}$$

$$x_{uv} \geqslant 0 \quad (u, v) \in A \tag{9}$$

The objective function (4) measures the total amount of fuel consumed in the system, where $g_{uv}(x_{uv}, p_u, p_v)$ denotes the fuel consumption cost at compressor station $(u, v) \in A_c$. For a single compressor unit the following function is typically used:

$$g^{(1)}(x_{uv}, p_u, p_v) = \frac{\alpha x_{uv}}{\eta}\left\{\left(\frac{p_v}{p_u}\right)^m - 1\right\},$$

where α and m are assumed constant and known parameters that depend on the gas physical properties, and η is the adiabatic efficiency coefficient. This adiabatic coefficient is a function of (x_{uv}, p_u, p_v) that is, in general, a complex expression, implicitly defined. A function evaluation of η requires solving a linear system of algebraic equations. In practice, though, polynomial approximation functions that fit the function relatively well and are simpler to evaluate are employed. In other cases, when the fluctuations of η are small enough, η can be assumed to be a constant.

For a compressor station (u, v) with n_{uv} identical compressor units hooked-up in parallel which is very commonly found in industry, the fuel consumption is given by:

$$g_{uv}(x_{uv}, p_u, p_v) = n_{uv} g^{(1)}(x_{uv}/n_{uv}, p_u, p_v).$$

When all n_{uv} units are fixed and operating we have a nonlinear programming (NLP) model. Treating n_{uv} as decision variables, leads to mixed-integer nonlinear programming (MINLP) models.

Constraints (5) establish the mass balance at each node. Constraints (6) denote the compressor operating limits, where D_{uv} denote the feasible operating domain for compressor $(u, v) \in A_c$. Eq. (7) states the relationship between the mass flow rate through a pipe and its pressure values at the end points under isothermal and steady-state assumptions, where R_{uv} (also known as the pipeline resistance parameter) is a parameter that depends on both the physical characteristics of the pipeline and gas physical properties. When the steady-state assumption does not hold, this relationship is a time-dependent partial differential equation which leads to transient models. Constraints (8) set the lower and upper limits of the pressure value at every node, and (9) set the non-negativity condition of the mass flow rate variables. Further details of this model can be found in Wu et al. [93].

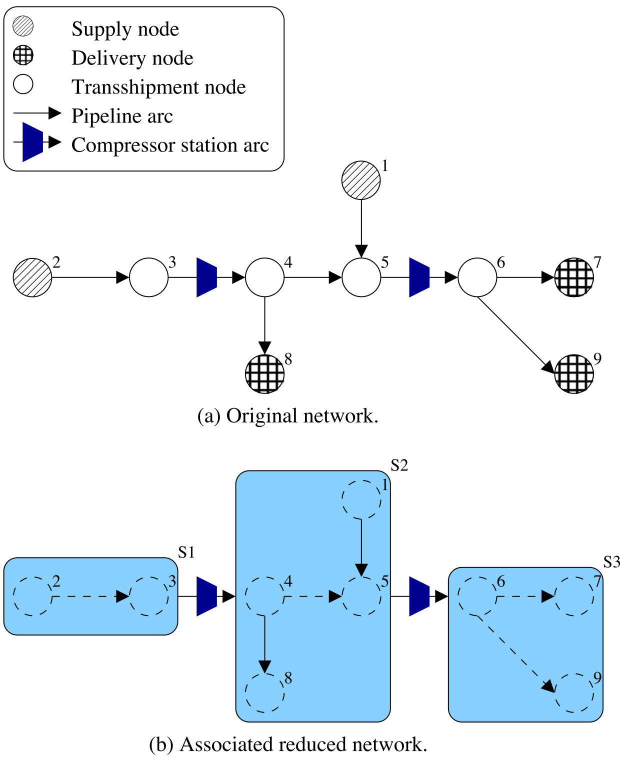

Fig. 7. Illustration of a reduced network.

5.3. Network topology

There are three different kinds of network topologies: (a) linear or gun-barrel, (b) tree or branched, and (c) cyclic. Technically, the procedure for making this classification is as follows. In a given network, the compressor arcs are temporarily removed. Then each of the remaining connected components are merged into a big super-node. Finally, the compressor arcs are put back into their place. This new network is called the *associated reduced network*. Fig. 7 illustrates the associated reduced network for a 9-node, 8-arc example. As can be seen, the reduced network has 3 supernodes (labeled S1, S2, S3) and 2 arcs (the compressor station arcs from the original network).

Types of network topologies:

Linear topology: This corresponds to a linear arrangement of the compressor station arcs, that is, when the reduced network is a single path.

Tree topology: This occurs when the compressors are arranged in branches through the system, that is, when the reduced network is a tree.

Cyclic topology: This happens when compressors are arranged forming cycles with other compressor stations. That is, it refers to a cyclic reduced network.

These different types of network topologies are shown in Fig. 8, were the original network is represented by solid line nodes and arcs, and the reduced network by dotted super nodes. Note that even though networks in Fig. 8(a) and (b) are not acyclic from a strict network definition, they are considered as non-cyclic pipeline network structures.

As it will be seen in the following section, the state of the art on steady-state systems establishes that linear and tree topologies are more tractable despite the nonconvexity of the problem. Since it has been shown that in this type of topologies, under certain conditions the flow variables can be uniquely determined [66], techniques such as dynamic programming have been successfully applied to solve for the discrete set of pressure variables. Cyclic structures are harder to solve, and, regardless of network topology, transient systems are even more challenging.

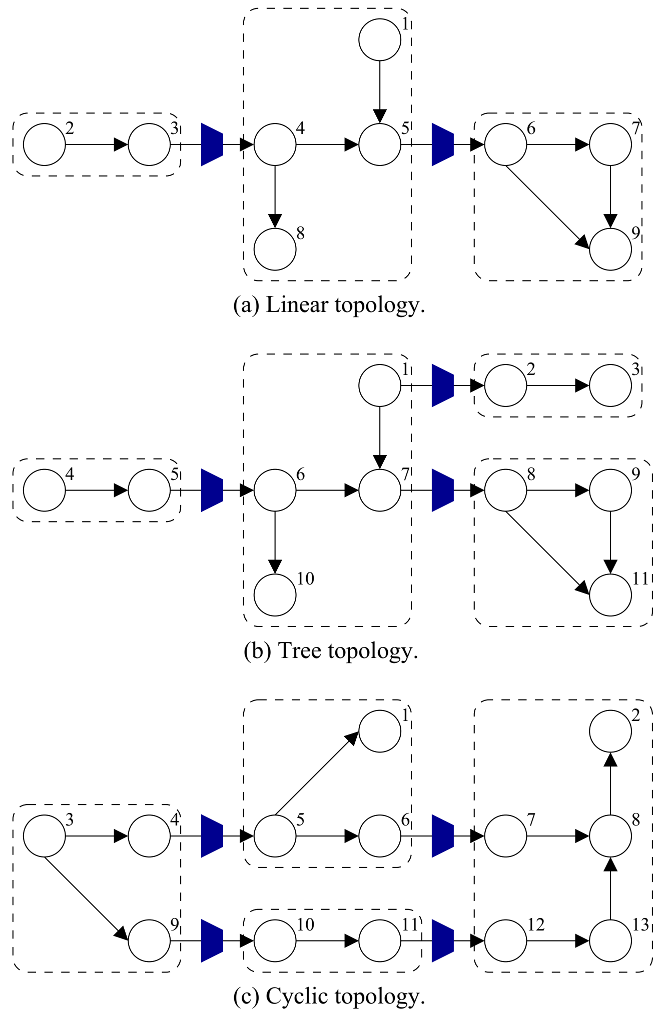

Fig. 8. Different kinds of pipeline network topologies.

5.4. Steady-state models

This section focuses on reviewing optimization models and approaches for steady-state problems. Descriptive simulation models are out of the scope of this work. Nonetheless, for research on simulation of natural gas networks or descriptive simulation models the reader is referred to the seminal work of Osiadacz [37], or the recent work of Zhu et al. [94], Herrán-González et al. [95], and Woldeyohannes and Majid [96].

Optimization of a compressor station has been studied previously by Mahmoudimehr and Sanaye [97] Osiadacz [98], Percell and Van Reet [99], and Wu et al. [100]. These works focused on a mathematical model in a single compressor unit. Later, Wu et al. [93] completed the analysis for the same problem, but considering several units within compressor stations. Krishnaswami et al. [31] present a systematic approach for operating the units of a compressor station to meet a specified line-pack profile. Nguyen et al. [101] present a comparison of three automation approaches for compressor selection. All these works study the behavior of a compressor station. In this survey we focus on the entire pipeline network optimization.

5.4.1. Model properties and lower bounds

Ríos-Mercado et al. [66] present a study of the properties of gas pipeline networks on steady-state, and exploit them to develop a technique that can be used to significantly reduce problem dimension without disrupting problem structure. This technique has been successfully used in many works on pipeline optimization.

Wu et al. [93] present an in-depth study of the underlying mathematical structure of the compressor stations of the MFCP. Then, based on this study, they propose two model relaxations (one in the compressor domain and another in the fuel cost function) and derive a lower bounding scheme. They present empirical evidence that shows the effectiveness of the lower bounding scheme. For the small problems, where optimal solutions were known, the proposed lower bound yield a relative optimality gap of around 15–20%. For a larger, more complex instance, it was not possible to find optimal solutions, but they were able to compute lower and upper bounds, finding a large relative gap between the two. They show this wide gap is mainly due to the presence of nonconvexity in the set of feasible solutions, since the proposed relaxations did a very good job of approximating the problem within each individual compressor station. Later, Borraz-Sánchez and Ríos-Mercado [69] compute a lower bound for cyclic instances obtaining relative optimality gaps of less than 16%, and, in most of the cases, less than 10%.

5.4.2. Methods based on dynamic programming

One of the most successful technique for addressing the MFCP is Dynamic Programming (DP). One of the main advantages of DP is that a global optimum is guaranteed to be found and that nonlinearity can be easily handled. Until very recently, its application had been practically limited to noncyclic networks, such as linear (also known as gun-barrel) or tree topologies. It is well known that in DP computation time increases exponentially with the dimension of the vector of state variables, commonly referred as the curse of dimensionality.

DP for pipeline optimization was originally applied to gun-barrel systems in the late 1960s. It has been one of the most useful techniques due to both its computational behavior and its versatility for handling nonlinearity on sequential systems. DP was first applied to linear systems by Wong and Larson [64] in 1968, and then applied to tree-structured topologies by Wong and Larson [102]. A similar approach was described by Lall and Percell [103] in 1990, who allow one diverging branch in their system.

In 1989, Luongo et al. [82] published a hierarchical approach that allowed for both cycles and branches of arbitrary complexity. This represented significant progress in terms of finally addressing the issue of real world pipeline configurations. Their technique was no longer pure DP. Basically, DP was used to optimally describe the pieces of the pipeline that were arranged in a sequential manner. This typically reduced the system to a much smaller combinatorial problem, without any possibility of a recursive DP solution. A sufficiently small instance could be solved exactly via enumeration; otherwise it was solved inexactly using simulated annealing. This hierarchical approach worked very well for some complex pipelines, but for others the computational cost was very high.

One of the most significant works on cyclic networks known to date is due to Carter [65] who developed a non-sequential DP algorithm, but limited to a fixed set of flows. This led to an interesting question of how to find the optimal setting of the flow variables and how to modify the current flow setting to obtain a better objective value. Extensions to work on cyclic systems addressing these issues were developed by Ríos-Mercado et al. [104].

Ríos-Mercado et al. [104] propose a heuristic solution procedure for fuel cost minimization on gas transmission systems with a cyclic network topology Their heuristic solution methodology is based on a two-stage iterative procedure. In a particular iteration, at a first stage, gas flow variables are fixed and optimal pressure variables are found via dynamic programming. At a second stage, pressure variables are fixed and an attempt is made to find a set of flow variables that improve the objective function by exploiting the underlying network structure. They tested their algorithm in some real-world instances provided by a Houston-based company. The first instance solved was a tree-structured system with 16 compressor stations, 56 pipes, and 64 total nodes (that is including supply, demand, and transshipment nodes). The second instance solved was single-cycle system with 6 compressor stations, 9 pipes, and 14 nodes. The third instance solved was a multi-cycle system with 17 compressor stations, 23 pipes, and 35 nodes. Empirical evidence supports the effectiveness of the proposed procedure by finding relative improvements ranging from 3.34% to 41.77% in the instances tested.

Borraz-Sánchez and Ríos-Mercado [105,69] propose a hybrid metaheuristic procedure that efficiently exploits the problem structure. This hybrid procedure combines very effectively a non-sequential dynamic programming algorithm for finding an optimal set of pressure variables for a fixed set of mass flow rate variables, and short-term memory tabu search procedure for guiding the search in the flow variable space. Empirical evidence over a number of instances supports the effectiveness of the proposed procedure outperforming a multi-start generalized reduced gradient (GRG) method both in terms of solution quality and feasibility. Furthermore, to assess the quality of the solutions obtained by the algorithm, a lower bound is derived. It is found that the solution quality obtained by the proposed lower bounding procedure is relatively good.

Borraz-Sánchez and Haugland [106] present a two-phase method for the MFCP. As suggested by previous work, they consider a procedure where each iteration consists of a flow improvement step and a pressure optimization step. Alternating between flow and pressure, one set of decision variables is kept fixed in each step. Still in agreement with previously suggested methods, the nonconvex subproblem of optimizing pressure is approximated by a combinatorial one. This is accomplished by discretization of the pressure variables. The contribution of their work is a method for solving the discrete version of the problem in instances where previously suggested methods fail. Unlike methods based on successive network reductions, their method does not make any assumptions concerning the sparsity of the network. By constructing a tree decomposition of the network, and applying dynamic programming to it, they were able to solve the discrete version of the pressure optimization problem without enumerating the whole solution space. By an adaptive discretization scheme, they obtain significant speed-up of the dynamic programming approach in comparison with fixed discretization. They tested their proposed solution method on a set of test instances, and compared the results to those obtained by applying both a global and a local optimizer to the continuous version of the problem. The experiments indicate that a method guaranteeing the global optimum in reasonable time seems unrealistic even for small instances. Further, discretizing the pressure variables and applying dynamic programming to a tree decomposition gives better results than applying a commercially available local optimization package.

The details on the DP formulation can be found in the referred works or in the work by Ríos-Mercado [107].

5.4.3. Methods based on gradient search

In 1987, Percell and Ryan [108] applied a different methodology based on a Generalized Reduced Gradient (GRG) nonlinear optimization technique for noncyclic structures. One of the advantages of GRG, when compared with DP, is that they can handle the dimensionality issue relatively well, and thus, can be applied to cyclic structures. Nevertheless, being a method based on a gradient search, there is no guarantee for a global optimal solution. Villalobos-Morales and Ríos-Mercado [109] evaluated preprocessing techniques for GRG, such as scaling, variable bounding, and choice of starting solution, that resulted in better results for both cyclic and noncyclic structures. Flores-Villarreal and Ríos-Mercado [110] performed an extensive computational evaluation of the GRG method over a large set of instances on cyclic structures with relative success.

5.4.4. Geometric programming approaches

Recently, Misra et al. [111] present a new approach based on geometric programming (GP) for the MFCP. They proved that for non-cyclic systems, the GP approach turns the MFCP into a convex optimization problem allowing for exact and efficient (polynomial time) solutions. A significant advantage of the GP method over the traditional dynamic programming approach derives from not having to discretize the node pressure and compression ratio variables. The GP approach also scales well, even in networks with a high degree of branching. They tested their approach on the Belgian natural gas network and the Transco pipeline network in the US, showing that their proposed geometric programming algorithm consistently outperforms DP in non-cyclic systems.

5.4.5. Linearization approaches

De Wolf and Smeers [68] take a different angle to the problem. They present a solution method based on piecewise linear approximations of the nonlinear flow-pressure relations. The approximated problem is solved by an extension of the Simplex method. The solution method is illustrated in an instance of the Belgium gas network, and solved some real-world cases. They compare their approach with other LP-based approach, called Successive Linear Programming (SLP). They found their proposed approach takes less time than SLP. They also found that, as the model is in general nonconvex, the choice of the starting point was crucial if one limits oneself to find only local solutions or upper bounds on the solution in global procedures. Thus, they devised a mechanism for generating the initial solution that was empirically shown to reduce running times by 50%.

Jin and Wojtanowicz [112] present a study aimed at optimizing a very large case study in China. The large size and complex geometry of network required breaking it down into smaller components, optimizing operations of the components locally, re-combining the optimized components into the network and optimizing the network globally. This four-step approach employed four different optimization methods to solve the problem: a penalty function method, pattern search, enumeration, and non-sequential dynamic programming. The results of applying global optimization show that the increase in gas throughput considerably reduces cost savings. For instance, a reduction of operational cost savings from 23% up to 1.2% was observed when increasing the gas rate from 67 to 90 million m^3/d. The study also shows that operation costs approach those found in current practice when compressor stations work at their maximum capacity. Hence, global optimization proves to be more effective when the gas pipeline system works at any mass flow rate other than its maximum rating, a typical case of present operation in Chinese gas networks.

5.4.6. Approaches for MINLP models

Pratt and Wilson [113] propose a successive mixed-integer linear programming method. Their algorithm solves the nonlinear optimization problem iteratively by linearizing the pressure drop-flow Eq. (7). Integer variables are included in the formulation for compressor unit selection, and the problem is solved using branch and bound.

Cobos-Zaleta and Ríos-Mercado [114] presented a solution technique based on an outer approximation with equality relaxation and augmented penalty algorithm OA/ER/AP for solving a mixed-integer nonlinear programming model, where an integer decision variable, representing the number of compressor units running within each station, is incorporated. They present satisfactory results as they were able to find local optima for many instances tested.

Martin et al. [115] incorporate binary decision variables to decide whether to use or not a compressor unit within a compressor station and whether to open or close valves. They describe some techniques for a piecewise linear approximation of the nonlinearities of the model resulting in a large mixed-integer linear program. They study sub-polyhedra linking these piecewise linear approximations and show that the number of vertices is computationally tractable yielding exact separation algorithms. They also present suitable branching strategies complementing the separation algorithms. They tested their method on three real-world instances provided by their industrial partner, E.ON Ruhrgas AG, a German gas company. The size of the instances range from 11 to 31 pipes and from 3 to 15 compressor stations. They observed that the piecewise linear approximation is accurate enough to guarantee globally optimal solutions.

Chebouba et al. [116] present an Ant Colony Optimization (ACO) algorithm for the MFCP with a variable number of compressor units within a compressor station. Part of the decision process involves determining the number of operating units in each compressor. The ACO algorithm [117] is a relatively new evolutionary optimization method to solve different combinatorial optimization problems. They tested their method on the Hassi R'mell-Arzew real-world pipeline network in Argelia consisting of 5 pipes, 6 nodes, 5 compressor stations, and 3 units in each compressor. They also built three additional cases with up to 23 compressor stations, and 12 compressor units in each compressor. They compare their method with a DP implementation. Their empirical work shows a good performance of the proposed method in noncyclic systems.

Tabkhi et al. [118] present a computational study of MFCP applied to a case study in the French company Gaz de France. The authors present a MINLP model where binary variables for representing pipeline flow direction are introduced. They used the GAMS/SBB solver for solving the MINLP model, which calls CONOPT for solving the NLP subproblems. The real-world case has 30 pipes and 6 compressor stations. To make the problem more tractable for the solver, the authors consider several different strategies for initializing some or all the binary variables. They also report on a sensitivity analysis discussing one of the particular strategies for initializing the binary variables.

Wu et al. [119] present a hybrid objective model with compressor switching constraints that aims at maximizing revenue and throughput while considering a weighting value to account for both optimization problems. The model is solved by means of a particle swarm optimization (PSO) algorithm that includes an adaptive inertia weight adjusting procedure to overcome premature convergence issues. A commercially available simulation software is used to provide the initial particles that satisfy the underlying model. The authors present a case study based on a Chinese gas pipeline system with a gun-barrel topological structure and four compressor stations. The proposed algorithm showed a fast convergence speed when compared with other extensions of the PSO algorithm.

Table 1 summarizes the research on steady-state models for pipeline network optimization. Entries are first sorted by model type, then by network topology, and then by chronological date of publication.

5.4.7. Other models

Carter et al. [120] describe a class of noisy optimization problems from the gas transmission industry, and propose an algorithms for their solution. The algorithms they consider are

Table 1
Summary of research on steady-state models.

Work	Model	N	Approach
Wong and Larson [64]	NLP	L	DP
Percell and Ryan [108]	NLP	L, T	GRG
Villalobos-Morales and Ríos-Mercado [109]	NLP	L, T	GRG
Misra et al. [111]	NLP	L, T	Geometric programming
De Wolf and Smeers [68]	NLP	T	Linearization
Lall and Percell [103]	NLP	T	DP
Luongo et al. [82]	NLP	T	Hierarchical, DP, SA
Wong and Larson [102]	NLP	T	DP
Borraz-Sánchez and Haugland [106]	NLP	C	NDP, tree decomposition, adaptive discretization
Borraz-Sánchez and Ríos-Mercado [69]	NLP	C	Tabu search and NDP
Carter [65]	NLP	C	NDP
Flores-Villarreal and Ríos-Mercado [110]	NLP	C	GRG
Jin and Wojtanoiwicz [112]	NLP	C	Penalty function, pattern search, NDP
Ríos-Mercado et al. [104]	NLP	C	Decomposition, DP
Chebouba et al. [116]	MINLP	L	ACO
Martin et al. [115]	MINLP	L	Linearization
Cobos-Zaleta and Ríos-Mercado [114]	MINLP	T	OA/ER/AP
Pratt and Wilson [113]	MINLP	C	Successive MILP, B&B
Tabkhi et al. [118]	MINLP	C	SBB/CONOPT
Wu et al. [119]	MINLP	L	PSO

Notation:
N = network topology (L = linear; T = tree; C = cyclic); Approach: ACO = ant colony optimization; NDP = non-sequential DP; SA = simulated annealing; SBB = standard branch and bound; PSO = particle swarm optimization.

implicit filtering [121], the global optimization algorithm DIRECT [122] and a new hybrid of implicit filtering and DIRECT, which attempts to capture the best features of the two. They consider minimizing the cost of fuel for the compressor stations in a gas pipeline network. This cost can be reduced by changing both flow patterns through the system and pressure settings throughout the system. In their model the problems have two flows as design variables. The flow variables can be either unknown inlet or outlet flows, or Kirchhoff's law representations of flow splits between different possible alternative paths. Once these boundary and loop flows have been specified, each evaluation of the objective function f involves solving a hierarchy of embedded optimization problems and associated simulation subproblems, and then evaluating the total fuel used at the solutions to the subproblems. The main decision variables in the subproblems are pressure settings throughout the pipeline system, and on/off settings for the large number of individual compressors throughout the system. The value of the objective function f is the total fuel used. The hierarchical optimization that is internal to the function evaluation involves solving a large combinatorial problem using non-sequential dynamic programming [65]. In this formulation, the pressure settings at the discharge side of each compressor station are first discretized into a set of discrete values covering the range of potentially attainable values for the equipment being simulated. Hydraulic analysis is used to propagate each discretized pressure at each station discharge forward in space, which establishes an implicit discretization of potential pressures at the suction side of each station as well. For any station (with specified flow) they pick each possible pair inlet and outlet pressures from the discretization and determine whether station operation is feasible and, if so, what is the fuel cost. This determination in itself is a substantial mixed integer nonlinear optimization and simulation problem [123]. Once the local consequences of operating each station at different combinations of inlet and outlet pressure have been computed, non-sequential dynamic programming is used to select the best possible combination of discretized pressures throughout the system while maintaining hydraulic integrity, satisfying the pressure drop equations between stations, and observing all equipment limitations. They apply the three methods to some instances from the gas pipeline industry and to a suite of test problems from the global optimization literature. They found that the performance of implicit filtering depends strongly on its starting point. When implicit filtering found a feasible point, then it performed much better than either DIRECT or the hybrid. However, for a large percentage of starting points implicit filtering did not find a feasible point. DIRECT did relatively poorly in all problems, needing a larger amount of function evaluations to get comparable results. For the suite of test problems from the literature, implicit filtering was trapped in a local minimum for a significant fraction of the runs. DIRECT was more robust, but not completely successful. They conclude that the hybrid algorithm offers the best compromise between low cost and robustness.

Wu et al. [124] consider a gas transportation problem in a distribution network rather than a transmission network. A pipeline network is generally established either to transmit gas at high pressure from coastal supplies to regional demand points (transmission network) or to distribute gas to consumers at low pressure from the regional demand points (distribution network). In this study, the distribution network is considered. The distribution network differs from the transmission one in a number of ways. Pipes involved in a distribution network are often much smaller and the network is simpler, having no valves, compressors or nozzles. In that paper, the authors introduce the problem of minimizing the cost of pipelines incurred by driving the gas in a distribute nonlinear network under steady-state assumptions. In particular, the decision variables include the length of the pipes' diameter, pressure drops at each node of the network, and mass flow rate at each pipeline leg. They establish a mathematical optimization model of this problem, and then present a global optimization approach, which is based on the GOP primal-relaxed dual decomposition method by Visweswaran and Floudas [125]. Their method is successfully tested on two real-world instances having 6 nodes and 5 pipes, and 13 nodes and 14 pipes.

Borraz-Sánchez and Haugland [44] extend previously suggested models by incorporating the variation in pipeline flow capacities with gas specific gravity and compressibility for a steady-state isothermal model. Flow capacities are modeled as functions of pressure, compressibility and specific gravity by the commonly-used Weymouth equation. In their work, the California Natural Gas Association method is used to model compressibility as a function of specific gravity and pressure. The sources feeding the transmission network do not necessarily supply gas with equal specific gravity. In their model, they assumed that when different flow streams enter a junction point, the specific gravity of the resulting flow is a weighted average of the specific gravities of entering flows. To handle the nonconvex NLP model, they propose a heuristic method based on an iterative scheme in which a simpler NLP model is solved in each iteration. Computational experiments are conducted in order to assess the computability of the model by applying a global optimizer, and to evaluate the performance of the heuristic approach. When applied to a wide set of test instances, the heuristic method provides solutions with deviations less than 10% from optimality, and in many instances turns out to be exact. They also report several experiments demonstrating that letting the compressibility and the specific gravity be global constants can lead to significant errors in the estimates of the total network capacity.

MohamadiBaghmolaei et al. [126] take a different angle at the MFCP. They argue that there might be some cases where accurate information about the process may not be available or the system may have a nonlinear time variable behavior. In such cases, due to the lack of information and difficulties in prediction of gas turbine and compressor efficiency, techniques which depend on experimental data such as artificial neural networks (ANNs) may be applied. In their work they use ANNs within a genetic algorithm to predict the relationship among the decisive parameters and minimize the fuel consumption in a pipeline network. They apply their approach to a case study in the south of Iran considering a linear system with four compressor stations. The comparison between the efficient total fuel consumption and the final delivery pressure predicted by ANN and conventional numerical models confirms the accuracy of the proposed method.

Gopalakrishnan and Biegler [127] study a Nonlinear Model Predictive Control (NMPC) formulation for optimizing the operational costs of gas pipeline networks. They use an economic NMPC formulation, which directly considers the compressor operating cost as the controller objective. Due to diurnal gas demands, the optimal operation is a cyclic steady state. The controller objective and terminal constraints are suitably defined to ensure asymptotic convergence and closed-loop stability of the cyclic steady state. It is shown through simulations that the performance of the economic NMPC formulation is better than a tracking NMPC. The inherent robustness of the formulation also ensures convergence to a region around the cyclic steady state when demand forecasts are inaccurate. The large scale NLP is also solved within a reasonable CPU time making it practical for online applications.

To the best of our knowledge, apart from the work of Wu et al. [119] where a hybrid objective model is presented along with a weighting value that accounts for both optimization problems, the only work on the MFCP from a biobjective optimization perspective is due to Hernandez Rodriguez et al. [128] who consider the minimization of fuel consumption and the maximization of gas mass flow delivery simultaneously. They present a computational comparison between a genetic algorithm (GA) coupled with a Newton–Raphson procedure and the well-known ϵ-constraint method for multiobjective programming. Additionally, a study of carbon dioxide (CO_2) emissions is carried out. In their empirical work, it was observed that the two methods obtain overlapping Pareto fronts, however, the one obtained by the GA is considerably larger than the one obtained from the ϵ-constraint method. Along the Pareto front provided by the GA, the CO_2 emissions vary from 1.1% to 1.8% of the natural gas flow delivery. In a related work, Alinia Kashani and Molaei [129] present multi-objective optimization model by considering three objectives: maximization of gas delivery flow, maximization of line pack, and minimization of operating costs. They represent the operating costs as the sum of the fuel consumption and CO_2 emissions costs. They present a NSGA-II algorithm which is a state-of-the-art GA for multi-objective combinatorial optimization problems. The results indicated that, since there was approximately a direct relation between the cost function and CO_2 emission, the Pareto points with lower operating cost resulted in minimum carbon dioxide emission and vice versa. A related study by Garcia-Hernandez and Brun [130] focuses on maximizing flow rate by keeping optimal conditions on the available compression power.

5.5. Transient models

Transient models are more challenging as the governing PDEs associated to the dynamics of the gas system must be taken into consideration. There has been certainly some research done from a descriptive perspective. Here we survey the most significant work related to the optimization of transient systems.

5.5.1. Hierarchical control approaches

Optimization techniques have also been applied for transient (time dependent) models. For instance, Larson and Wismer [131] propose a hierarchical control (HC) approach for a transient operation of a gun-barrel pipeline system. Osiadacz and Bell [132] suggest a simplified algorithm for the optimization of the transient gas transmission network, which is based on a HC approach. The HC approach for transient models can be found in Anglard and David [133], Osiadacz [134], and Osiadacz and Swierczewski [135]. Some degree of success has been reported from these approaches as far as optimizing the compressor station subproblem. However, these approaches have limitations in globally optimizing the minimum cost.

One of the most significant early efforts to address transient flow in natural gas pipeline systems from an HC standpoint was due to Osiadacz [136] who developed an algorithm based on hierarchical control and network decomposition. Local problems were solved using a gradient search technique. The subsystems are coordinated using a *goal coordination method* to find the global optimum. He formulated discrete state equations for the case in which output pressures are treated as elements of the control vector. The algorithm was tested using part of the National Grid of UK containing 23 nodes, 13 pipelines, 3 compressor stations, 2 storage supply nodes, and 1 source. A time frame of 24 h with time discretization steps of 2 h were established. The results were somewhat similar to those obtained by an alternate algorithm based on sequential quadratic programming due to Furey [137]. The maximum discrepancy found was in the order of 15%. The authors indicate that dynamic instances over 24 h could not be solved exactly in reasonable times. The authors conclude by suggesting their proposed method based on decomposition-coordination is suitable for parallel computing.

5.5.2. Mathematical programming approaches

Early work on transient optimization of natural gas pipeline systems is due to Mantri et al. [138]. They develop a transient gas optimization model that minimizes the cost of transporting natural gas over time periods in which line-pack and throughput are changing due to designated fluctuations in supply and demand. The major component of their optimization engine is based on the GRG method and dynamic programming. Tao and Ti [139] derive a method for transient analysis in a gas pipeline network. Traditionally, the governing equations for transient analysis of a gas pipeline system involve two partial differential equations, which are normally solved by complex numerical methods. The authors extend the electric analogy method by combining resistance and capacitance, which leads to a first order ordinary differential equation and an alternative way of solving the transient problem. The proposed method was found more efficient than previous approaches. Later Ke and Ti [140] use the same analogy to an electrical system to develop a new model. Empirical evidence shows that solutions obtained under this new model are compatible with those using previous models. This new model is found more tractable. Osiadacz and Chaczykowski [141] present a comparison between isothermal and non-isothermal models for transient flow in natural gas pipeline systems.

Ehrhardt and Steinbach [142] address a transient pipeline optimization problem. They present appropriate space and time discretizations to obtain a large-scale nonlinear programming problem (NLP). This large-scale NLP is solved by the general-purpose NLP code SNOPT in combination with the automatic differentiation add-on SnadiOpt. They tested their approach on a relatively small network with three compressor stations considering different scenarios.

Aalto [143] present a study on real time optimization of a natural gas pipeline in transient conditions. He points out that many pipeline systems are, however, only mildly nonlinear even in large transients such as compressor station (CS) shutdown or startup. A dynamic, receding horizon optimization problem is defined, where the free response prediction of the pipeline is obtained from a pipeline simulator and the optimal values of the decision variables are obtained solving an approximate Quadratic Programming (QP) problem where the cost function is the energy consumption of the CSs. The problem is extended with discrete decision variables, the shutdown/start-up commands of CSs. A Mixed Logical Dynamical (MLD) system is defined, but the resulting Mixed Integer QP problem is shown to be very high-dimensional. Instead, a sequence of QP problems is defined resulting in an optimization problem with considerably smaller dimension. The receding horizon optimization is tested in a simulation environment and comparison with data from a true natural gas pipeline shows 5–8% savings in compressor energy consumption.

Mahlke et al. [144] present a simulated annealing metaheuristic for the transient natural gas network optimization problem. For this transient problem, they present a highly complex mixed integer nonlinear program. They relax the equations describing the gas dynamics in pipes by adding these constraints combined with appropriate penalty factors to the objective function. A suitable neighborhood structure is developed for the relaxed problem where time steps as well as pressure and flow of the gas are decoupled. They tested their method on three real-world instances provided by the German gas company E.ON Ruhrgas AG. The range of the size was from 11 to 31 pipes and from 3 to 15 compressor stations. They obtained reasonably good results in very competitive running times.

Domschke et al. [145] apply an implicit box scheme to the isothermal Euler equation to derive an MINLP for the transient MCFP. The model is solved by means of a combination of (i) a novel mixed-integer linear programming approach based on piecewise linearization and (ii) a classical sequential quadratic program applied for given combinatorial constraints. Their empirical work reveals that better approximations to the optimal control problem can be obtained by using solutions of the SQP algorithm to improve the MILP. Moreover, iteratively applying these two techniques improves the results even further. In this regard, recent methodological results on the solution of such piecewise linear representation systems by Vielma and Nemhauser [146] and by Rebennack and Kallrath [147,148] may be worthy of further investigation.

More recently, Zavala [36] proposes a discretization framework for the PDEs governing the dynamics of the gas system by applying a Finite Difference scheme in space and by using an implicit Euler scheme in time for the continuity and momentum equations. The discretization is used in a two-stage stochastic model that involves recourse actions before the end of the planning horizon. The author claims that the two-stage structure is more restrictive but also computationally more tractable.

6. Research challenges

The natural gas industry keeps evolving and thus a greater flexibility in the day-to-day gas transport operations is required. While posing huge opportunities for improvements, the natural gas infrastructure has to convey her ideas through analytical methods. With technological breakthrough observed during the last decades, these methods can now become applicable and can be developed by the scientific community.

From the optimization perspective, there are still quite a few areas that pose a wide range of challenges to the scientific community. Given the nonconvex nature of the problem, it is evident that global optimization techniques are necessary for handling this type of problems. A closely related problem is that of electricity transportation [149], where the AC power flow equations are also nonconvex. In both fields, global optimization methods are tremendously challenged and current state-of-the-art methods cannot solve problems of practical relevance to optimality. However, both fields may indeed benefit from methodological advances in one of the areas. In the meantime, the development of metaheuristics and its integration with classical optimization methods have proven very successfull for obtaining approximate solutions of very good quality.

Although in practice, gas transport operations are defined by inherently transient processes, that is, models depending on time, we assume that the problem is in steady-state. That is, the mathematical models provide solutions for pipeline systems that have been operating for a relative large amount of time. Another of the most significant challenges to the natural gas transport industry is how to integrate and to solve in the analysis transient models. More precisely, by conducting a steady-state study, we consider the gas flow decision variables in the system to be independent of time. This allows the use of algebraic equations to describe the behavior of natural gas through the pipeline network. A transient analysis requires the use of partial differential equations to describe the continuity, energy, and momentum equations that relate the decision variables, such as gas flow, velocity, density, pressure, and temperature, as a function of time. Due to the challenge imposed by the transient case, while increasing the number of variables, as well as the inherent complexity of the problem, works on this area are still in a developing phase. See Section 5.5.

The majority of the works discussed here are based on deterministic models, i.e., where each parameter is assumed known in advance. There is an evident need of stochastic models and approaches to handle those cases where the variation of the parameters (such as demand or supply) is so high that deterministic assumptions no longer hold. In this paper, we have reviewed some of the few works starting to face this challenge, particularly in compressor station optimization networks and pooling systems. In this regard, simulation–optimization schemes such as the recent study by Fasihizadeh et al. [150], may also prove helpful and worthy of future investigation. Naturally, studying and addressing stochastic MINLP and nonconvex NLP models represents a tremendous challenge as well.

Concerning pipeline capacity, the question of how to handle excess in the maximum capacity in pipelines while meeting strict transportation contracts poses a significant issue for the natural gas industry. Analytical models encompassing the optimization of the pipeline capacity release are required. The volume variability also introduces significant configuration challenges in the natural gas transport, particularly in compressor stations operation.

From the gas quality perspective, the improvements not only concern customer demands satisfaction, but also represent a higher impact on pipeline infrastructure. From the operational perspective, for example, the theory of optimization can become a blunt instrument in gas quality estimates on the acid formation from sulfur compounds that may lead to pipeline corrosion. The difficulty stems from the fact that constituents can vary seasonally or even more frequently. Safety and reliability are also main concerns for the natural gas transport industry.

Moreover, most of the works discussed in this paper are confined to irreversible flows in steady-state, i.e., the gas can flow through a pipeline in only one direction. The authors inherently assume that valves are present to restrict the direction of flow. However, steady-state flow models considering reversible flows may represent a significant contribution to those pipeline networks that connect their major lines with storage facilities, in which the flow in either direction may be allowed.

In steady-state models, the gas flow is considered isothermal at an inlet average effective temperature. This is a common practice in which authors assume that a heat transfer with the surroundings in the pipeline system causes the temperature to remain constant. Works on this area pose a significant challenge due to the inherent complexities associated to the gas temperature. In these works, it is also assumed that the transmission lines are composed of horizontal pipelines. In practice, these systems have frequent changes in their elevation. Hence, a special attention must be paid into the necessary correction factors to compensate the changes in elevation.

There is another class of problems that have gained particular relevance since the deregulation of the industry, which occurred in the mid 1990s in ths US and in the mid 2000s in Europe. Rather than focusing on pipeline optimization problems, these changes have led to issues more closely related to the marketing of the natural gas. While previously network operator and gas vendor were united, they were forced to split up into independent companies. The network has to be open to any other gas trader at the same conditions, and free network capacities have to be identified and publicly offered in a non-discriminatory way. In this regard, Fügenschuh et al. [151] present an excellent review and discussion of these new class of problems. This includes the validation of nominations (see [152] for a more in-depth study), that asks for the decision if the network capacity is sufficient to transport a specific amount of flow, the verification of booked capacities and the detection of available freely allocable capacities, and the topological extension of the network with new pipelines or compressors in order to increase its capacity. Additionally, we have seen some works in other natural gas marketing problems (see [5] for a review on some of these problems); nonetheless, we believe this is a tremendous area of opportunity for problems in natural gas transmission systems.

Finally, one of the major challenges to efficiently exploit the natural gas supplies arises from the limitation of the optimization techniques, which are already developed in theory, but to less extent applicable in practice due to considerably strong assumptions. For success in the increase in the demand, more than a few successful optimization tools capable of responding to changing conditions in a rational manner are required. This would certainly make the use of existing natural gas transmission systems more efficient, resulting in significant economic compensations beyond the expected levels for the natural gas industry. We expect that this trend will speed up towards a more promising future with the continual contribution of the scientific community.

Acknowledgements

We thank the three anonymous referees for their remarks that helped improve the quality of this paper. The research of the first author was supported by the Mexican Council for Science and Technology, Grant CONACyT CB-2011-01-166397.

References

[1] NaturalGas.Org Web Site; January 2012. <http://naturalgas.org/>.

[2] Sánchez-Úbeda EF, Berzosa A. Modeling and forecasting industrial end-use natural gas consumption. Energy Econ 2007;29(4):710–42.

[3] U.S. Energy Information Administration. Annual energy outlook 2011 with projections to 2035. Technical report DOE/EIA-0383(2011), U.S. Department of Energy; April 2011. <http://www.eia.gov/forecasts/aeo/>.

[4] U.S. Energy Information Administration. International energy outlook 2011. Technical report DOE/EIA-0484(2011), EIA, Office of Communication; September 2011. <http://www.eia.gov/ieo/pdf/0484(2011).pdf>.

[5] Zheng QP, Rebennack S, Iliadis NA, Pardalos PM. Optimization models in the natural gas industry. In: Rebennack S, Pardalos PM, Pereira MVF, Iliadis NA, editors. Handbook of power systems I. Energy systems. Berlin, Germany: Springer-Verlag; 2010. p. 121–48.

[6] Hübner M, Haubrich H-J. Long-term pressure-stage comprehensive planning of natural gas networks. In: Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, editors. Handbook of networks in power systems II. Energy systems. Berlin Heidelberg: Springer; 2012. p. 37–59.

[7] Li X, Armagan E, Tomasgard A, Barton PI. Long-term planning of natural gas production systems via a stochastic pooling problem. In: American control conference (2010-ACC). IEEE; 2010. p. 429–35. ISBN: 978-1-4244-7426-4.

[8] Nord Stream. The new gas supply route to europe. Electronic Newsletter; 01 September 2008. <http://media.nord-stream.com/media/news/facts/en/2008/09/the-new-gas-supply-route-to-europe_20080901.pdf>.

[9] Dempsey J. Gazprom plans to re-route controversial European pipeline. The New York Times, International Herald Tribune; 23 August 2007. <http://www.iht.com/articles/2007/08/23/news/pipeline.php?page=1>.

[10] Broeker RJ. A new process for the transportation of natural gas. In: Proceedings of the 1st international conference on liquified natural gas, Chicago; 1968. p. 1–11.

[11] Ffooks R. Natural gas by sea: the development of a new technology. Witherby Seamanship, London, UK, 2nd ed.; 1993.

[12] Ginzburg Y. ANG storage as a technological solution for the "chicken-and-egg" problem of NGV refueling infrastructure development. In: Proceedings of 23rd world gas conference, Amsterdam, The Netherlands; 2006.

[13] Foss MM. Introduction to LNG: an overview on liquefied natural gas (LNG), its properties, organization of the LNG industry and safety considerations. Technical report, Center for Energy Economics, Jackson School of Geosciences, University of Texas, Austin; January 2007.

[14] U.S. Environmental Protection Agency. Greenhouse gas emissions reporting from the petroleum and natural gas industry. Technical report, U.S. Environmental Protection Agency, Climate Change Division, Washington, DC; 2010.

[15] Harrison MR, Shires TM, Wessels JK, Cowgill RM. Methane emissions from the natural gas industry. Technical report EPA/600/SR-96/080, U.S. Environmental Protection Agency, Center for Environmental Research Information National Risk Management Research Laboratory, Research Triangle Park, NC; 1997.

[16] Mokhatab S, Poe WA, Speight JG. Handbook of natural gas transmission and processing. Houston: Gulf Professional Publishing; 2006.

[17] Mokhatab S, Santos SP, Cleveland T. Compressor station design criteria. Pipeline Gas J 2007;234(6):26–32.

[18] Akhtar MS. Selection and optimization of centrifugal compressors for oil and gas applications. Paper presented at the GPA Europe Spring Meeting, Bergen, Norway; May 1992.

[19] Kurz R, Ohanian S, Lubomirsky M. On compressor station layout. In: ASME conference proceedings – vol. 4: Turbo Expo 2003; 2003. p. 1–10. Paper GT2003-38019.

[20] Kurz R. Considerations on compressor station layout. Pipeline Gas J 2007;234(9):22–6.

[21] Santos SP. Series or parallel arrangement for a compressor station: a recurring question that needs a convincing answer. In: Proceedings of the 36th PSIG annual meeting, Palm Springs; October 2004.

[22] Menon ES. Gas pipeline hydraulics. Boca Raton: CRC Press; 2005.

[23] Contesse L, Ferrer JC, Maturana S. A mixed-integer programming model for gas purchase and transportation. Ann Oper Res 2005;139(1):39–63.

[24] Zwitserloot R, Radloff A. Strategic use of gas storage facilities. In: Bausch A, Schwenker B, editors. Handbook utility management. Berlin, Germany: Springer-Verlag; 2009. p. 417–32 [chapter 25].

[25] Neumann A, Zachmann G. Expected vs. observed storage usage: limits to intertemporal arbitrage. In: Cretì A, editor. The economics of natural gas storage: a European perspective. Berlin, Germany: Springer-Verlag; 2009. p. 13–29 [chapter 2].

[26] Holland A. Optimization of injection/withdrawal schedules for natural gas storage facilities. In: Ellis R, Allen T, Petridis M, editors. Applications and innovations in intelligent systems XV. London, UK: Springer; 2008. p. 287–300.

[27] Holland A. A decision support system for energy storage traders. In: Coyle L, Freyne J, editors. Artificial intelligence and cognitive science. Lecture notes in computer science, vol. 6206. Berlin, Germany: Springer; 2010. p. 291–2.

[28] Welch TH, Smith JG, Rix JP, Reader RD. Meeting seasonal peak demands for natural gas. J Oper Res Soc 1971;22(s1):93–106.

[29] de Nevers N, Day A. Packing and drafting in natural gas pipelines. J Petroleum Technol 1983;35(3):655–8.

[30] Carter RG. Rachford Jr HH. Optimizing line-pack management to hedge against future load uncertainty. In: Proceedings of the 35th PSIG annual meeting, Bern, Switzerland; October 2003. Paper 0306.
[31] Krishnaswami P, Chapman KS, Abbaspour M. Compressor station optimization for linepack maintenance. In: Proceedings of the 36th PSIG annual meeting, Palm Springs; October 2004.
[32] Frimannslund L, Haugland D. Line pack management for improved regularity in pipeline gas transportation networks. In: Martorell S, Guedes-Soares C, Barnett J, editors. Safety, reliability and risk analysis: theory, methods and applications, vol. 4. London, UK: CRC Press; 2008. p. 2963–9.
[33] Borraz-Sánchez C. Optimization methods for pipeline transportation of natural gas. PhD thesis, University of Bergen, Bergen, Norway; 2010.
[34] Frimannslund L, El Ghami M, Alfaki M, Haugland D. Solving the pooling problem with LMI relaxations. In: Cafieri S, Tóth BG, Hendrix EMT, Liberti L, Messine F, editors. Proceedings of the Toulouse global optimization workshop, Toulouse, France; September 2010. p. 51–4.
[35] Haugland D. An overview of models and solution methods for pooling problems. In: Bjørndal E, Bjørndal M, Pardalos PM, Rönnqvist M, editors. Energy, natural resources and environmental economics. Energy systems. Berlin, Germany: Springer; 2010. p. 459–69. ISBN: 978-3-642-12066-4.
[36] Zavala VM. Stochastic optimal control model for natural gas networks. Comput Chem Eng 2014;64:103–13.
[37] Osiadacz AJ. Simulation and analysis of gas networks. Houston: Gulf Publishing Company; 1987.
[38] CRANE. Flow of fluids: through valves, fittings and pipe. Technical paper 410M, Crane Company, New York; 1982.
[39] Modisette JL. Equation of state tutorial. In: Proceedings of the 32th PSIG annual meeting, Savannah; October 2000.
[40] Dranchuk PM, Abou-Kassem JH. Calculations of z factors for natural gases using equations of state. J Canadian Petroleum Technol 1975;14(3):34–6.
[41] Katz DL, Cornell D, Kobayashi R, Poettmann FH, Vary JA, Elenbaas JR, et al. Handbook of natural gas engineering. New York: McGraw-Hill; 1959.
[42] Dranchuk PM, Purvis RA, Robinson DB. Computer calculations of natural gas compressibility factors using the Standing and Katz correlation. In: Proceedings of the petroleum society of Canada annual technical meeting, Edmonton, Canada; May 1973. ISBN: 978-1-55563-673-9.
[43] Gopal VN. Gas z-factor equations developed for computers. J Oil Gas 1977;75(32):58–60.
[44] Borraz-Sánchez C, Haugland D. Optimization methods for pipeline transportation of natural gas with variable specific gravity and compressibility. TOP 2013;21(3):524–41.
[45] Davisson EG. A computer program for flow calculations. Technical report ORNL–TM–1093, Oak Ridge National Laboratory, Oak Ridge; February 1965.
[46] Savidge JL. Compressibility of natural gas. Intellisite Technical Library, Paper 1040. p. 12–26. <http://help.intellisitesuite.com/Hydrocarbon/papers/1040.pdf>.
[47] Benedict M, Webb GB, Rubin LC. An empirical equation for thermodynamic properties of light hydrocarbons and their mixtures i. Methane, ethane, propane and n-butane. J Chem Phys 1940;8(4):334–45.
[48] Adhya N, Tawarmalani M, Sahinidis NV. A Lagrangian approach to the pooling problem. Indust Eng Chem Res 1999;38(5):1956–72.
[49] Foulds LR, Haugland D, Jørnsten K. A bilinear approach to the pooling problem. Optimization 1992;24(1–2):165–80.
[50] Alfaki M, Haugland D. Strong formulations for the pooling problem. J Global Optim 2013;56(3):897–916.
[51] Haverly CA. Studies of the behavior of recursion for the pooling problem. ACM SIGMAP Bull 1978;25:19–28.
[52] Aggarwal A, Floudas C. A decomposition strategy for global optimization search in the pooling problem. ORSA J Comput 1990;2(3):225–35.
[53] Floudas CA, Visweswaran V. A global optimization algorithm (GOP) for certain classes of nonconvex NLPs – I. Theory. Comput Chem Eng 1990;14(12):1397–417.
[54] Almutairi H, Elhedhli S. A new Lagrangian approach to the pooling problem. J Global Optim 2009;45(2):237–57.
[55] Audet C, Brimberg J, Hansen P, Digabel SL, Mladenovic N. Pooling problem: alternate formulations and solution methods. Manage Sci 2004;50(6):761–76.
[56] Misener R, Floudas CA. A review of advances for the pooling problem: modeling, global optimization, and computational studies. Appl Comput Math: Int J 2009;8(1):3–22.
[57] Ben-Tal A, Eiger G, Gershovitz V. Global minimization by reducing the duality gap. Math Program 1994;63(1–3):193–212.
[58] Tawarmalani M, Sahinidis NV. A polyhedral branch-and-cut approach to global optimization. Math Program 2005;103(2):225–49.
[59] Sherali HD, Adams WP. A reformulation-linearization technique for solving discrete and continuous nonconvex problems. Dordrecht, The Netherlands: Kluwer; 1999.
[60] Li X, Armagan E, Tomasgard A, Barton PI. Stochastic pooling problem for natural gas production network design and operation under uncertainty. AIChE J 2011;57(8):2120–35.
[61] Li X, Tomasgard A, Barton PI. Decomposition strategy for the stochastic pooling problem. J Global Optim 2012;54(4):765–90.
[62] Tawarmalani M, Sahinidis NV. Global optimization of mixed-integer nonlinear programs: a theoretical and computational study. Math Program 2004;99(3):563–91.
[63] Ahuja RK, Magnanti TL, Orlin JB. Network flows. Englewood Cliffs: Prentice Hall; 1993.
[64] Wong PJ, Larson RE. Optimization of natural-gas pipeline systems via dynamic programming. IEEE Trans Autom Control 1968;AC-13(5):475–81.
[65] Carter RG. Pipeline optimization: dynamic programming after 30 years. In: Proceedings of the 30th PSIG annual meeting, Denver; October 1998.
[66] Ríos-Mercado RZ, Wu S, Scott LR, Boyd EA. A reduction technique for natural gas transmission network optimization problems. Ann Oper Res 2002;117(1–4):217–34.
[67] De Wolf D, Smeers Y. Mathematical properties of formulations of the gas transmission problem. SMG Preprint 94/12, Université Libre de Bruxelles, Bruxelles, Belgium; 1994.
[68] De Wolf D, Smeers Y. The gas transmission problem solved by an extension of the simplex algorithm. Manage Sci 2000;46(11):1454–65.
[69] Borraz-Sánchez C, Ríos-Mercado RZ. Improving the operation of pipeline systems on cyclic structures by tabu search. Comput Chem Eng 2009;33(1):58–64.
[70] Bakhouya B, De Wolf D. The gas transmission problem when the merchant and the transport functions are disconnected. Working paper, Université du Littoral, Dunkerque, France; January 2007.
[71] Kalvelagen E. An improved formulation for a gas transportation problem. Working paper, GAMS Development Corporation, Washington, DC; April 2003.
[72] Borraz-Sánchez C, Haugland D. A tree decomposition algorithm for minimizing fuel cost in gas transmission networks. In: Kacem I, editor. Proceedings of the 39th international conference on computers & industrial engineering (CIE 2009). IEEE; 2009. p. 244–9. ISBN: 978-1-4244-4136-5.
[73] O'Neill RP, Williard M, Wilkins B, Pike R. A mathematical programming model for allocation of natural gas. Oper Res 1979;27(5):857–73.
[74] Wilson JG, Wallace J, Furey BP. Steady-state optimisation of large gas transmission systems. In: Osiadacz AJ, editor. Simulation and optimization of large systems. Oxford, UK: Clarendon Press; 1988. p. 193–207.
[75] Fourer R. A simplex algorithm for piecewise-linear programming I: Derivation and proof. Math Program 1985;33(2):204–33.
[76] GAMS Development Corporation. GAMS: the solver manuals. GAMS Development Corporation, Washington, DC; 2008.
[77] Abbaspour M, Chapman KS. Nonisothermal transient flow in natural gas pipeline. J Appl Mech 2008;75(3):181–8.
[78] Chaczykowski M. Transient flow in natural gas pipeline – the effect of pipeline thermal model. Appl Math Modell 2010;34(4):1051–67.
[79] Belyaev BM, Gushchin AA, Shenbrot IM. State of the art and development prospects in metrological support to natural gas metering. Measure Tech 1999;42(10):1004–7.
[80] Belyaev BM, Patrikeev VG. Influence of variations of the composition of natural gas from different fields on the results of mass and volumetric flow rate measurement under standard conditions in gas distribution organizations. Measure Tech 2003;46(5):462–5.
[81] Bahadori A, Mokhatab S, Towler BF. Rapidly estimating natural gas compressibility factor. J Natural Gas Chem 2007;16(4):349–53.
[82] Luongo CA, Gilmour BJ, Schroeder DW. Optimization in natural gas transmission networks: a tool to improve operational efficiency. Technical report, Stoner Associates, Inc., Houston; April 1989.
[83] Babonneau F, Nesterov Y, Vial J-P. Design and operations of gas transmission networks. Oper Res 2012;60(1):34–47.
[84] Costa ALH, de Medeiros JL, Pessoa FLP. Optimization of pressure relief header networks: a linear programming formulation. Comput Chem Eng 2000;24(1):153–6.
[85] El-Shiekh TM. The optimal design of natural gas transmission pipelines. Energy Sources, Part B 2013;8(1):7–13.
[86] Marcoulaki EC, Papazoglou IA, Pixopoulou N. Integrated framework for the design of pipeline systems using stochastic optimisation and GIS tools. Chem Eng Res Des 2012;90(12):2209–22.
[87] Mariani O, Ancillai F, Donati E. Design of a pipeline: optimal configuration. In: Proceedings of the 29th PSIG annual meeting, Tucson; October 1997.
[88] Osiadacz AJ, Górecki M. Optimization of pipe sizes for distribution gas network design. In: Proceedings of the 27th PSIG annual meeting, Albuquerque; October 1995.
[89] Sanaye S, Mahmoudimehr J. Optimal design of a natural gas transmission network layout. Chem Eng Res Des 2013;91(12):2465–76.
[90] Tsal RJ, Gordon E, Simpson KO, Olson RR. Optimal gas pipeline design via dynamic programming with variable stages. In: Proceedings of the 18th PSIG annual meeting, New Orleans; October 1986.
[91] Zhou CC, Liu P, Pei Z. A superstructure-based mixed-integer programming approach to optimal design of pipeline network for large-scale CO2 transport. AIChE J 2014;60(7):2442–61.
[92] Üster H, Dilaveroğlu Ş. Optimization for design and operation of natural gas transmission networks. Appl Energy 2014;133:56–69.
[93] Wu S, Ríos-Mercado RZ, Boyd EA, Scott LR. Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. Math Comput Modell 2000;31(2–3):197–220.
[94] Zhu G-Y, Henson MA, Megan L. Dynamic modeling and linear model predictive control of gas pipeline networks. J Process Control 2001;11(2):129–48.
[95] Herrán-González A, De La Cruz JM, De Andrés-Toro B, Risco-Martín JL. Modeling and simulation of a gas distribution pipeline network. Appl Math Modell 2009;33(3):1584–600.

[96] Woldeyohannes AD, Majid MAA. Simulation model for natural gas transmission pipeline network system. Simul Model Pract Theory 2011;19(1):196–212.

[97] Mahmoudimehr J, Sanaye S. Minimization of fuel consumption of natural gas compressor stations with similar and dissimilar turbo-compressor units. J Energy Eng 2014;140(1):04013001.

[98] Osiadacz AJ. Nonlinear programming applied to the optimum control of a gas compressor station. Int J Numer Methods Eng 1980;15(9):1287–301.

[99] Percell PB, Van Reet JD. A compressor station optimizer for planning gas pipeline operation. In: Proceedings of the 21st PSIG annual meeting, El Paso; October 1989.

[100] Wu S, Boyd EA, Scott LR. Minimizing fuel consumption at gas compressor stations. In: Chen JJ-W, Mital A, editors. Advances in industrial engineering applications and practice I, Cincinnati, Ohio; 1996. p. 972–7. International Journal of Industrial Engineering.

[101] Nguyen HH, Uraikul V, Chan CW, Tontiwachwuthikul P. A comparison of automation techniques for optimization of compressor scheduling. Adv Eng Softw 2008;39(3):178–88.

[102] Wong PJ, Larson RE. Optimization of tree-structured natural-gas transmission networks. J Math Anal Appl 1968;24(3):613–26.

[103] Lall HS, Percell PB. A dynamic programming based gas pipeline optimizer. In: Bensoussan A, Lions JL, editors. Analysis and optimization of systems. Lecture notes in control and information sciences, vol. 144. Berlin: Springer-Verlag; 1990. p. 123–32.

[104] Ríos-Mercado RZ, Kim S, Boyd EA. Efficient operation of natural gas transmission systems: a network-based heuristic for cyclic structures. Comput Oper Res 2006;33(8):2323–51.

[105] Borraz-Sánchez C, Ríos-Mercado RZ. A hybrid meta-heuristic approach for natural gas pipeline network optimization. In: Blesa MJ, Blum C, Roli A, Sampels M, editors. Hybrid metaheuristics. Berlin, Germany: Springer; 2005. p. 54–65.

[106] Borraz-Sánchez C, Haugland D. Minimizing fuel cost in gas transmission networks by dynamic programming and adaptive discretization. Comput Indus Eng 2011;61(2):364–72.

[107] Ríos-Mercado RZ. Natural gas pipeline optimization. In: Pardalos PM, Resende MGC, editors. Handbook of applied optimization. New York: Oxford University Press; 2002. p. 813–25 [chapter 18.8.3].

[108] Percell PB, Ryan MJ. Steady-state optimization of gas pipeline network operation. In: Proceedings of the 19th PSIG annual meeting, Tulsa; October 1987.

[109] Villalobos-Morales Y, Ríos-Mercado RZ. Preprocesamiento efectivo de un problema de minimización de combustible en sistemas de transporte de gas natural. Revista Ingeniería Sistemas 2005;19:79–103 [in Spanish].

[110] Flores-Villarreal HJ, Ríos-Mercado RZ. Computational experience with a GRG method for minimizing fuel consumption on cyclic natural gas networks. In: Mastorakis NE, Stathopulos IA, Manikopoulos C, Antoniou GE, Mladenov VM, Gonos IF, editors. Computational methods in circuits and systems applications. Athens, Greece: WSEAS Press; 2003. p. 90–4.

[111] Misra S, Fisher MW, Backhaus S, Bent R, Chertkov M, Pan F. Optimal compression in natural gas networks: a geometric programming approach. IEEE transactions on control of network systems 2014. forthcoming (<http://arxiv.org/pdf/1312.2668.pdf>).

[112] Jin L, Wojtanowocz AK. Optimization of large gas pipeline network – a case study in China. J Canadian Petroleum Technol 2010;49(4):36–43.

[113] Pratt KF, Wilson JG. Optimisation of the operation of gas transmission systems. Trans Inst Measure Control 1984;6(5):261–9.

[114] Cobos-Zaleta D, Ríos-Mercado RZ. A MINLP model for minimizing fuel consumption on natural gas pipeline networks. In: Proceedings of the XI Latin-Ibero-American conference on operations research, Concepción, Chile; October 2002.

[115] Martin A, Möller M, Moritz S. Mixed integer models for the stationary case of gas network optimization. Math Program 2006;105(2–3):563–82.

[116] Chebouba A, Yalaoui F, Smati A, Amodeo L, Younsi K, Tairi A. Optimization of natural gas pipeline transportation using ant colony optimization. Comput Oper Res 2009;36(6):1916–23.

[117] Dorigo M, Di Caro G. The ant colony optimization metaheuristic. In: Corne D, Dorigo M, Glover F, editors. New ideas in optimization. London, UK: McGraw-Hill; 1999. p. 11–32.

[118] Tabkhi F, Pibouleau L, Hernandez-Rodriguez G, Azzaro-Pantel C, Domenech S. Improving the performance of natural gas pipeline networks fuel consumption minimization problems. AIChE J 2010;56(4):946–64.

[119] Wu X, Li C, Jia W, He Y. Optimal operation of trunk natural gas pipelines via an inertia-adaptive particle swarm optimization algorithm. J Natural Gas Sci Eng 2014;21:10–8.

[120] Carter RG, Gablonsky JM, Patrick A, Kelley CT, Eslinger OJ. Algorithms for noisy problems in gas transmission pipeline optimization. Optim Eng 2001;2(2):139–57.

[121] Gilmore P, Kelley CT. An implicit filtering algorithm for optimization of functions with many local minima. SIAM J Optim 1995;5(2):269–85.

[122] Jones DR. DIRECT global optimization algorithm. In: Floudas CA, Pardalos PM, editors. Encyclopedia of optimization, vol. 1. Dordrecht, The Netherlands: Kluwer; 2001. p. 431–40.

[123] Carter RG. Compressor station optimization: computational accuracy and speed. In: Proceedings of the 28th PSIG annual meeting, San Francisco; October 1996.

[124] Wu Y, Lai KK, Liu Y. Deterministic global optimization approach to steady-state distribution gas pipeline networks. Optim Eng 2007;8(3):259–75.

[125] Visweswaran V, Floudas CA. Computational results for an efficient implementation of the GOP algorithms and its variants. In: Grossmann IE, editor. Global optimization in engineering design. Nonconvex optimization and its applications. Dordrecht, The Netherlands: Kluwer; 1996. p. 111–54.

[126] MohamadiBaghmolaei M, Mahmoudy M, Jafari D, MohamadiBaghmolaei R, Tabkhi F. Assessing and optimization of pipeline system performance using intelligent systems. J Natural Gas Sci Eng 2014;18:64–76.

[127] Gopalakrishnan A, Biegler LT. Economic nonlinear model predictive control for periodic optimal operation of gas pipeline networks. Comput Chem Eng 2013;52:90–9.

[128] Hernandez Rodriguez G, Pibouleau LG, Azzaro Pantel C, Domenech S. Optimization of gas transmission networks under energetic and environmental considerations. Int J Chem Reactor Eng 2010;8:1–31. Paper A136.

[129] Alinia Kashani AH, Molaei R. Techno-economical and environmental optimization of natural gas network operation. Chem Eng Res Des 2014;92(11):2106–22.

[130] Garcia-Hernandez A, Brun K. Energy usage in natural gas pipeline applications. J Eng Gas Turbines Power 2012;134(2):1–9.

[131] Larson RE, Wismer DA. Hierarchical control of transient flow in natural gas pipeline networks. In: Proceedings of the IFAC symposium on distributed parameter systems, Banff, Alberta, Canada; 1971.

[132] Osiadacz AJ, Bell DJ. A simplified algorithm for optimization of large-scale gas networks. Opt Control Appl Methods 1986;7(1):95–104.

[133] Anglard P, David P. Hierarchical steady state optimization of very large gas pipelines. In: Proceedings of the 20th PSIG annual meeting, Toronto; October 1988.

[134] Osiadacz AJ. Dynamic optimization of high pressure gas networks using hierarchical systems theory. In: Proceedings of the 26th PSIG annual meeting, San Diego; October 1994.

[135] Osiadacz AJ, Swierczewski S. Optimal control of gas transportation systems. In: Proceedings of the 3rd IEEE conference on control applications, Glasgow, UK; August 1994. p. 795–6. ISBN: 0-7803-1872-2.

[136] Osiadacz AJ. Hierarchical control of transient flow in natural gas pipeline systems. Int Trans Oper Res 1998;5(4):285–302.

[137] Furey BP. A sequential quadratic programming-based algorithm for optimization of gas networks. Automatica 1993;29(6):1439–50.

[138] Mantri VB, Preston LB, Pringle CS. Transient optimization of a natural gas pipeline system. In: Proceedings of the 17th PSIG annual meeting, Albuquerque; October 1985.

[139] Tao WQ, Ti HC. Transient analysis of gas pipeline network. Chem Eng J 1998;69(1):47–52.

[140] Ke SL, Ti HC. Transient analysis of isothermal gas flow in pipeline network. Chem Eng J 2000;76(2):169–77.

[141] Osiadacz AJ, Chaczykowski M. Comparison of isothermal and non-isothermal pipeline gas flow models. Chem Eng J 2001;81(1):41–51.

[142] Ehrhardt K, Steinbach MC. Nonlinear optimization in gas networks. In: Bock HG, Kostina E, Phu HX, Rannacher R, editors. Modeling, simulation and optimization of complex processes. Berlin: Springer; 2005. p. 139–48.

[143] Aalto H. Optimal control of natural gas pipeline networks: a real-time, model-based, receding horizon optimisation approach. Saarbrücken, Germany: VDM Verlag; 2008.

[144] Mahlke D, Martin A, Moritz S. A simulated annealing algorithm for transient optimization in gas networks. Math Methods Oper Res 2007;66(1):99–115.

[145] Domschke P, Geißler B, Kolb O, Lang J, Martin A, Morsi A. Combination of nonlinear and linear optimization of transient gas networks. INFORMS J Comput 2011;23(4):605–17.

[146] Vielma JP, Nemhauser GL. Modeling disjunctive constraints with a logarithmic number of binary variables and constraints. Math Program 2011;128(1–2):49–72.

[147] Rebennack S, Kallrath J. Continuous piecewise linear delta-approximations for univariate functions: computing minimal breakpoint systems. J Optim Theory Appl 2014. forthcoming (doi:http://dx.doi.org/10.1007/s10957-014-0687-3).

[148] Rebennack S, Kallrath J. Continuous piecewise linear delta-approximations for bivariate and multivariate functions. J Optim Theory Appl 2014. forthcoming (doi:http://dx.doi.org/10.1007/s10957-014-0688-2).

[149] Frank S, Steponavice I, Rebennack S. Optimal power flow: a bibliographic survey i. Energy Syst 2012;3(3):221–58.

[150] Fasihizadeh M, Sefti MV, Torbati HM. Improving gas transmission networks operation using simulation algorithms: case study of the National Iranian Gas Network. J Natural Gas Sci Eng 2014;20:319–27.

[151] Fügenschuh A, Geißler B, Gollmer R, Hayn C, Henrion R, Hiller B, et al. Mathematical optimization for challenging network planning problems in unbundled liberalized gas markets. Energy Syst 2014;5(3):449–73.

[152] Pfetsch ME, Fügenschuh A, Geißler B, Geißler N, Gollmer R, Hiller B, et al. Validation of nominations in gas network optimization: models, methods, and solutions. Optim Methods Softw 2015;30(1):15–53.